STRESS CONCENTRATIONS: THEIR EFFECT ON DESIGN FOR REPEATED LOADING

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INTRODUCTION

Changes of section, such as fillets, grooves, oil holes, keyways, and the like, are necessary in many machine parts. These are sources of stress concentration when a part is under load. Stress concentrations may also occur near bolts, pins, rivets, spot welds, and other discrete fasteners in joints of structural members. Flaws, inclusions, and other discontinuities in a metal may also interrupt the stress pattern under load. The general term "stress raiser" has been coined to describe any such irregularity or inhomogeneity which produces a local concentration of stress in a loaded part.

Stress raisers are particularly harmful in a machine part or structural component which must withstand repeated or varying loads without failure by fatigue.

An example from a laboratory test illustrates the importance of stress raisers in fatigue. Figure 1 shows the failures of a riveted lap joint:

(a) The failure in the upper photograph occurred at a maximum load of 4600 pounds in a static tension test. In this test, the joint failed by shear of the rivets.

(b) The lower photograph shows a failure which occurred after about 80,000 alternations of load from 250 pounds to 1000 pounds. This fatigue failure did not occur in the rivets, but consisted of cracking of the sheet material along the lines of stress concentration around the revets.

Thus, in these tests, stress concentration in the sheet material caused fatigue failure to occur, not only at a much lower maximum load than that at which static failure would have occurred, but at a different location in the joint.

It is not difficult to understand such behavior qualitatively. In the static test, the highly stressed sheet material around the rivets deformed plastically, and the high stresses passed on to other regions before failure occurred. Under repeated loading at lower levels, where most of the sheet material was stressed elastically, yielding did not take place so extensively. Highly localized deformation started a fatigue crack before extensive redistribution of the stresses relieved the stress concentration.

While this concept of the effect of stress raisers is fairly simple, quantitative allowance for stress raisers in the design of parts may be extremely difficult. The objective of this paper is to summarize some of the considerations that enter into such design.

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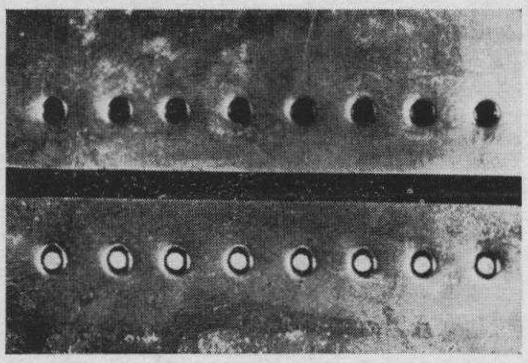
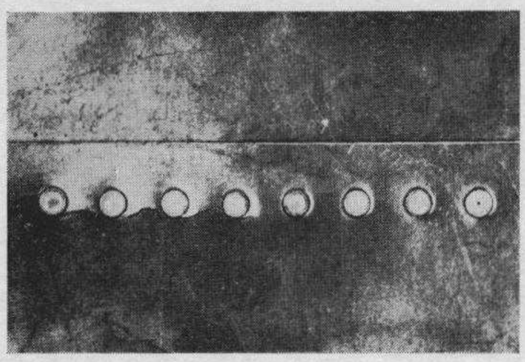


(a) Shear failure found in static tests



(b) Cracking of sheet found in fatigue tests

Fig. 1. A "NOTCH" EFFECT IN FAILURE OF RIVETED LAP JOINT

MATERIAL AS A FACTOR IN DESIGN

It is usual practice in the theory of elsticity to assume elastic behavior of materials below some limiting stress. The assumption of isotropic and homogeneous materials is also customary. However, experimental evidence casts doubt on the validity of these three assumptions.

Figure 2 illustrates stress-strain curves observed in typical engineering materials. Curve A is characteristic of brittle materials such as cast iron. Curves B and C are characteristic of ductile materials, such as low carbon steels (Curve B) and some alloy steels and nonferrous alloys (Curve C). These curves, though obviously different in appearance, have several features in common. For insance, the initial portion of each curve (o to a) is linear. Stress is directly proportional to strain. Beyond point "a", the elastic limit, each curve deviates from a straight line until failure (point "b"). In this region, strains are no longer proportional to stresses.

