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APPLICATION OF STATISTICAL ANALYSIS IN SINTERING AND BLAST FURNACE IRON MAKING

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Statistical techniques are used to represent a physical situation functionally and/or to confirm a hypothesis. Statistical techniques are of considerable help in analyzing the performance of a complex system, such as a blast furnace, which is subjected to a large number of inter-influencing variables.

The present paper reports, in brief, the statistical analysis of the performance data of the blast furnace 'X' and the sintering plant 1 of Bhilai Steel Plant used for routine production. Anamolies, which need to be handled before drawing any conclusion are discussed, and these are handled too in the present analysis. The conclusions, derived, have been applied to set the priority for the operator guidance and reference. The important conclusions are : (i) considerable saving in coke rate and increase in productivity are achieved by eliminating limestone and reducing the content of -10 mm size fraction in the ferrous burden, and (ii) the productivity of the sintering stand as well as the quality of sinter improves with increase in basicity to a limit.

INTRODUCTION

Statistical techniques have been in use in every field of life sciences. These are used to represent a physical situation functionally and/or to confirm a hypothesis[1]. Quite often we come across a multivariate system which is subjected to a large number of inter-influencing variables, some of them may act in uncontrolled manner. Data, which serve the basis for the functional relationship within the system, can be obtained either from a specified sample or a specially designed experiment. Sometimes we come across a situation where a specific experiment may not be permissible. In that case, inferences are to be drawn based on the 'regression of the system working'. The techniques of multivariate regression analysis consists essentially in expressing one variable, known as 'dependent variable', as a function of a number of 'independent variables' and implies that the value of the former is in some sense, a result of the value of the latter. The log data, however, may include anomalies due to the nature of collection and it is desirable to handle them before drawing a conclusion.

In iron making, a blast furnace is a multivariate system, where a large number of inter influencing variables interact simultaneously. In these cases, statistical analysis of the operating data is of considerable value in a number of ways[2]: (i) it provides a standard with which to appraise current performance of a blast by taking into account the

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external conditions, (ii) it helps in estimating the effect of changes in raw materials supplies, burden preparation, operation practices etc. These are perhaps not accountable on purely thermochemical treatment, and (ii) studies of variation in furnace economy and productivity by this technique may assist in making comparison of the furnace of different size and design.

The work in this field, especially that of Flint[3], has provided a foundation for the evaluation of various changes in the raw material and operating practices. However, a similar comprehensive analysis of an Indian blast furnace has hardly been undertaken. The characteristics of the raw material used and the operational parameters are rather unique in the Indian blast furnace practice which could be the reason for this gap in knowledge. The author's analytical work on the blast furnace[4] and the sintering plant[5] at Bhilai is perhaps a step in this direction. The present paper is an extension of the previous works and is based on our experience as regards the statistical analysis of these systems. The objectives of the present work were as follows:

(i) to discuss anomalies, which generally appear with such analysis and to find out the techniques of their handling,

(ii) to establish the empirical equations illustrating more specifically,
  - the role of burden variables in the blast furnace performance and
  - the role of raw mix composition and chemistry of sinter in the sinter quality, and

(iii) to set the priority for the operators guidance under the Indian condition of iron making practice.

ANALYTICAL TECHNIQUES

The analytical techniques employed in the present work includes the mass balance over the system and the statistical analysis of the data. Details of the statistical techniques, employed in the present work, are reported elsewhere[6-9].

Sintering Plant

The sintering plant performance indices with their respective symbols, the independent variables, X1 to X7, their average values, coefficients of variation and the range studies are listed in Table I. Some of the variables have been defined in rather unconventional way. Strand productivity has been calculated on the basis of +10 mm size fraction of sinter. Details as regards other variables have been mentioned elsewhere[5]. The yearly average data for the previous 16 years of the sintering plant No. 1 at Bhilai Steel Plant have been analyzed through the statistical techniques and the linear and multiple linear regression equations are established. The analysis also includes: (i) error analysis of data by material balance, (ii) analysis of the residuals, (iii) computation of the 95% confidence intervals for the regression coefficients, and (iv) test of null hypothesis.

Blast Furnace

The blast furnace performance indices, y1 to y3, the independent variables, x1 to x18, their average values, coefficient of variation and the ranges studied, are listed in Table II. Monthly average data for the previous 16 years of the blast furnace 'X' at Bhilai have been analyzed through the statistical techniques and the linear and multiple linear regression equations were established. The analysis also includes : (i) error analysis of data by material balance, (ii) analysis of residuals, (iii) isolation of the inter-influence amongst the explanatory variables, and (iv) setting up the reduced model vis-a-vis generalized model.

RESULTS

Sintering Plant

Fig. 1 typically shows the residual plots for the relationship between the productivity and the basicity of sinter. It is apparent from the figure that the standardized residuals are randomly distributed about zero in between -2 and +2. This range is within the acceptable limit[6]. This fact suggests that the linear relationship could be expected between the strand productivity and the basicity of sinter.

Table III shows the summary of the statistical analysis established for the linear relationship. Salient observations are reported here :

(a) The slope does not change its sign within the 95% confidence interval.
(b) The null hypothesis is rejected at the 0.05 level of significance showing that there exists definite relationship between the dependent and the independent variables.
(c) The effect of the individual factor, as it is apparent from the regression coefficient, could be explained by theory.

Table IV shows the multiple linear regression equations relating the sintering indices with the process parameters. The correlation coefficients (r) of these equations are also mentioned in the table. In addition, Table IV shows the observed and the predicted values for the two situations, namely, when the observed index was minimum and when it was maximum.

Blast Furnace

Table V typically shows the important multiple linear equations relating the performance indices with the independent variables. These are based on analysis of monthly average data. The correlation coefficients (r) and the F-ratio are also shown in the Table. These indices have been defined elsewhere[4]. Equations depicted in Table V are the reduced model for which the null hypothesis has been tested. It has been observed that the response variables in the present analysis could be represented by a lesser number of significant independent variables. The reduced module has got advantages since these are simple and they provide expression in simple way[6]. Table V also shows the observed and predicted values of the carbon rate and productivity for the two situations: one, when the observed index was minimum during a particular year and the other when it was maximum.

