# On the calculation of economic cross-section for large D.C. busbars

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BUSBARS for aluminium reduction cells carry direct current from one cell to another and the passage of current results in power losses due to resistance. Electrical resistance for a given material being proportional directly to length and inversely to area of cross-section, the power losses can be reduced by shortening the length of busbars and increasing the area of cross-section. The length of busbars is determined by the design of the cell and facility in operation. The cross-section can be increased within practical limits but, for a given length, an increase in the cross-section will involve higher capital expenditure as more material will be used. In short, the initial investment in busbars varies directly with the area of cross-section of the busbars and subsequent operating costs, i.e. power, lost as heat due to resistance, varies inversely as the area of cross-section. The total costs incurred over the operating life of busbars are the initial investment followed by the stream of operating costs and the choice of cross-section should intend to minimise the total costs.

The total cost T has, as is mentioned above, two components, namely Tc and To. Tc is a function of the area of cross-section A and represents the cost incurred in installing busbars while To is a function $\frac{1}{A}$ and represents the cost of power losses in busbars during the years of operation of the plant. The objective is to determine A opt which would minimise.

$$T = Tc\ (A) + To\ \left(\frac{1}{A}\right) \qquad \ldots \qquad .. \ (1)$$

## Types of costs involved

### *Capital costs*

Initially a cash outlay is required for purchase and installation of busbars. This outlay is the capital cost of busbars. In case a company manufactures the busbars, the only 'out-of-pocket' cost is the cost of manufacture.

**SYNOPSIS**

*The calculation of economic cross-section for DC busbars, based on the minimisation of total costs e. g. cost of conductor and a stream of costs for power losses in the conductor, is quite well known. Certain complications arise in quantifying the cost of conductor and cost of the power lost. In the literature it is customary to express the cost of conductor in terms of an equivalent annual cost. In this paper the investment in conductors and the cost of power losses, over the life of the project, are discounted to the base year at a continuous interest rate. This method, apart from being fundamentally more rigorous, affords some advantage in calculation. The cash flow calculations are formulated mathematically and a compact formula for economic cross-section is derived based on factors applicable to Indian conditions. This would spare an engineer the need to be familiar with cash flow calculations. Another advantage of this derivation is that sensitivity of the economic cross-section to changes in various factors could be readily investigated.*

*For aluminium smelters, under certain conditions, the economic cross-section based on minimisation of total costs only determines the lower limit of cross-section. By increasing the cross-section of the busbars, there would be a decrease in the voltage drop between the cells and, the total line voltage being constant, more cells could be installed. The increase in output of the line would fetch additional revenue. The decision to increase the cross-section of busbars should be based on the rate of return expected on the total incremental investment in busbars and additional cells. This concept is explored in the paper.*

By appropriating this material for its own use, the company does not earn profit on it. Therefore, the actual profit foregone on the sale of this material, if it can be sold, is a cost to the company. However, this cost cannot be treated as a capital cost, only as a decrease in profits for that year. During the first year of the operation of the plant the Government gives a tax write-off in the form of development rebate and

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from the same year savings in taxes result from depreciation allowed on these assets year after year. At the end of operating life of the plant the scrap value of busbars is recovered. Cost Tc will, therefore, be

Tc = Capital cost + after tax value of profits foregone—tax savings due to depreciation and development rebate—after tax value of scrap recovery ... (2)

*Operating cost*

During the operation of the smelter, some power will be dissipated in the busbars. The charges for this power dissipated will be included in the operating cost of the smelter. Therefore, the power loss in busbars involves a recurring operating cost rather than a capital cost. The component To of the total cost T will be

To = After tax value of charges for power loss in busbars ... (3)

*Time value of money*

The flow of cash concerning Tc and To is spread over the life of the project. Rupees at different points of time have different degrees of importance. This arises from the concept that money can earn a return or at least an interest as soon as it is made available. A sum of money available in future is given decreased importance by 'discounting' it at an interest rate. The idea essentially is that a smaller sum available now will earn an interest at compound rate and grow equal to that sum in future. To express the value of money relative to the same time base all cash flows are discounted at a compound interest rate, to arrive at their present value. For discounting, the interest rate could be charged annually or continuously at infinitesimal time intervals. Since cash flows more or less continuously in and out of the enterprise, it may be argued that it is fundamentally more rigorous to charge interest at a continuous rate although the difference in the two methods of compounding makes a very small difference in the final result. In this paper all cash flows have been discounted to the year before the starting up of operation using continuous interest rate.

