PHOTOELASTIC METHOD FOR STRESS ANALYSIS

By

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Photoelasticity is an experimental method for two dimensional stress analysis which uses optical effect to determine mechanical stresses and their distribution.

The discovery of photoelastic effect is credited to Sir David Brewster who published in 1816 an account that clear glass when stressed and examined in polarised light exhibited coloured patterns. The corresponding theory was developed by Neuman, Maxwell, Wertheim and other noted physicists. In the engineering world, this science first appeared around 1900 and was developed mainly by Professors A. Mesnager, E.G. Coker and L.N.G. Filon. Prof. Coker made engineering applications of photoelasticity possible mostly through introduction of celluloid for models, replacing costly and difficult-to-machine glass models and the use of monochromatic light. Notable among other important workers are Professors Föppl, Frocht and Neuber. In recent years the development of new synthetic resins possessing desirable photoelastic characteristics, has helped to enlarge applications of the method to a wider variety of problems.

The use of 'Polaroid disc' has considerably reduced the cost of the necessary equipment. The two-dimensional photoelasticity can also be applied to three dimensional problems. Recently developed methods namely 'Freezing Method' invented by Oppel in 1937 and developed by Mönch, Hiltscher's 'Converging Light Method' and 'scattered light method' have been devised for the direct application of photoelasticity to three dimensional models.

A brief review will be given of the relationship existing between stresses at a given point of a plate.

A stress system which consists only of normal stresses σx and σy, and shear stresses τxy, τyx, which are functions of x and y only, is defined as *a two-dimensional or plane stress system*, as in Fig 1. It can only exist in thin plates and is closely approximated by a thin flat model, loaded in the plane of the model.

These stresses cause in any plane passing through the given point and inclined at an angle θ to the axis, a normal stress $\sigma\theta$ and a shearing stress $\tau\theta$ of different intensities.

The normal stress is maximum or minimum when $\frac{d\sigma\theta}{d\theta} = o$ and for this case ($\tau\theta = 0$) the shearing stress vanishes. These maximum and minimum stresses are called *principal stresses*, they are acting on principal planes, perpendicular to each other, they are denoted by p and q. The photoelastic effects are related only to principal stresses.

Furthermore we have $p + q = \sigma x + \sigma y = \text{const.}$ and $\frac{1}{2}(p-q) = \tau max$. This last result is of special significance in photoelasticity and can be found photoelastically.

Photoelasticity requires the use of polarised light. The light is plane polarised when the light vector remains constant both in magnitude and in direction. When the tip of the vector traces out an ellipse or a circle the light is respectively elliptically or circularly polarised. All types of polarised light enter into the study of photoelasticity.