It is notable that the onset of plastic deformation may occur at different stresses or strains for different materials. Of equal importance are the various capacities each material has for plastic deformation. Such behavior directs the use of particular materials in certain engineering applications.

Materials stressed in the region from o to a are stressfree and strainfree when the external load is released. On the other hand, materials stressed in the region from a to b no longer are stressfree or strainfree. Fot instance, tension members exhibit residual deformation and, in the engineering sense, are stressfree; however, a plastically bent beam will have both residual stresses and residual strains when the bending moment is released.

The former case is illustrated in Figure 2. A tension member of material C, originally stressed to a value "c", shows residual tensile strain "od" when the external load is removed.

Working stresses usually are limited to elastic stresses in design applications. Sometimes this may not be possible because of: (1) insufficient knowledge of working loads, (2) inadequacy of stress analysis, and (3) fabrication effects. In many cases where ductile materials are used, this limitation may not be necessary. On initial load application, localized yielding may occur and redistribute the stress to less stressed material. Stress peaks in regions of discontinuities may be alleviated somewhat.

An example of stress redistribution is illustrated in Figure 3. The material assumed is one whose stress-strain diagram is similar to that of material C in Figure 2. The figure shows a sheet specimen with symmetrical edge notches loaded axially with load P. Considering the

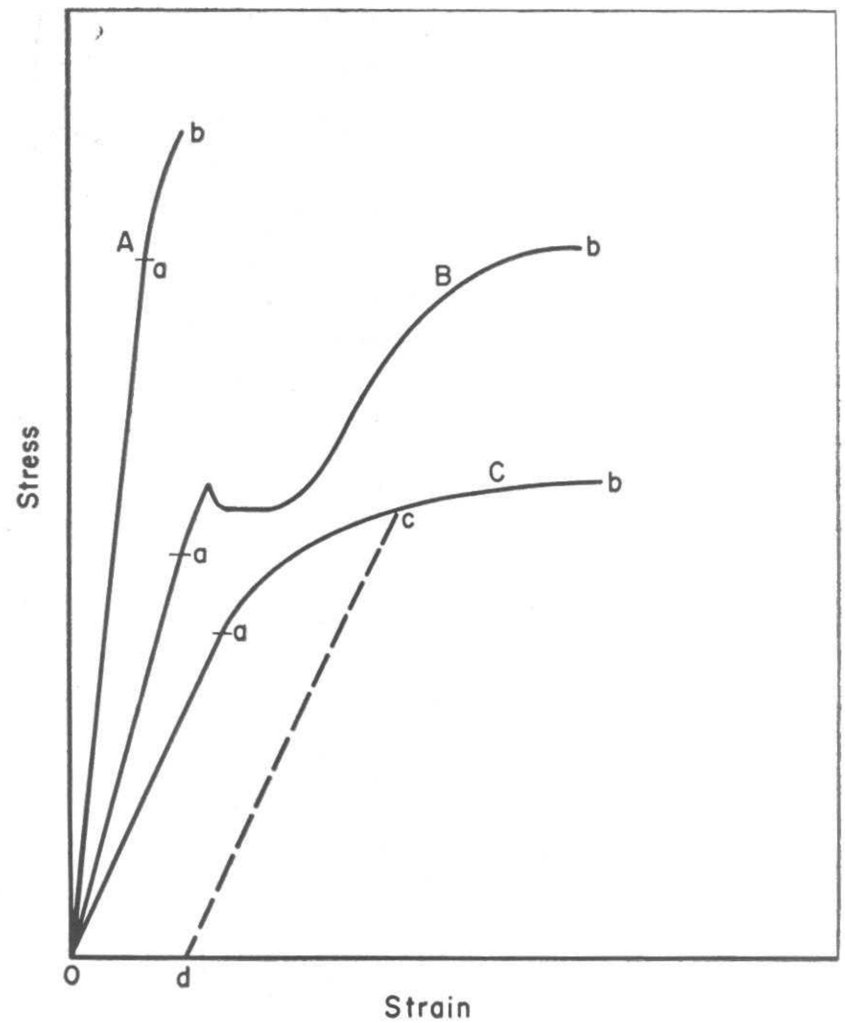


FIGURE 2. STRESS-STRAIN CURVES OF TYPICAL ENGINEERING MATERIALS

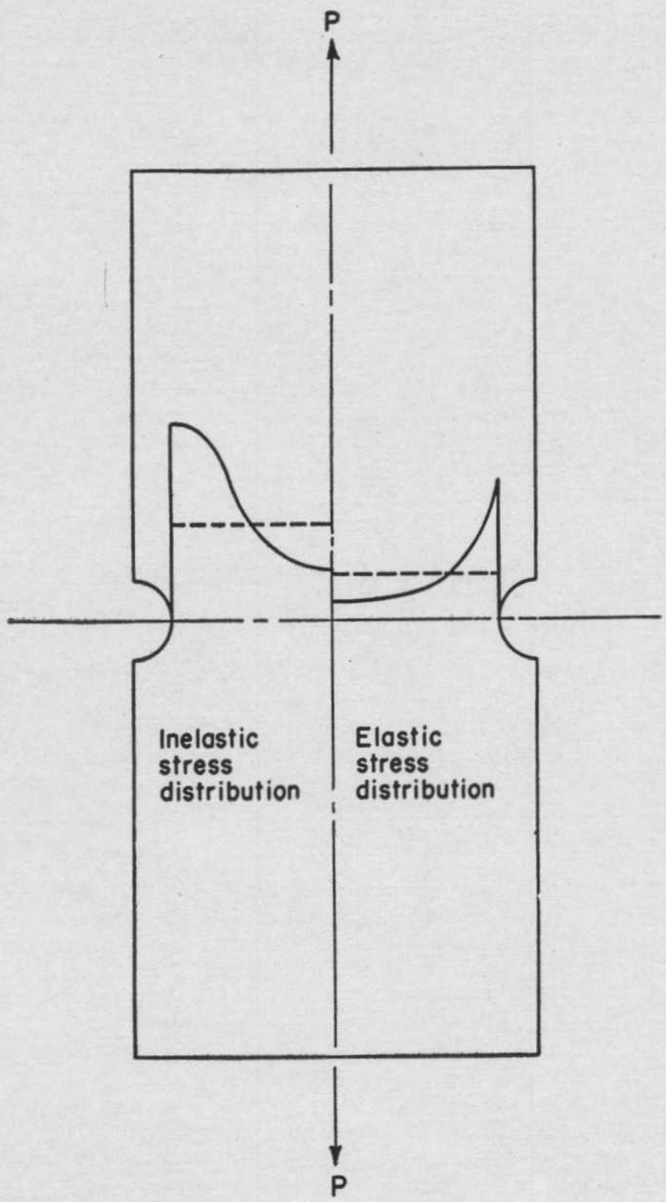