### Derivation of expression for economic cross-section

Let Cm be the cost of busbars per unit volume (material, fabrication and installation),

$P_1$ profit foregone per unit volume as a proportion of cost per unit volume,

$P_2$ development rebate allowed as a proportion of the capitalised cost,

$P_3$ salvage value of busbars less its book value at the end of operating life, as a proportion of original book value,

D rate of depreciation allowed,

t the tax rate for the company,

A the maximum (*not* optimum) cross-sectional area along the length of the rod (the area of cross-section of the busbar may vary along its length),

L the length of busbar,

Ve volume shape factor such that VeA is the 'effective' area of cross-section of the busbar and the volume is given by LAVe

i interest rate, which is the minimum interest rate at which the company can borrow or if ownership funds are available the interest rate which these can earn,

n the life of the project in years.

Using these notations the components of Tc as given in Eq. (2) are as follows :

Capital cost = Ve LACm
After tax value of profits foregone = $tp_1$ Ve LACm

After tax value of scrap recovery = $tp_3$ Ve LACm$e^{-in}$

Tax savings due to development rebate = $tp_2$ Ve LACm$e^{-i}$

In several countries there is a choice of the methods of depreciating assets for tax evaluation, e.g. straight-line, declining balance, double declining balance, sum of the year digits. In India for the purpose of tax evaluation the assets are allowed to be depreciated on the basis of reducing balance, in which each year a fixed rate of depreciation is applied on the undepreciated balance. The sum of the present value of tax savings due to depreciation over the life of the projects is given by (see Appendix Ia).

$$\frac{tDe^{-i}\left[1-(1-D)^{n}\ e^{-ni}\right]}{1-(1-D)\ e-i}$$

According to equation (2)

$$Tc = Ve\ LACm\left[1 + tp_1 - tp_2\,e^{-i} - tp_3\,e^{-in} - tDe^{-i}\,\frac{\{1-(1-D)^{n}e^{-ni}}{1-(1-D)e^{-i}}\right]$$

$$= Ve\ LACm\ Y \qquad \ldots \qquad \ldots \quad (4)$$

where $Y$ is the expression inside parenthesis.

Now the electrical resistance R, for a given material, is proportional directly to length L and inversely to area of cross-section A.

$\mathrm{R}=\frac{\rho L}{A}$ where $\rho$ is the specific resistance of the material, and changes with temperature.

The loss of power in the busbars, on account of resistance, is given by

Power $\text{loss}=\frac{\rho L}{A}S^2$

where S is the strength of current flowing through it. It is common occurrence in the smelters that the strength of current is increased progressively to get higher production. The busbars could be designed for maximum current anticipated to flow through the busbars or alternatively to consider it increasing at a steady rate up to a maximum.

Let Cp be the charges per unit power per year,

Pe power shape factor. The strength of current may vary along the length of the busbars due to number of taps along its length. This factor is calculated such that $\frac{\rho L\,\mathrm{PeS}^2}{\mathrm{A}}$ gives the actual power lost.

The annual operating cost for power lost is :

$$\frac{\rho L}{A}S^2\ Pe\ Cp$$

Usually long-term agreements are entered into for the purchase of power from utilities but if the power charges are expected to increase as well as the current the expression for To would be

$$To=Pe\ t\ \frac{\rho L}{A}\sum_{r=1}^{n}Cp_r\,S_r^{2}\,e^{-ir}$$

where Cpr and Sr are the power charges and current strength in $\mathrm{r}^{\mathrm{n}}$ year. If Cp can be assumed to be constant over the life of plant and maximum anticipated strength of current is S, the expression for *To* (see Appendix 1b) is:

$$To=t\ \frac{\rho L}{A}Cp\,S^2\,Pe\,\frac{e^{-i}-e^{-(n+1)i}}{1-e^{-i}}$$

$$=\frac{\rho L}{A}Cp\ S^2\ Pe\ Z \qquad \ldots \qquad \ldots\ (5)$$

where $Z=\frac{e^{-i}-e^{-(1+n)i}}{1-e^{-i}}\ t$

The expression for the total cost $T$ will be

$$T=Ve\ LACm\ Y+Pe\ \frac{\rho L}{A}Cp\ S^2Z \qquad \ldots\ (6)$$

The total costs $T$ being a function of $A$ the conditions for minimum $T$ are

$$\frac{\partial T}{\partial A}=0 \text{ and } \frac{\partial^2 T}{\partial A^2}>0$$