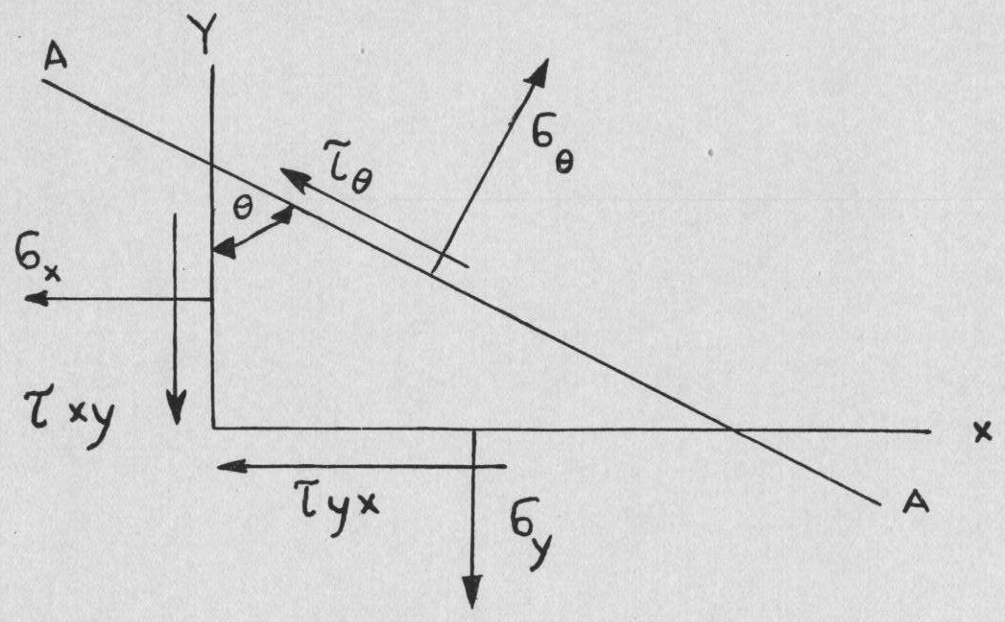

FIG 1:- THE TWO-DIMENSIONAL OR PLANE STRESS SYSTEM.

Plates made of certain crystalline material such as mica have the permanent property called *double refraction or "birefringence,"* of resolving the light into two components defined as ordinary and extra-ordinary rays. Both of them are plane polarised and transmitted on planes at right angles but with different velocities. Since when they emerge from the crystaline plate they are out of phase, the corresponding phase differences is called permanent phase retardation, and denoted Rp.

Almost all transparent isotropic materials such as glass, celluloid and many synthetic resins, when subjected to stress, become temporary crystals and produce double refracting effect on a beam of light. On release of the load the birefringence disappears. The light is polarised in the direction of principal stress axes and is transmitted only on the planes of principal stresses.

The velocity of transmission in each plane is different and depends on the intensity of the principal stresses. The transmission of light obeys the following law, which rests upon experimental evidence : e

$$R_t = C.\ t.\ (p\text{-}q)$$

Where R_t—temporary linear retardation between two rays in wave lengths.

c —constant known as stress-optic coefficient.

(p—q) —difference between principal stresses.

t —thickness of the plate.

This is the fundamental law of photoelasticity. The optical system employed to produce the polarised light and to interpret the photoelastic effect in terms of stress is called polariscope. The simplest form of polariscope consists of:

1. Source of light.
2. Polariser (a device converting ordinary light into plane polarised)
3. Model with loading frame.
4. Another polariser called analyser.
5. Screen or camera.

The beam of light emerging from any polarising prism is plane polarised. *The vibration of light is then in planes parallel to the principal plane* of the entering face of this prism. It is obvious that when the principal planes of both the polariser and analyser are parallel all the light will get through, and when they are at *right angle*, that is *crossed*, no light will get through, in accordance with vectorial law.

The light from the source, plane polarised by the polariser reaches the model. The unstressed model will not bring any change and the image of the model on the screen with crossed polariser and analyser will remain dark. But when we apply a load, the stressed model will behave like a crystal and will immediately resolve the light into two components in the directions of principal stresses moving with different velocities. This will create a phase difference between the emerging rays due to which the emerging light will be generally eliptically polarised. Again, the analyser lets through only the components of these two rays in its principal plane and brings them to interference, giving on the screen the *stress pattern.* Fig. 2. The stress pattern represents the basic data which the polariscope furnishes and all further numerical computations of stresses are based upon the stress pattern.

Let us consider for example the case of *pure compression* (or tension). Then $\sigma_x = 0$, $\sigma_y = q$, $R_t = c.t\ (p\text{-}q) = \text{-}c.t.\ \sigma_y$. It is clear that, when $\sigma_x = \sigma_y = 0$, no light is transmitted. When the load is gradually increasing from o, the image on the screen gradually brightens and reaches maximum of illumination, when $R_t = \frac{\lambda}{2.}$ When $R_t = \lambda$, a dark image is restored. By further addition of the load, the image will alternate between unifrom brightness and uniform darkness. This is the stress pattern of a compression model.