FIGURE 3. EFFECT OF LOAD ON STRESS DISTRIBUTION OF A NOTCHED SPECIMEN

minimum width section, the sress-raising effect of the nothces will cause peak stresses to occur at the notch root. If P is such that the maximum stress is less than the elastic-limit stress, the stress distribution will be as indicated on the right side of the figure (the dotted line represents the average stress, $s=P/A$, on the cross section). If P is large enough to cause yielding at the notch root, the stress distribution is as indicated on the left side of the figure. Note that yielding at the notch root has rounded off the stress peak.

The above example indicates the more gross type of departure from simple elastic material behavior which occurs frequently in machines and structures. This behavior may be important in static-load design as well as in repeated load design.

It must be realized also that the polycrystalline metallic alloys, which are of main importance as structural materials, are really neither homogeneous nor isotropic. Flaws and inclusions cause inhomogeneity, and orientation effects from rolling and forging produce anisotropy, even on a macroscopic scale. On a finer scale, differences in metallurgical phases, variations in orientation of grains and crystallites, and similar factors produce microsopic inhomogeneity and anisotropy. Even a single crystal is seldom isotropic (the modulus of elasticity of iron differs almost twofold in different crystallographic directions). These departures from simple elastic behavior may be reflected in the mechanical behavior of parts: in drift, set, and hysteresis of spring members, and so on. As noted in the following section, there is some evidence that even microscopic factors may be important in the fatigue life of a member—particularly in regions of high stress gradients at sharp notches.