The first condition gives

$$Ve\ L\ Cm\ Y-Pe\ \frac{\rho L}{A^2}\ Cp\ S^2\ Z=0$$

$$A^2\ \text{opt}=\frac{Pe}{Ve}\ \frac{Cp}{Cm}\ \frac{Z}{Y}\ \rho S^2$$

$$\text{Aopt}=S\ \sqrt{\frac{Pe}{Ve}\ \frac{Cp}{Cm}\ \frac{Z}{Y}}\ \sqrt{\rho} \qquad \ldots\ (7)$$

Total minimum cost Topt is

$$\text{Topt}=2\ \sqrt{VePe}\ \sqrt{CpCm}\ \sqrt{ZY}\ \sqrt{\rho}\ \ LS \qquad \ldots\ (8)$$

As expressed by Kelvin's Law for the optimum cross-section

$$Tc=To=\ \sqrt{VePe}\ \sqrt{CpCm}\ \sqrt{ZY}\ \sqrt{\rho}\ LS \ \ldots\ (9)$$

**Calculation of economic cross-section**

The economic cross-section can be calculated by substituting appropriate values in equation (8). To facilitate calculation, the values of Z, Y and $\sqrt{\frac{\mathrm{Z}}{\mathrm{Y}}}$ have been estimated for different interest rates and different life of the project and are given in Appendices 2, 3 and 4 respectively. Z was estimated on following assumptions :

1. Electrical machinery (having no moving parts) is entitled to a depreciation of 10% for three-shift operation.
2. Development rebate for priority industries in India is 35% up to March 31, 1970 and 25% thereafter.

3. Salvage value of busbars at the end of operating life of plant is 50% of the original book value.
4. The undepreciated value at the end of project is written off.
5. The prevalent tax rate for industry is 50%.
6. $P_1=0$. However if $p_1\neq 0$ the value of Y can easily be modified as is explained below.

In order to read the values of Y and Z from the tables, values of n and i should be known. For the cost of capital (i) it would be better to take the advice of the Accounts Department. If the manufacturer of aluminium foregoes some profit to manufacture busbars for its own use then $p_1t$ should be added to the value of Y given in Appendix 2. The value of $\sqrt{\frac{Z}{Y}}$ can then be estimated. If this is not the case then the value of $\sqrt{\frac{Z}{Y}}$ can be read directly from Appendix 4.

Power shape factor Pe and volume shape factor Ve can be estimated from ohm's law and the geometry of busbars taking into account the number of taps from the busbars.

An example is solved to illustrate the use of this formula.

(a) Cost of purchase and installation of aluminium busbars Rs 7000/tonne
Cost of power Rs 300 per KW year
Anticipated maximum amperage 65000 A

Economic cross-section is calculated for anode riser in aluminium smelter for which Ve = 1 and Pe = 1.

Let $r=10\%$

$n=20$ years and development rebate $=35\%$

$\rho=3{\cdot}1046\times10^{-6}$ ohms cm at 40°C

Cv $=$ Rs. $18{\cdot}9\times10^{-3}$ per cc(Density of Al 2·70 gm/cc)

Cp $=$ Rs. $300\times10^{-3}$ per watt year

From Appendix 3 $\sqrt{\frac{Z}{Y}}=2{\cdot}713$

$$\text{Aopt}=65\times10^{3}\times2{\cdot}713\ \sqrt{\frac{300\times10^{-3}\times3{\cdot}1046\times10^{-6}}{18{\cdot}9\times10^{-3}}}$$

$$=176{\cdot}345\ \sqrt{49{\cdot}2794}$$

Aopt $=1238$ sq. cm

$$\text{Total Cost}=2\ \sqrt{18{\cdot}9\times10^{-3}\times300\times10^{-3}}\ \sqrt{2{\cdot}29646}$$

$$\sqrt{3{\cdot}1046\times10^{-6}\times65\times10^{-3}\times L}$$

$$=2\times10^{-6}\ \sqrt{40425\times65\times10^{-3}\times L}$$

$=82{\cdot}68$ Rs per cm of busbar.