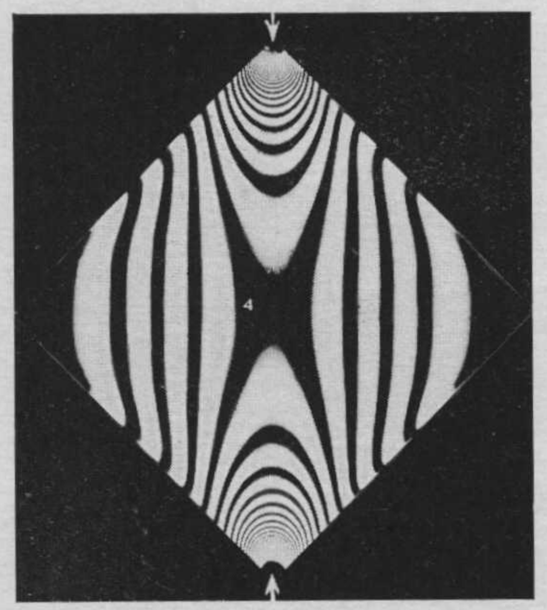

FIG. 2. Stress Pattern of a Square Block, Subjected to Diagonal Concentrated Loads.

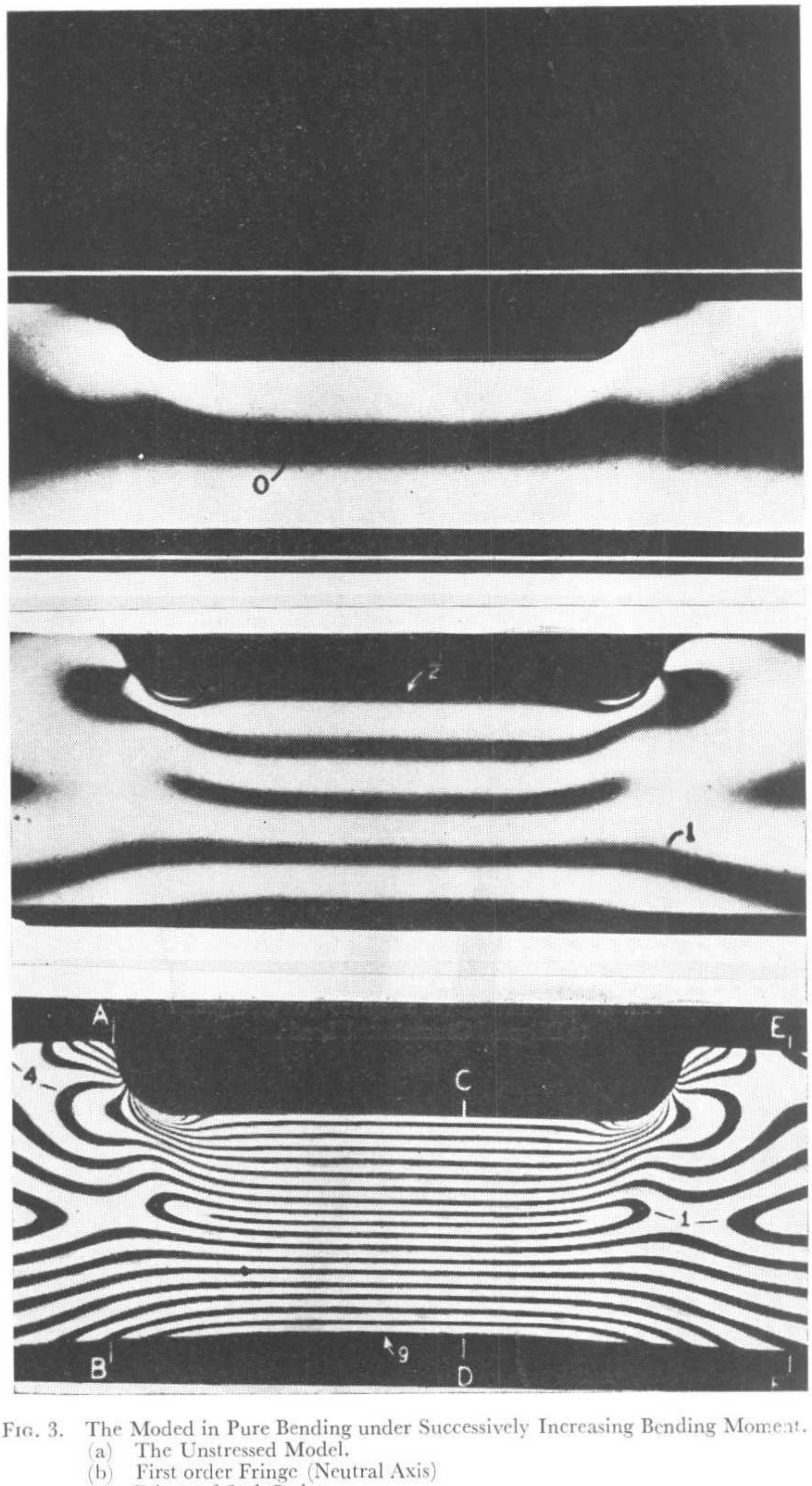

Fig. 3. The Moded in Pure Bending under Successively Increasing Bending Moment.
(a) The Unstressed Model.
(b) First order Fringe (Neutral Axis)
(c) Fringe of 2nd Order.
(d) At the Boundary Fringe of 9th Order.

When the model is subjected to *pure bending*, we have from the theory of elasticity :

$\sigma x = O, \tau xy = O$

$\sigma y = p$

and $\boxed{\sigma y = \frac{M.y}{I} = p}$

When : M—Benging moment

I—Moment of Inertia

y—distance from the neutral axis

If at a point P, distant y from the neutral axis $R_t = \lambda$ (Wave length), then all points on the horizontal line, parallel to the neutral axis, passing through P, would cause the same phase difference $R_t = \lambda$, and result in a black band on the screen.

Hence $R_t = n. \lambda$ represents the system of black bands.

Halfway between black bands would be bright bands, caused by a phase difference of $\lambda/2$, or of and odd number of half λ. Hence the Stress pattern in pure bending figures 3-*a*, *b*, *c*, *d*, consists of straight, parallel and equidistant black bands, known as fringes. In terms of stresses a fringe becomes the locus of points of constant principal stress difference, and, what follows, also a locus of points of constant maximum shear.

The retardation at any point in the model is defined as *fringe order n* at that point. That means, *n is the number of fringes* which pass through the point during the application of loads, increasing from o to their maximum. Fringe order generally varies from point to point in the model. In terms of fringe order the stress optic law becomes : $\boxed{\tau max = n.F}$

$$R_t = c.t\ (p-q) = n.\ \lambda, \quad \tau max = \tfrac{1}{2}(p-q) \text{ and } \tau max = \frac{n.\ \lambda}{2c.t}, \text{ let } \frac{\lambda}{2\ c.t} = F,$$