GEOMETRICAL STRESS-RAISERS

Theoretical Stress Concentration Factor

Figure 4 shows the theoretical stress distribution in the neighborhood of a notch in a sheet specimen under tensile load. This distribution of stress was calculated by the theory of elasticity, assuming homogeneous, isotropic, and perfectly elastic material. The maximum tensile stress is the longitudinal stress just at the edge of the notch. In this case, the maximum stress is twice as high as the average longitudinal stress (the nominal P/A stress) across the center section. The ratio of this maximum stress to the nominal stress (2.0 in this example) is called the "theoretical stress-concentration factor", K_t, of the notch.

It may be shown, by the theory of elasticity, that (for ideal materials) K_t is constant for a particular notch under a particular type of loading. The theoretical stress-concentration factor is independent of load level* and of material, and depends only on the geometry of the member and the type of load applied (tension, bending, torsion). The value of K_t is a measure of the severity of a stress raiser and affords the designer an idea of actual peak stresses (in contrast to nominal stresses, computed by common engineering formulas).

Values of K_t for many geometrical notch forms have been computed and are available in published literature (see, particularly, References 1 to 5). In some cases, designer must contend with an unsymmetrically notched part for which no evaluation of K_t is available. In such cases, recourse to texts on notch-stress calculation (Reference 3) or to experimental methods such as photoelasticity (Reference 6) or the use of stress coat and wire strain gages (References 7 and 8) may be helpful.

Inclusions and flaws in a metal may give rise to stress concentration, either on account of their geometrical effect or on account of a transition in material properties. Stress-concentration factors for such "metallurgical stress raisers" are difficult to define or to evaluate. At present, the main expedients with regard to these are: (1) use of empirical fatigue test data to determine the magnitude of fatigue-strength reduction caused by common defects, and (2) inspection and quality control to avoid such defects in critical regions.

* For maximum stress within the elastic limit.