ANAMOLIES AND THEIR HANDLING

Choice of Variables

For an effective representation of the system functionally, the variables need to be carefully selected. For a complex system, such as a blast furnace, which is subjected to a large number of inter-influencing variables it is difficult to include all the variables. The underlying problems are:

Adhocism in the relationship

The two variables, namely, X1 and X2 may be related to each other because they are mutually correlated to a third variable X3. Although, it would not be surprising, for example, to obtain a high positive correlation between the number of literate persons in a city and the incidence of crime in that city, one cannot conclude that the crime may be reduced by prohibiting the literacy. Since, both the variables depend upon the size of the population of that city, and it is the mutual relationship with a third variable (population size) which produces the positive correlation.

In multiple regression equations, each coefficient purports to show the effect on the dependent variable. Therefore, great care must be taken in the choice of independent variables, if the coefficient are to have a rational physical meaning.

Inter-influence between the explanatory variables

In a multivariate system there may be many examples where the explanatory variables are not orthogonal. The lack of orthogonality may be due to the following reasons:

(a) The nature cause and effect inter relationship amongst the explanatory variables. For example, higher percentage of fluxed/super fluxed sinter in the burden requires less raw limestone to be charged in the furnace. The sign of the partial regression coefficient on the specific consumption of limestone (X13) in equation (iv) (Table V) is reverse to that of theoretical effect. It is because the variable has been inter-influenced by the charge rate as well as the specific sinter consumption; both have appeared in the equation. Such inter-influence is usually identified through an examination of the correlation coefficients matrix. The simple equations, obtained through the analysis of such matrix are also mentioned in Table V. These show that the limestone directly charged in the furnace is closely correlated with the specific consumption of sinter ($r = -0.93$). It is also correlated with the charge rate ($r = 0.664$).

When the two independent variables are closely related to each other, it may be essential for better reliability of the partial regression coefficient that only one variable is included in the equation. For instance, in the present analysis, the variable, the limestone consumption could be replaced by the variable, the specific consumption of sinter because they are closely correlated ($r = -0.93$). However, in order to avoid anomalies, such replacement should be justified on theoretical considerations. Conversely, the theoretical consideration should be substantiated from the analysis of the actual data. For instance, required quantity of lime can be incorporated through greater percentage of sinter in the burden. Therefore, the replacement of limestone consumption by the sinter consumption could also be theoretically justified. Conversely, it is also theoretically justified that limestone can be eliminated from the blast furnace burden by using the same quantity of sinter but at higher basicity. However, the replacement of limestone consumption by the basicity of sinter need to be substantiated from the plant data. In the present case the basicity of sinter has varied not so significantly (coefficient of variation = 8.49, Table II) as compared to the specific consumption of sinter (c.v. = 36.70, Table II). Additionally the correlation between the limestone consumption and the basicity of sinter ($r = -0.41$) is not high as compared to the same for the specific sinter consumption ($r = -0.93$).

Theoretically, the thermal requirement of the furnace increase with the addition to the burden which may be:

- with increased calcination, and
- with increased slag formation.

In the present case of blast furnace operation, the charge rate increases with the consumption of limestone directly charged (Eq. (v), Table V). This is because of the fact that for every kg of solid weight of limestone, approximately two third kg of CO_2 is also charged into the furnace. From the direct correlation (the correlation coefficient is +0.664), it seems that the variable charge rate could be replaced by the weight of limestone (and, ultimately by the specific consumption of sinter). However, this may not be permissible because of the following reasons: (i) the addition of the burden is not only contributed by the CO_2 associated by the limestone, but also by the gangue constituents associated with the other burden materials, (ii) the correlation coefficient between these variable is not so strongly high.

(b) The apparent inter-relationship amongst the explanatory variables and masking the effect. The present analysis shows that while the ash content in coke increased during the period 1967 to 1978, the coke rate decreased. This was made possible by the significant reduction in the level of limestone directly charged in the furnace through increased consumption of superfluxed sinter. Apparently, equations (iii) and (iv) do not include the variable, ash content in coke. This was due to the fact that the effect of this variable has been masked by the specific consumption of limestone. It appears from the correlation coefficient matrix that the correlation between coke rate and ash content in coke is negative (-0.25, Table VI). However, it would be ambiguous to interpret that the coke rate has decreased with increase in ash content in coke merely on this basis. Similarly, the sign of the partial correlation coefficient on Fe content in sinter (X5) in equation (iii) or (iv) is reverse to the sign of the theoretical effect. This variable has been inter-influenced by the variables, namely, specific sinter consumption and reducibility of sinter. Equation for the productivity, equation (x), apparently has the negative sign of the partial regression coefficient on top gas pressure. This should be reverse to the sign of the theoretical effect. The positive effect of increasing the top gas pressure on productivity has been masked by an increase in the fines content in lump iron ore as well as sinter, both have adverse effect on productivity. The above examples are typical, there may be many examples like the above in case of a complex system. So, it is necessary to isolate the inter-influence before drawing any conclusion. Table VI shows that the signs of the partial correlation coefficients on these variables change conducive to those of theoretical effects when the infer-influences were isolated.

Choice between alternatives

While analyzing the data, we often come across a situation where a choice has to be made as to which variable need to be selected. For instance, the productivity of a blast furnace is usually reported based on the working volume as well as the useful volume of the furnace. The working volume is calculated where the height of the furnace is measured from the tuyere level to the stroke line, whereas the useful volume includes the hearth volume also. Which productivity should we consider for the assessment? The present analysis shows that while these could be expressed by -12 mm content in iron ore lump, and blast rate; the production based on the working volume of the furnace has a better correlation with these variables than that based on the useful volume (Table V; Eq. ix and xii). The choice of productivity based on working volume is also theoretically justified, as the latter includes the hearth volume which has little role in the reduction process.

The burden rate is the amount of burden materials, in kg to produce 1 t of hot metal. Coke is not included in this. The coke rate increased with increase in the burden rate, because the furnace need extra thermal requirement with the addition of the gangue constituents and limestone to the burden. Coke also contains the ash which need additional thermal requirement. This fact is not accounted for when the coke (or carbon) rate is expressed in terms of the burden rate. On the other hand, if the coke rate is expressed in terms of the charge rate, the latter includes the coke rate, which is the dependent variable. In other words, considering either the charge rate or burden rate as an independent variable has merit as well as demerit.