where F is defined as *model fringe value* and *is a fucnction of the material of the model and of the wave length.* As we see the fringe order is of utmost importance in photoelasticity. If however we use white light the colours will be extinguished not simultaneously, but in succession and given on the screen complementary colours. For tension specimen, R_t is the same for every point, the stress pattern has therefore the same colour all over. For model in *pure bending we get in one stress pattern* bands of *all* the *complementary colours*, as we have simultaneously a large range of stresses. *These coloured bands straight and parallel to the neutral axis*, are known as **isochromatics.**

All along the unloaded boundary of the moel there exists only *one principal stress, tangent to the contour*, whilst the other is zero. *Hence the stress pattern for a two-dimensional model furnishes directly the numerical value* of *the existing stress on the free boundary.* The sign can also be determined photoelastically, Fig. 4. *This provides one of the most important practical applications of photoelasticity particularly for investigations of parts subjected to fatigue loading* in which case failure is likely to begin as a surface crack, originating in a region of high tensile stress.

At all points in the model, *where* the *principal stresses have constant direction, parallel to the principal plane of polariser*, the crossed *analyser will extinguish the light.*

The locus of these points is defined as **isoclinic curve** Fig. 5. Using white light we shall get *black* isoclinics superimposed over coloured isochromatics. By rotating simultaneously polariser and analyser, we shall get isoclinics of various parameter. On the basis of isoclinic curves, known as *principal stress trajectories or isostatics*, can be constructed by a pure graphical process. Isostatics are also of frequent use in the photoelasticity. *Isoclinics and isochromatics give us the differences between and the direction of principal stress within the boundary of the model.*

If it is desired to evaluate each stress independently, we must use auxiliary data. But in most cases the stresses at interior points are of academic interest only, since the maximum stress occurs at boundaries of the model.