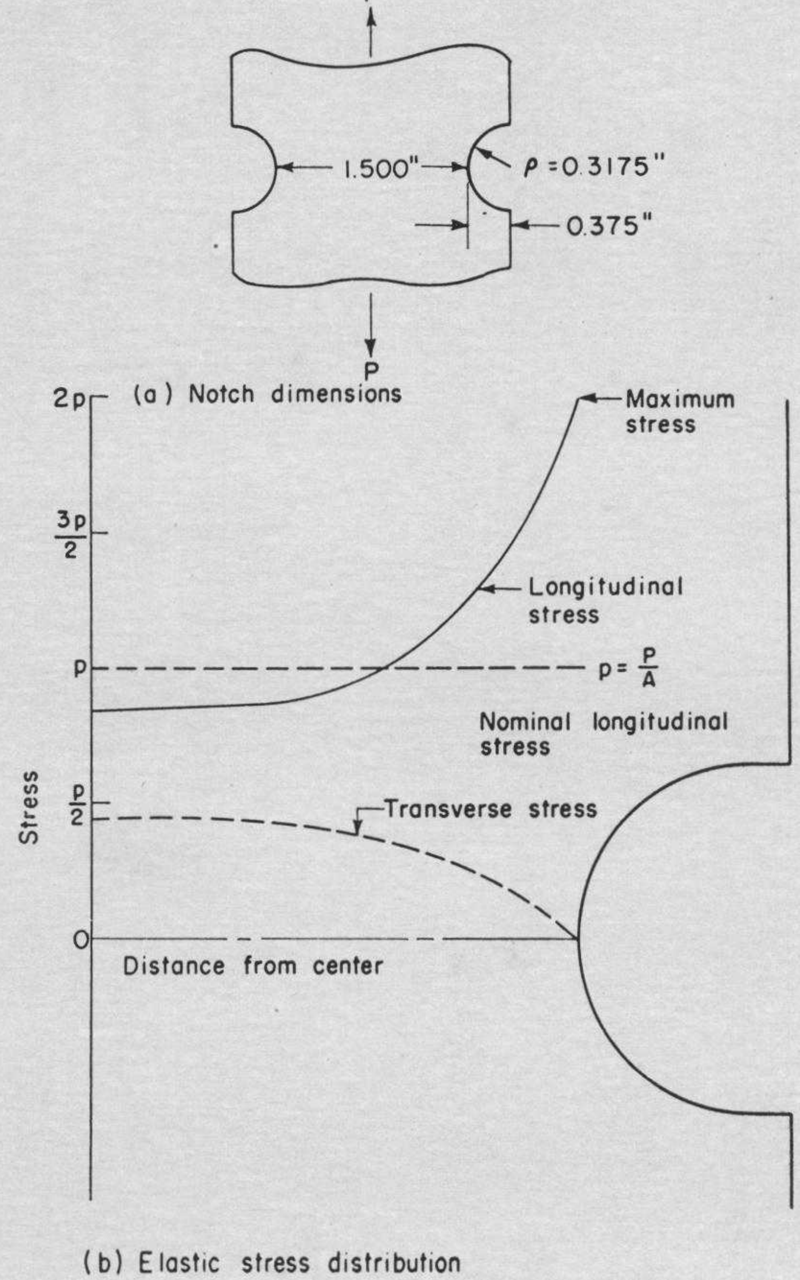


FIGURE 4. THEORETICAL ELASTIC STRESS DISTRIBUTION IN A PARTICULAR NOTCH

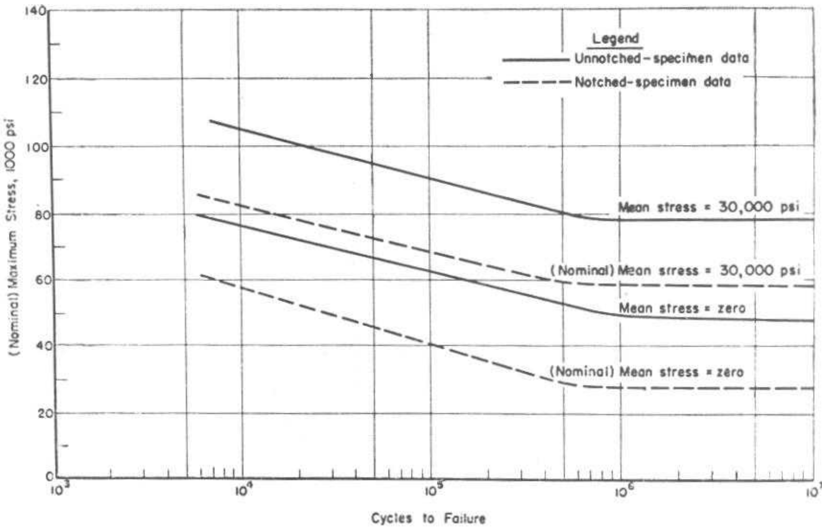


FIGURE 5. S-N CURVES FOR NOTCHED AND UNNOTCHED SAE 4130 STEEL SHEET (AXIAL LOADING)

Fatigue-Strength-Reduction Factor

Figure 5 shows some results of laboratory axial-loading fatigue tests of sheet specimens of normalized SAE 4130 steel (Reference 10). Unnotched specimens had a net section of 1 inch ×0.075 inch. Notched specimen dimensions are shown in Figure 4. The S-N curves show the significant reduction in fatigue strength (in terms of nominal stress) caused by the notch.