Choice for Data Points

Once the independent variables are selected for a particular response, their data points are to be considered based on the criteria mentioned: (i) number of data points to be considered, and (ii) frequency and duration of data points to be measured and considered.

The criteria for the best fit of the equation is not only the correlation coefficient, but also the number of data points considered for statistical analysis. For example, in the extreme case, a straight line can be drawn with the correlation coefficient, 1, when the data points are only two. Similarly, the three data points can always be represented by a quadratic equation. For the equation to be meaningful, it is desirable that a considerable large and relevant data points are analyzed and the correlation coefficient to be examined for its significance. In the present work, the regression equations were established on analysis of monthly average data of the blast furnace for the 16 years. Considerably high number of data points as well as theoretical justification of the regression coefficients (discussed in section 5) results in a higher degree of confidence in expressing these equations.

Another criterion, which should be kept in mind, is how frequently the data points are measured and considered for analysis and what is their range? For instance, the residence time of a blast furnace is considerably high. Therefore, through analysis of daily average data, it is not possible to assess the true output of iron from a blast furnace due to the variables, amount of materials remaining in the furnace at the end of cast. The time unit of one month considered in the present analysis is a compromise. Longer period than a month have the disadvantage that the number of sets of data readily available becomes too small for reliable assessment of the significance of the results. Moreover, the averaging effect of a longer period will tend to obscure the relationship sought.

In order to predict the response variable from the equation care must be taken to examine whether the values of independent variables fall within the range of their variation (based on which the equation has been developed). Moreover, in order to assess the effect of an independent variable, the selection of the period should be such that the variable has undergone a significant change during that period. For instance, in the present analysis the effect of ash content in coke rate has not been reflected. This is due to the masking of the effect by another strong variable, namely, weight of limestone in the burden which has undergone a significant change (coefficient of variation = 47.4%) as compared to the ash content in coke (C.V. = 5.1%) during the period considered.

Reliability of Data

Once a regression equation is established based on a certain set of data points, one equation is generally asked, which seems to be rather embarrassing, but very pertinent one, is how far the data are reliable? An analyst is liable to answer this question since the accuracy of the functional relationship lies solely on the reliability of the data. Quite often we come across a situation where the past data are analysed in order to check a hypothesis. In that case it may not be possible to verify the data in person. Nevertheless, it can be done so for the present set of data and a justification can be set about the reliability of the data considered. Another important aspect is examining the reliability by the mass and energy balances. The mass balance has been adopted in the present analyses as described below.

Sintering plant

By following the mass balance, the mass fraction of the two major components, namely, Fe and CaO were calculated from the material input. These were compared with the reported mass fractions of these components in sinter following the chemical analysis. It was observed that the difference between the above two was not significant.

The monthly production figure for sinter was calculated from the input of raw materials. This figure was compared with the reported monthly consumption figure of sinter in the blast furnace. Likewise, the difference between the above two was not significant.

The error involved in chemical analysis was estimated by analyzing duplicate samples of raw materials and sinters having different basicities. It was observed that the difference in values for the chemical constituents, namely, Fe and CaO was not significant and that for the constituents, namely SiO_2, Al_2O_3, and MgO was higher. However, the contribution of error through the analysis of these latter components would be marginal since these are in smaller quantities in sinter.

Blast Furnace

Mass balances for the components, namely, iron, carbon and oxygen were performed for the 28 sets of randomly collected monthly average data. It was estimated that the difference between the input iron and output iron was within the range of -5 +6%. The observed specific blast rate had been 10 to 20% higher than that computed through the carbon and oxygen balances. This shows the leakage through the tuyeres.

Statistical Inference

Having established a regression equation it is desirable to know how best the equation represents the physical situation functionally or how best the equation can predict the performance based on a set of log data. Strength of a relationship is generally expressed in terms of the correlation coefficient. The commonly adopted way to express the coefficient of correlation (r) is as follows:

$$r2 = \pm 1 - \frac{[\Sigma(Y-Y)^2]}{[\Sigma(Y-\overline{Y})^2]} \qquad (1)$$

where, Y = observed value of the dependent variable, Y' = estimated value of the dependent variable using the equation, Y = average value of the dependent variable. The value of the coefficient falls within the range of -1 and +1. The confidence in expressing the relationship increases as the coefficient approaches the value of either -1 or +1. In other words, Y approaches the value of Y. However, there are certain anomalies which need to be handled before drawing any conclusion from the equation. Let us examine these in the following paragraph:

Firstly, it must be confirmed that r is an estimate of the strength of the linear relationship between the random variables only, not the curvilinear relationship. As illustrated, in Fig. 2, r may be close to '0' even there is strong relationship between the two random variables[8].

Secondly and most importantly, a significant correlation does not necessarily imply a 'casual' relationship between the two random variables. Both the variables may have a mutual relationship with a third variable which may produce a strong correlation between the two variables.

Thirdly, a large value of the correlation coefficient or a significant 't' statistics does not essentially insure that the data has been fitted well. To emphasize this point, Anscombe[9] has constituted four data sets, each with a distinct pattern, but, each having the same sets of summary statistics. The data and graphs are reproduced in Table VII and Fig. 3 respectively. So, an analysis, which is exclusively based on an examination of summary statistics, would have been unable to detect the difference in the pattern and thereby leading to an incorrect analysis.

Analysis of the residuals is a powerful tool to handle such anamolies. Fig. 1, for example, shows that the residuals are randomly distributed about zero in between -2 and +2. Therefore, it is evident that the relationship between the productivity of sintering strand and the basicity of sinter could be represented by the linear equation mentioned in Table IV.

DISCUSSION

Blast Furnace : Factors Affecting Coke Rate

Carbon rate increases significantly with increase in the charge rate. It is reported that addition to the burden has significant effect on coke rate[10-12]. In the present case, an increase in the charge rate has been mainly reflected through an increase in the limestone consumption.

Calcination of limestone inside the furnace increases the thermal requirement[11]. The limestone could be eliminated from the burden through the addition of greater quantity of superfluxed sinter.

An increase in blast temperature increases the sensible heat, and therefore, saves the coke rate considerably. The saving in the coke rate, achieved in the present furnace, is comparable with that reported in literature[2,3,11].