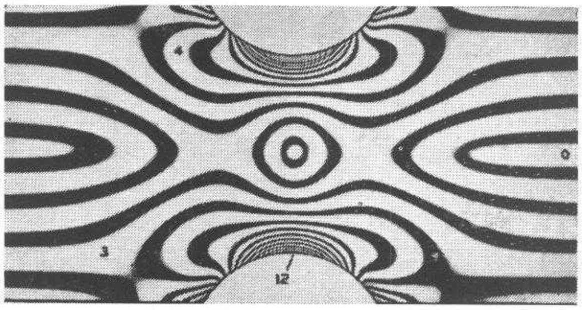

FIG. 4. Stress Pattern of a Beam with grooves, subjected to Pure Bending. Showing the Boundary Stresses.

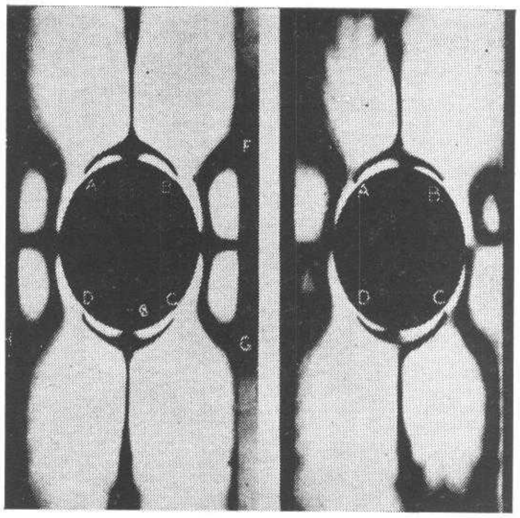

FIG. 5. Photographs of Isoclinics in a Bar with a Hole, Subjected to Axial Tension.

One of the best and quickest method for securing the auxiliary data is the measurement of lateral strain by means of mechanical Lateral Extensometer. In addition there are numerous analytical methods, involving the use of differential equations of equilibrium and compatibility.

Many of these methods furnish the value of the sum of principal stresses (p+q), which combined with (p—q) furnished by isochromatics, give each stress separately.

If we examine photoelastically a *three dimensionally stressed* model, we only see the summarized optical effect of all stresses along the path of ray. If we intend to find the stresses at every point within the model, we introduce a new concept of *secondary principal stresses*, defined by p′ and q′, resulting not from the actual stresses, but from their components, which lie in a plane normal to the ray. *It is because only the stresses normal to the ray produce photoelastic effect.*

Thus the stress optic law in three dimensions may be stated in the following form:

"The polarised beam of light upon entering the three-dimensional model is resolved into components which are parallel to the secondary principal stresses, corresponding to the given ray at the point of entrance."

$R'_t = c.t' (p' - q')$, where t′ is component of the thickness of the plate in the direction of entering ray, and represents the actual path of light.

There are three different photoelastic methods for a three—dimensional stress analysis. One is a modern "Stress Freezing" method, developed by Oppel and Monch in 1937, based upon the *amazing phenomenon of permanent deformation in certain resins.* A heated model loaded and slowly *cooled under load to the room temperature exhibits a photoelastic stress pattern which is "frozen" in the model.* Then the model can be cut into slices without any affect upon the pattern, and examined photoelastically. This method is applied when the directions of principal stresses are known, for example in pure torsion. Another application of frozen stress pattern is found in the study of dynamic stresses due to rotation. For the analysis of stresses, when the directions of principal stresses are unknown, serves the "Scattered light method" developed by Weller in 1932, and the "Convergent light method" employing the crystallographic technique of convergent light, developed by Dr. R. Hiltscher in 1937.