(b) If Rs 6000 per tonne is the cost of installed busbars and on each tonne of busbar used a profit of Rs 1000 is foregone then

$$p_1=\frac{1000}{6000}=\tfrac{1}{6}=0{\cdot}1667$$

$$tp_1=0{\cdot}08335$$

Value of Y from Appendix 3

$$Y=0{\cdot}55863+0{\cdot}08335=0{\cdot}64198$$

$$\frac{Z}{Y}=\frac{4{\cdot}11087}{0{\cdot}64198}=6{\cdot}4032\ \therefore\ \sqrt{\frac{Z}{Y}}=2{\cdot}530\text{ approx.}$$

$$\text{Aopt}=65\times10^{-3}\times2{\cdot}530\times\ \sqrt{\frac{300\times10^{-3}\times3{\cdot}1046\times10^{-6}}{16{\cdot}20\times10^{-3}}}$$

$$=164{\cdot}45\ \sqrt{57{\cdot}4926}$$

$=1247$ sq. cm.

(c) The economic cross-section at different values of temperature can be obtained by the use of well known relationship :

$$\rho t_2=\rho t_1\ [1+0{\cdot}00403(t_2-t_1)]$$

Substituting the value of $\rho$ at different temperatures in the expression for Aopt in the example (a) above, the values of Aopt for different temperatures are as follows : For a 70°C rise the Aopt has increased by approximately 12·7%, i.e. 1·8% increase for every 10°C rise in temperature.

| Temp. in °C | Aopt in sq. cm. |
|---|---|
| 33 | 1215 |
| 40 | 1238 |
| 50 | 1260 |
| 60 | 1284 |
| 70 | 1305 |
| 80 | 1327 |
| 90 | 1348 |
| 100 | 1370 |

(d) If the total cost is plotted for different values of area of cross-section then near the value of Aopt the shape of curve is relatively flat. This reveals that slight deviations from Aopt do not affect the total cost to the same degree. Aopt is most sensitive to the value of amperage. The sensitivity of Aopt for other parameters can be explored by the use of the formula and Appendices 2, 3 and 4.

## Discussion

In the foregoing a method is presented for estimating the economic cross-section for D.C. busbars. Choice of perimeter that is the breadth and thickness is still open. Method described in (2) for estimating the temperature rise in busbars shows that for a given cross-section and a given temperature rise the current carrying capacity increases with an increase in perimeter. Also for the same total cross-section a number of leaves with some space between them will be able to carry a higher current for the same temperature rise, which is evident as heat losses both from radiation and convection will be more. It is quite possible that use of standard size of busbars may be more economical than off standard size determined on the basis of calculations for Aopt. But it may be worthwhile to evaluate the total costs with standard sizes.

There is one situation under which minimisation of total costs may be contra to the maximisation of revenue. If there is sufficient demand for aluminium but the power supply available to the smelter is limited then there may be an incentive to use as much power to produce aluminium as economics can justify. The cost of power dissipated, in this case, cannot be taken at the rate at which it is supplied by the utility because there is a potential for using it to produce more aluminium from it. By dissipating power we are losing this opportunity. The intention is to increase profits rather than reduce costs. By increasing the cross-section of the busbars some power can be saved which will permit operation of a few more pots. The increase in the output of the line would fetch additional revenue. The problem in this case is to decide the extent to which the cross-section could be increased over the cross-section estimated by minimising costs. A possible approach to this problem can be as follows. First, the total power dissipated in the busbars of economic cross-section is calculated. Then using an estimate of the power that could be utilised from busbars a new cross-section is estimated. If the incremental investment in heavier busbars and additional pots gives a good rate of return then the new cross-section is economic. This economic cross-section can be determined by successive determination of yield on incremental investment.

## References

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