The adverse effect of ash content in coke on coke rate has been masked by the limestone consumption (section 4.1 b). Isolation of the inter-influence, through analysis of correlation coefficient matrix has resulted in equation (vii) which shows that coke rate increase with an increase in coke ash.

The present analysis shows that an increase in the fines content in the ferrous burden increases the carbon rate. This corroborates the Flint's finding[1]. The variable Fe content in sinter has been inter-influenced by the reducibility of sinter and apparently by specific consumption of sinter. Therefore, the sign of the partial regression coefficient on this variable is reverse to the sign of theoretical effect. The effects of these factors on coke rate have been discussed in detail elsewhere[4].

Blast Furnace : Factors Affecting Productivity

The present analysis shows that the most significant factor which influenced the productivity was -12 mm size fraction in lump ore. Earlier work of Bokaro and Durgapur shows that the permeability decreases with increase in the fines content in burden[13]. The productivity was also adversely affected by -10 mm content in sinter. The present analysis shows that blast rate was the next most significant variable which influenced productivity. Earlier work has shown a linear relationship between productivity and blast rate[13]. The wind acceptability increase with increase in permeability of the burden. This results in an increase in productivity.

Other factors, which have affected the productivity of the furnace, are reducibility of sinter, Fe content in sinter and its chemical consistency. The effects have been discussed in detail elsewhere[4].

Sinter Plant : Factors Affecting its Indices

The present analysis shows that an increase in the basicity of sinter increases the strand productivity, improves the yield and strength of sinter. The ratio RO/RI has been considered as a measure of strength in the absence of any (past) reliable data on it. It has been established through the laboratory experiments that the ratio has been inversely proportional to the shatter index of sinter. Increase in strength and reducibility of sinter with basicity is possible due to increase in the calcium ferrites and a more favourable morphology. The findings arrived through the present analysis corroborates those in the literature[14,15].

PRIORITY DETERMINATION THROUGH STATISTICAL ANALYSIS

For effective and better functioning of a multivariate system, it may be often desirable to set priority : short term, medium term and long term. The philosophy of setting the priority lies on choosing the easier approach out of many of improved performance. Gradually the harder approach could be attempted.

The results arrived at through the statistical analysis in the present work have been applied to formulate the basis for priority determination in case of the blast furnace and the sintering plant. These may be applicable to the blast furnace practice in India or elsewhere in abroad where the burden characteristics and operating practices are not so coherent. The adverse performance of the furnace is in relation to: (i) high ash content and low strength of coke, (ii) high proportion fines in the burden reducing the wind acceptability, (iii) high alumina slag practice and high slag volume, and (iv) no auxiliary fuel injection and low blast temperature, etc.

Blast Furnace

For this purpose, the factors which affect the performance can be categorized, rather arbitarity, into two:

(a) Factors which are controllable with difficulty : These factors are guided by the constraints from the nature. For example, the variables like, alumina content of iron ore, ash content of coke, etc. depend upon the raw materials from the nature. It requires an elaborate treatment of raw materials to bring down the level of these variables. Sometimes it may not be permissible, of course, under economic compulsion.

(b) Factors which are easily controllable : These are mostly within the control of operators. For example, the granulometric composition of the burden materials, sinter basicity etc. could be changed by an operator.

The priority can be set so that the factors, which are easily controllable are concentrated first.

The present statistical analysis has shown that the most significant variable for the coke rate and productivity, respectively are, (i) weight of raw limestone in the burden, and (ii) the content of -12 mm size fraction in lump ore. Other variables, which are statistically significant (and easily controllable by an operator) are: (iii) the content of -10 mm size fraction in sinter, (iv) temperature of the hot blast, and (v) reducibility of sinter etc.

Fig. 4 typically shows the steps for the improvement of the furnace performance. The expected benefits, as quantitatively established through the statistical analysis, by controlling the significant variables are also shown in the figure. The variables mentioned above, may be considered for their control as a short term measure. This is because they could be controlled easily. For instance, limestone can be eliminated from the burden through the greater use of superfluxed sinter. On the other hand, measures like, optimization of slag regime, reduction in ash content in coke etc. could be considered on long term basis. This does not mean that the variables are not important. Such demarcation, however, may be essential because of the fact that these could not be easily controlled.

Sintering Plant

The present analysis has shown that the indices, namely, reducibility, strength and yield of sinter improves with increasing the basicity upto the range considered. Therefore, an increase and control of the basicity of sinter may be considered, as a short term measure, for improved quality of sinter and its positive effect in the blast furnace.

CONCLUSION

Important conclusions of the present work are summarized below:

(a) Statistical techniques can be applied to represent a multivariable system functionally. However, care should be exercised as it may suffer from anomalies in selection of variables, collection of data and the inferences drawn. In order to make the conclusions obtained through such analysis more effective, it is desirable to handle these anomalies. Such anomalies and the techniques of their handling have been illustrated, typically, for the sintering plant and the blast furnace.

(b) The quantitative relationships, established through the statistical analysis have been able to formulate a short term priority for the improved performance of the blast furnace as well as that of the sintering plant. These are (i) eliminating raw limestone from the burden by its incorporation through sinter, (ii) reducing the content of -10 mm size fractions from lump ore and sinter, and (iii) improving the reducibility as well as strength of sinter by modifying the mix chemistry and sintering condition.

The levels of limestone and fines in the ferrous burden in the Indian blast furnaces are considerably high. Therefore, there is a considerable scope to improve the furnace performance by controlling these parameters.