The quantitative measure of the effect of a notch on the fatigue strength of a member often is described in terms of a "fatigue-strength-reduction factor." For fully reversed loading (nominal mean stress = zero), this factor is:

$$K_f = \frac{\text{Maximum stress for unnotched specimen at N cycles}}{\text{Maximum nominal stress for notched specimen at N cycles}}$$

For example, for the case illustrated in Figure 5,

$$K_f = \frac{48.5}{27} = 1.8 \text{ at } 10^7 \text{ cycles, and}$$

$$K_f = \frac{76}{57} = 1.35 \text{ at } 10^4 \text{ cycles.}$$

When the loading is not fully reversed, it is necessary to specify further the fatigue-strength-reduction factor; there does not seem to be complete agreement as to the manner of doing this.†

In the example shown in Figure 5, it may be noted that for long lifetimes and fully reversed loading K_f closely approximates K_t. It would simplify many design problems if this were always true. However, there is considerable evidence that factors other than the theoretical stress-concentration factor influence the fatigue-strength reduction caused by geometrical stress raisers.

Figure 6 shows fatigue-strength reduction factors, at long lifetimes in rotating-bending fatigue of grooved round bars, plotted as a function of theoretical stress concentration factor. Note:

(1) K_f is usually less than K_t (actually the few exceptions may be ascribable to experimental errors).

† For example. one may define Kf as the ratio of nominal maximum stresses at a specified nominal mean stress. Two alternatives are: the ratio of nominal stress amplitudes at specified nominal mean stress, or the ratio of nominal maximum stresses at a specified ratio of minimum load to maximum load. Other possibilities exist.

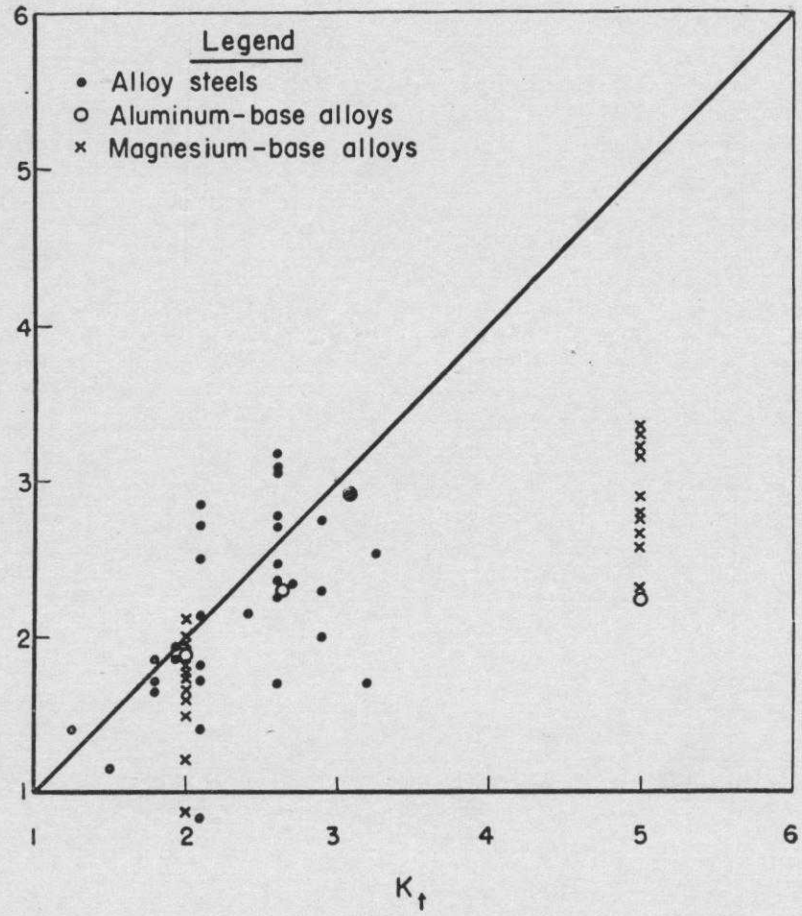


FIGURE 6. VARIATION OF K_f WITH K_t

K_f at 10,000,000 cycles of rotating bending; all notches circumferential grooves

(2) There is a general trend for K_f to increase with increasing K_t.
(3) The ratio of K_f to K_t appears to decrease as K_t becomes large.
(4) There is considerable scatter of points.

These observations imply that design based on assuming K_f as great as K_t usually will be conservative and, in some case, may be overconservative.

Effect of Notch Size and Stress Gradient.