Rational design is inseparably connected with the ability to evaluate the stresses, produced by a given set of loads. The great majority of machine failures are caused by faulty design, which was based on average stresses, ignoring or neglecting exact stress distribution, especially the presence of stress concentration in abrupt change in section, such as in fillets, grooves, holes etc. The experimental methods constitute a recourse and furnish the needed information. The application of photoelastic method may be classified as follows :—

1. *For problems* involving contours of irregular, form, *which cannot be solved analytically, photoelasticity is of untold value in/determination of stresses* and *in evaluation of stress concentration.* Fig.6. *This is one of the most practical contribution of the photoelasticity.*

2. As a tool to aid in machine design, this method can be utilised *for locating regions of low stresses*, from which the material may be removed, for the purpose of reducing weight and cost of the material.

3. Stress analysis in engineering structures.

4. Studies of dynamic stresses.

Other advantages of the method are, that it provides means of accurate stress determination comparable to results obtained with precise strain-gage techniques; readily obtaining qualitative results for the determination of changes in stress distribution, by minor alterations in shape of the model (a sequence of cuts), to aid in the process of developing and selecting an optimum design. Fig. 7.

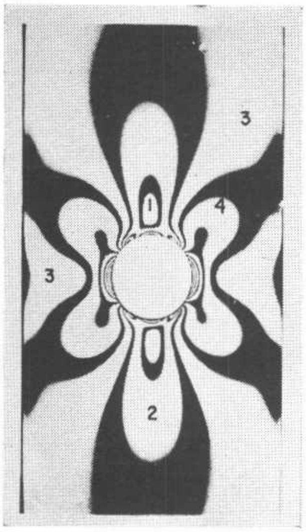

FIG. 6. Stress Concentration around a Hole in a Bar Subjected to Axial Tension.

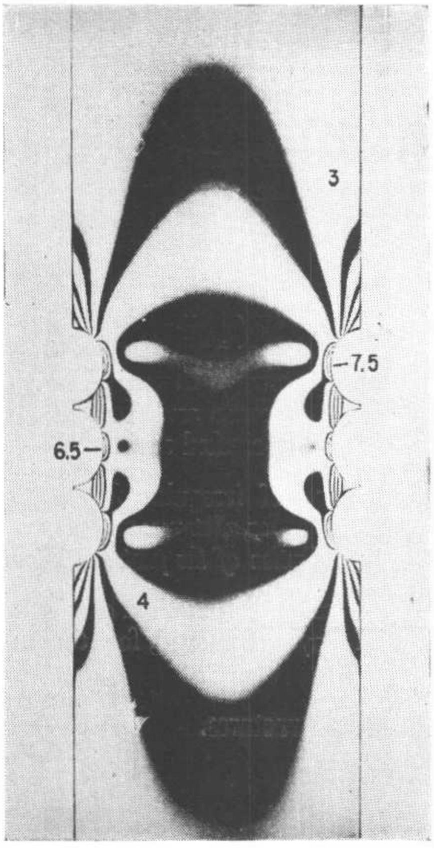

FIG. 7. Stress Pattern of a Bar in Pure Tension. The Max. Stress for 3 grooves is smaller than for one groove.

The conclusions, concerning stress distribution, obtained from photoelastic models are directly applicable to structural materials, because in all cases of plates with *simply connected boundaries the stress distribution in independent of the elastic constants of material,* and can be applied to structure of any isotropic material. *It is more convenient however, for practical purpose, to determine some non dimensional magnitudes, such as from factor* $\alpha = \frac{\mathfrak{S}\ \text{maximum}}{\mathfrak{S}\ \text{normal}}$: by testing of fillets, grooves etc., or other influence magnitudes such as $\beta = \frac{\mathfrak{S}}{P/F}$, where P/F is an arbitrary stress of reference, as *these magnitudes are the same both for models and prototypes.* Today photoelasticity has grown to the full stature as a powerful technical instrument of not only *qualitative*, but *first of all* of *quantitative* stress *analysis,* which for two-dimensional state of stress, at least, exceeds all other methods in reliability, scope and practicability.

There is no method by which the complete exploration of principal stresses, let alone the stresses on free boundaries, can be determined with the same speed and accuracy and at such a small cost, as the photoelastic method. Nor is there a method which has the same visual appeal by vividly portraying the entire stress situation in the model with *one pattern.*