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Table I

LIST OF DEPENDENT AND INDEPENDENT VARIABLES WITH THEIR AVERAGE VALUE (X), COEFFICIENT OF VARIATION (100σ/X) AND OPERATIVE RANGE STUDIED. (YEARLY AVERAGE DATA POINTS UNLESS OTHERWISE STATED)

Symbol	Variables	Unit of measurement	Value Average (X)	Value C.V. 100σ/X	Range Min.-Max.
A.	Dependent Variables				
I	Input of raw mix to the strand (Monthly avg. data n = 91)	T/m^2 hr.	2.308	5.24	1.98-2.52
P	Strand productivity	-do-	0.664	9.07	0.56-0.76
Y	Yield of sinter (also monthly avg. data n = 87)	% %	45.64 38.31	7.18 1.95	31.7-38.9 36.4-39.9
RQ/RI	Ratio, return output to return input	-	1.708	6.18	1.57-1.89
R	Reducibility of sinter (also monthly avg. data for sixteen years)	%	47.01 47.05	3.29 3.97	42.7-48.6 37.5-51.9
B.	Independent Variables				
X_1	Suction at wind main (monthly avg. data n = 91)	m.m.w.c.	607	7.38	510-705
X_2	Basicity of sinter $(CaO + MgO)/(SiO_2 + Al_2O_3)$ (also monthly avg. data for sixteen years)	- -	1.98 1.98	7.18 8.49	1.73-2.21 1.45-2.37
X_3	Specific coke consumption	kg/T	121.2	6.36	111-135
X_4	-3 mm fraction in coke	%	90.94	1.57	87.8-92.9
X_5	-3 mm fraction in flux	%	90.36	1.89	87.1-92.7
X_6	-1 mm fraction in ore fines (monthly avg. data n = 87)	%	51.95	4.42	46.1-57.7
X_7	Efficiency of utilization of machine (monthly avg. data n = 20)	%	81.85	5.54	71.4-89.5

Table II

LIST OF DEPENDENT AND INDEPENDENT VARIABLES SHOWING THEIR AVERAGE VALUE (X), COEFFICIENT OF VARIATION (100σ/X) AND APPROXIMATE OPERATIVE RANGE STUDIED.

Symbol	Variables	Unit of measurement	Value		
			Average (X)	C.V. 1005/X	Range Min.-Max.
A.	**Variables taken into analysis of the monthly average data :** Dependent Variables				
Y_1	Carbon rate	Kg/tHm	634.80	6.08	568-795
Y_2	Productivity	t/m^3 of working volume. day.	1.24	11.50	0.80-1.56
Y_3	Productivity	-do-	1.01	10.48	0.77-1.22
B.	Independent Variables				
X_1	Sinter consumption	kg/tHm	681	36.70	0-1298
X_2	Basicity of sinter $(CaO + MgO)/(SiO_2 + Al_2O_3)$	-	1.98	8.49	1.45-2.37
X_3	Reducibility of sinter	%	47.05	3.97	37.5-51.9
X_4	MgO content in sinter	%	5.02	11.51	3.99-6.24
X_5	Fe content in sinter	%	46.2	2.89	42.7-51.6
X_6	FeO content in sinter	%	9.49	7.86	5.51-12.3
X_7	Consistency factor in chemical composition $(CaO-SiO_2)$ of sinter	%	71.65	12.38	45.3-90.3
X_8	Consistency factor in chemical composition (Fe) of sinter	%	66.14	19.88	41.4-95.0
X_9	Sinter return in the process of sintering	%	31.2	5.25	26.7-36.3

X_{10}	-10 mm content in sinter	%	42.33	6.29	36.14-46.10
X_{11}	40-60 mm size fraction in sinter	%	2.68	19.24	2.0-4.9
X_{12}	-12 mm content in iron ore lump	%	20.19	25.65	12.2-32.0
X_{13}	Weight of raw lime stone in burden	kg/tHm	220.96	47.41	12-554
X_{14}	Charge rate	kg/tHm	2889	4.05	2679-3245
X_{15}	Blast rate	Nm^3/min	1778	5.56	1495-2140
X_{16}	Average top pressure	Atm.	0.71	21.10	0.36-0.95
X_{17}	Average blast temperature	^oC	777.6	19.24	600-902
X_{18}	Ash content in coke	%	24.96	5.13	22.6-28.0

Table III

SUMMARY OF STATISTICAL ANALYSIS

Sl.No.	Relationship	Within 95% confidence interval		Whether null hypothesis is rejected at the 0.05 level of significance
		The range of regression coeff. 'b'	Type of correlation (within the whole range)	
A	**Input of raw mix to the strand with**			
1.	Suction at the wind main	0.0015 ± 0.005	Positive	Yes
B	**Strand productivity with**			
1.	Yield of sinter	0.021 ± 0.09	Positive	Yes
2.	Basicity of sinter	0.373 ± 0.125	Positive	Yes
3.	Product, basicity of sinter and sp. coke consumption	0.0018 ± 0.0006	Positive	Yes
C	**Yield of sinter with**			
1.	Ratio, RO/RI	-20.32 ± 7.35	Negative	Yes
2.	Basicity of sinter	13.73 ± 6.30	Positive	Yes
3.	Sp. coke consumption	0.245 ± 0.129	Positive	Yes
4.	-3 mm fraction in coke	1.35 ± 0.88	Positive	Yes
5.	-3 mm fraction in coke	0.71 ± 0.67	Positive	Yes
6.	-1 mm fraction in ore fines	0.099 ± 0.067	Positive	Yes
7.	Efficiency of utilisation of machine	0.096 ± 0.064	Positive	Yes
8.	Product, basicity of sinter and sp. coke consumption	0.0756 ± 0.0246	Positive	Yes

D	Ratio, RO/RI with			
1.	Basicity of sinter	-0.508 - 0.308	Negative	Yes
2.	-3 mm fraction in flux	-0.04 ± 0.02	Negative	Yes
3.	Sp. coke consumption	-0.009 ± 0.006	Negative	Yes
4.	-3 mm fraction in coke	0.05 ± 0.04	Negative	Yes
E	Reducibility of sinter with			
1.	-3 mm fraction in coke	0.94 ± 0.63	Positive	Yes
2.	Basicity of sinter (monthly average data)	7.68 ± 4.15 5.05 ± 1.40	Positive Positive	Yes Yes
3.	-3 mm fraction in flux	0.55 ± 0.34	Positive	Yes

Table IV

EFFECT OF THE PROCESS PARAMETERS ON THE PERFORMANCE ANALYSIS OF THE SINTERING PLANT. RESULTS OF MULTIPLE LINEAR REGRESSION ANALYSIS

S.No.	Equations	Calculated index value against observed value			
		obs.	calc.	obs.	calc.
a.	$P = 1.543 + 0.148\ X_2 + 0.0026\ X_3 + 0.0177\ X_5,\ r = 95.9\%$	0.565	0.504	0.760	0.745
b.	$Y = -1.436 + 10.24\ X_2 + 0.138\ X_3,\ r = 86.6\%$	31.69	33.59	38.87	39.26
c.	$RO/RI = 6.11 - 0.00586\ X_3 - 0.041\ X_5,\ r = 86.8\%$	1.569	1.634	1.887	1.831
d.	$R = 40.51 + 11.17\ X_2 - 0.129\ X_3,\ r = 88.0\%$	42.70	43.95	48.64	42.29

Table V

EFFECT OF PROCESS PARAMETERS (x's) ON CARBON RATE (y1) AND PRODUCTIVITY BASED ON WORKING VOLUME (y1) AND THAT BASED ON USEFUL VOLUME (y2) OF THE FURNACE

(a)	Carbon rate (y1), kg/tHm
(i)	$y1 = -110 + 0.258\ x14$; $r = 0.77$, $F = 281$
(ii)	$y1 = 40.3 + 0.242\ x14 - 0.133\ x17$; $r = 0.80$, $F = 169$
(iii)	$y1 = -389.2 + 0.298\ x14 - 0.017\ x1 - 0.186\ x17 + 7.821\ x5 + 2.066\ x12 - 41.463\ x2$; $r = 0.847$ (minimum value : observed = 602, predicted = 602) (maximum value : observed = 701, predicted = 711)
(iv)	$y1 = -110.8 + 0.298\ x14 - 0.181\ x1 - 0.186\ x17 - 0.42\ x13 + 7.82\ x5 + 2.07\ x\ 12 - 41.46\ x2 - 2.36\ x9$; $r = 0.913$ (minimum value : observed = 602, predicted = 607) (maximum value : observed = 701, predicted = 698)
(v)	$x14 = 2725 + 0.741\ x13$; $r = 0.664$, $F = 151$
(vi)	$x13 = 466.8 - 0.39\ x1$; $r = 0.93$, $F = 1230$
(vii)	$y1 = -220 + 3.0\ x18 + 0.27\ x14$; $r = 0.776$
(b)	Productivity (y2), t/m^3 of working volume per day
(viii)	$y2 = 1.62 - 0.019\ x12$; $r = 0.68$, $F = 165$
(ix)	$y2 = 0.815 - 0.018\ x12 + 0.0004\ x15$; $r = 0.745$, $F = 119$
(x)	$y2 = 1.348 - 0.365\ x16 - 0.011\ x12 + 0.0006\ x15 + 0.017\ x3 - 0.013\ x10 - 0.0003\ x14$; $r = 0.846$ (minimum value : observed = 1.01, predicted = 1.11) (maximum value : observed = 1.42, predicted = 1.44)
(c)	Productivity (y3), t/m^3 of useful volume per day
(xi)	$y3 = 1.25 - 0.0118\ x12$; $r = 0.577$, $F = 96$
(xii)	$y3 = 0.50 - 0.0011\ x12 + 0.0004\ x15$; $r = 0.691$, $F = 87$

Table VI

LIST OF SIMPLE CORRELATION COEFFICIENT AND PARTIAL CORRELATION COEFFICIENTS

Relationship	Value of correlation coefficient
(A) Carbon Rate (kg/tHm)	
(i) with ash in coke	
Simple correlation coefficient	-0.251
Partial correlation coefficients with respect to charge rate	+0.139
With respect to charge rate and blast temperature	+0.122
(ii) with Fe content in sinter	
Simple correlation coefficient	+0.326
Partial correlation coefficient with respect to sinter consumption	+0.118
With respect to sinter consumption and reducibility of sinter	+0.020
(B) Productivity (t/m working volume.day)	
(i) with sinter consumption	
Simple correlation coefficient	-0.455
Partial correlation coefficients with respect to -12 mm content in iron ore lump	+0.177
With respect to -12 mm content in iron ore lump and -10 mm content in sinter	+0.090
(ii) with top pressure	
Simple correlation coefficient	-0.365
Partial correlation coefficients with respect to -12 mm content in iron ore lump	-0.052
With respect to -12 mm content in iron ore lump and -10 mm content in sinter	-0.067

Table VII

FOUR DATA SETS HAVING SAME VALUES OF SUMMARY STATISTICS
(SOURCE ; ANSCOMBE[9])

	x1	y1	x2	y2	x3	y3	x4	y4
01	10	6.04	10	9.14	10	7.46	8	6.54
02	8	6.95	8	8.14	8	6.77	8	5.76
03	13	7.58	13	8.74	13	12.74	8	7.71
04	9	8.81	9	8.77	9	7.11	8	8.84
05	11	8.33	11	9.26	11	7.81	8	8.87
06	14	9.96	14	8.10	14	8.84	8	7.04
07	6	7.24	6	6.13	6	6.08	8	5.25
08	4	4.26	4	3.10	4	5.11	19	12.5
09	12	10.84	12	9.13	12	8.15	8	5.56
10	7	4.82	7	7.26	7	6.42	8	7.41
11	5	5.68	5	4.74	5	3.73	8	4.86

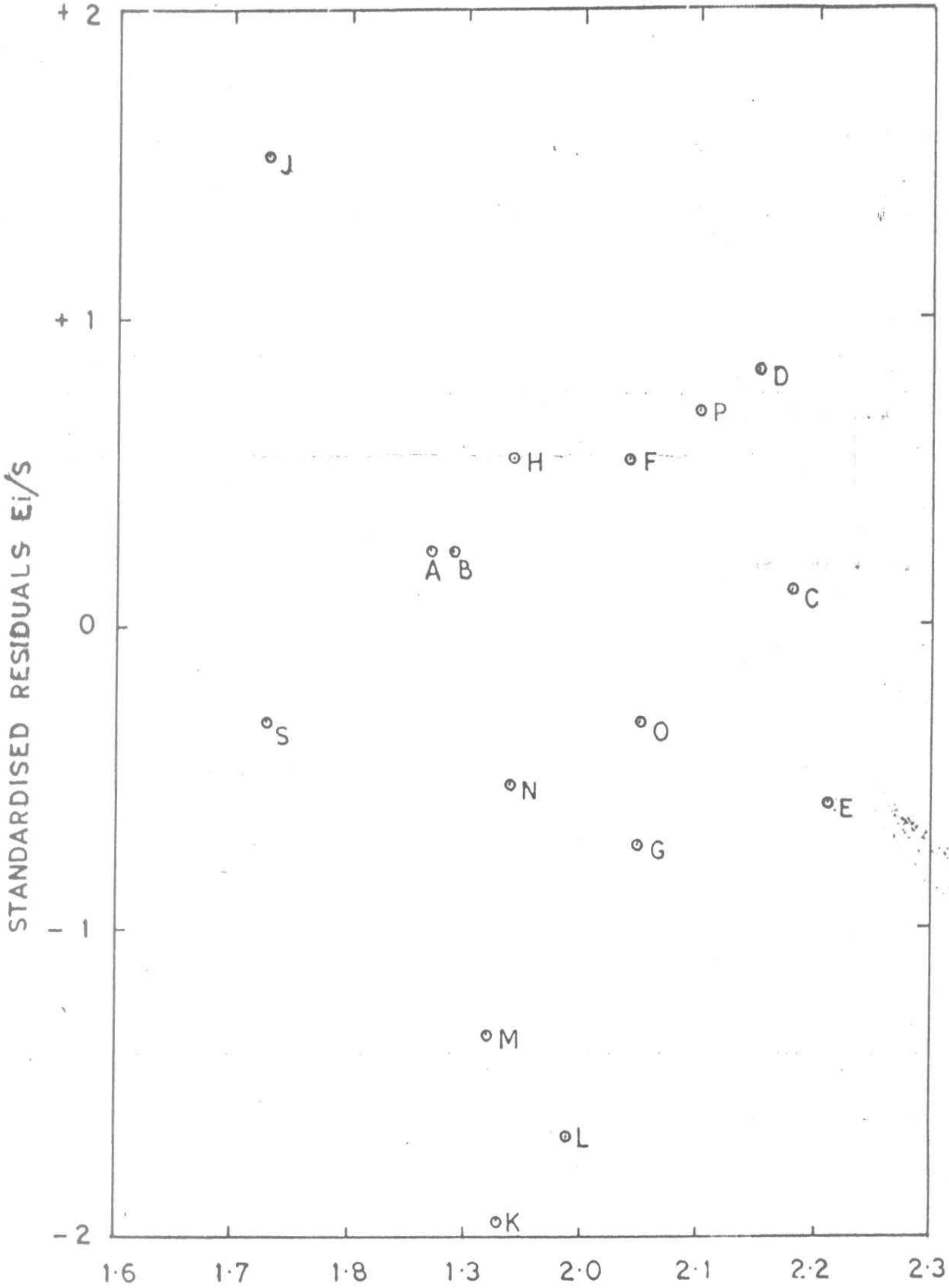

FIG. 1 : A typical analysis of residual

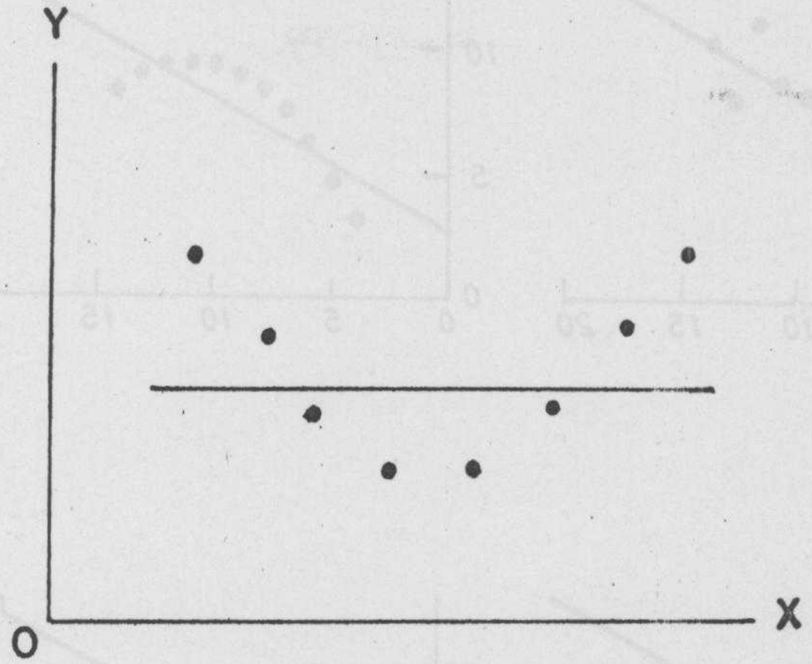

FIG. 2 NONLINEAR RELATIONSHIP WHERE CORRELATION COEFFICIENT IS ZERO.

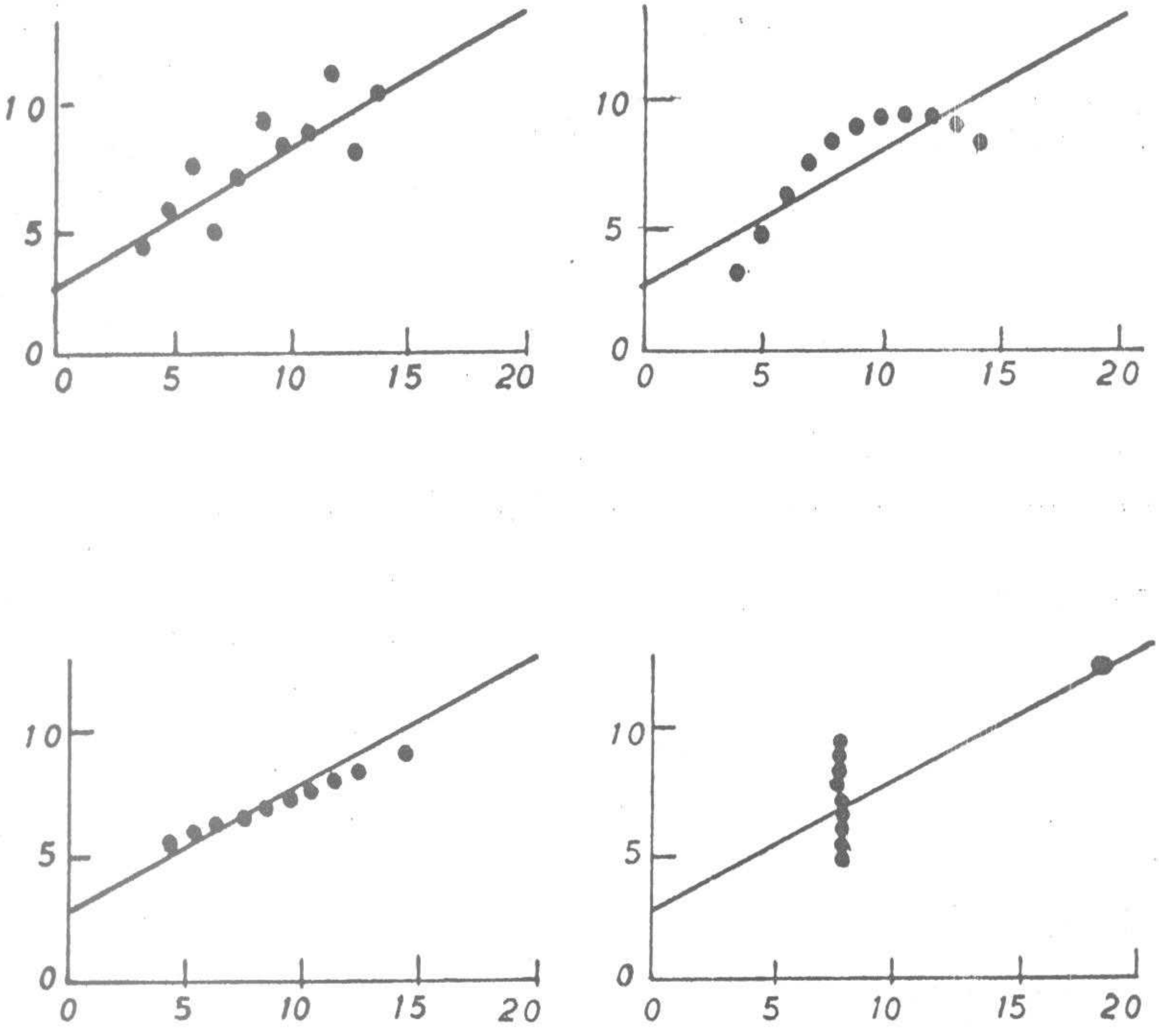

FIG.3 PLOT OF THE DATA (X,Y) WITH THE FITTED LINE FOR FOUR DATA SETS (SOURCE: ANSCOMBE [9])

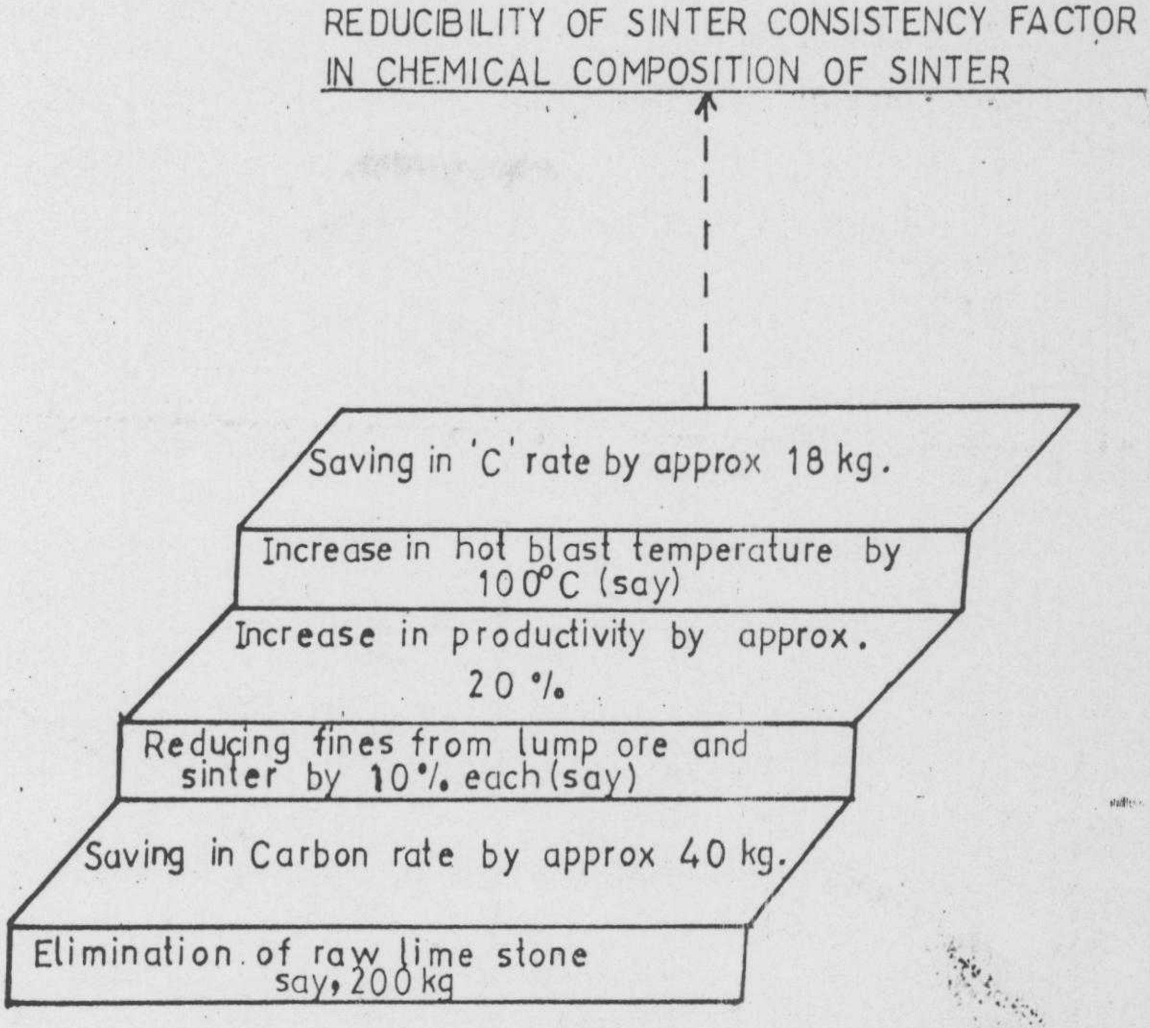

FIG. 4 STEPS FOR IMPROVEMENT IN THE FURNACE PERFORMANCE.