It was noted, in the data shown in Figure 6, that K_f was much less than K_t for very large K_t. Notches with large K_t were sharp grooves with small radius. There is additional evidence that small-radius notches produce relatively small fatigue-strength reduction. Figure 7 shows, in illustration, the results of some axial loading fatigue tests on sheet specimens notched with holes of various diameters (Reference 11). Note that, for small radius of hole, the fatigue-strength reduction factor is relatively low, while the theoretical stress-concentration factor is high.

There is, at present, no complete theory of the effect of notch size and shape upon fatigue-strength reduction. Neuber (Reference 3) suggests that whenever the stress gradient is high for a length ρ' over which the anisotropy and inhomogeneity of material cannot be neglected, the effective stress-concentration factor is less than K_t. For a notch of radius, ρ, he suggests the factor.

$$K_N = 1 + \frac{K_t - 1}{1 + \sqrt{\rho'/\rho}}.$$

The value of ρ' is expected to be different for different materials; Neuber suggests that ρ' be evaluated experimentally. The dotted line in Figure 7 indicates K_N (computed with ρ' arbitrarily assumed as 0.05 inch) in comparison with K_t and K_f. In this example, as in several other cases, Neuber's relation is compatible with the general trend of experimental data. However, the range of validity of this relation has not been established.

Effect of Plastic Deformation at Notch Root.

In addition to the effect of notch severity (as indicated by K_t) and notch size (as indicated by notch radius, ρ), other factors influence the fatigue-strength reduction.

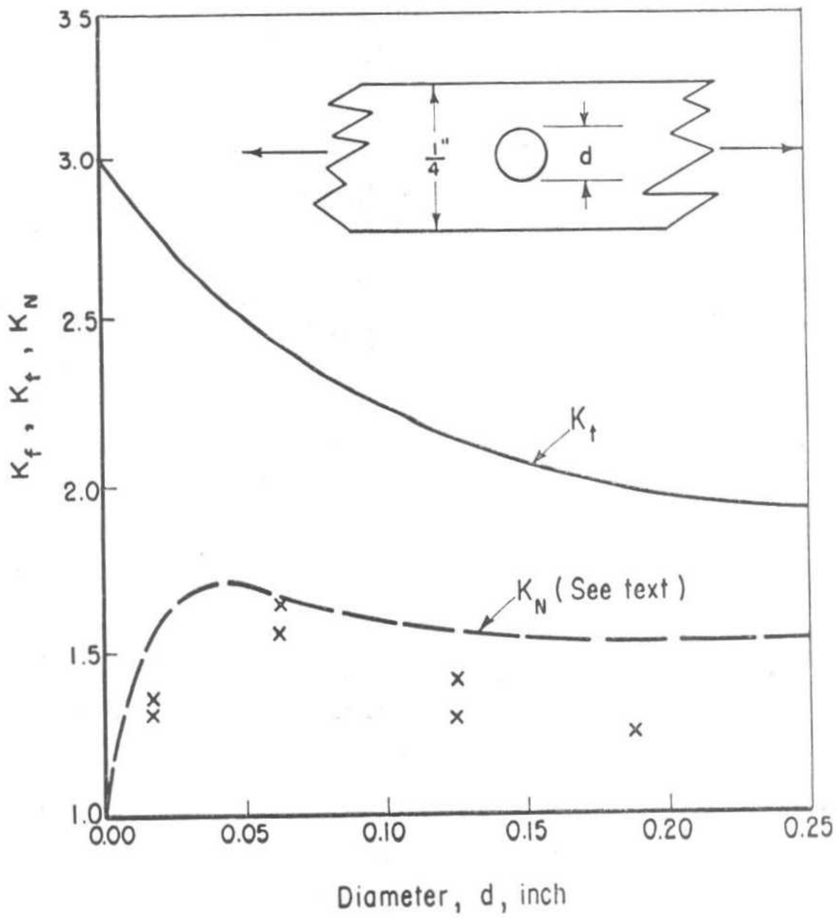


FIGURE 7 VARIATION OF FATIGUE-STRENGTH REDUCTION WITH NOTCH RADIUS

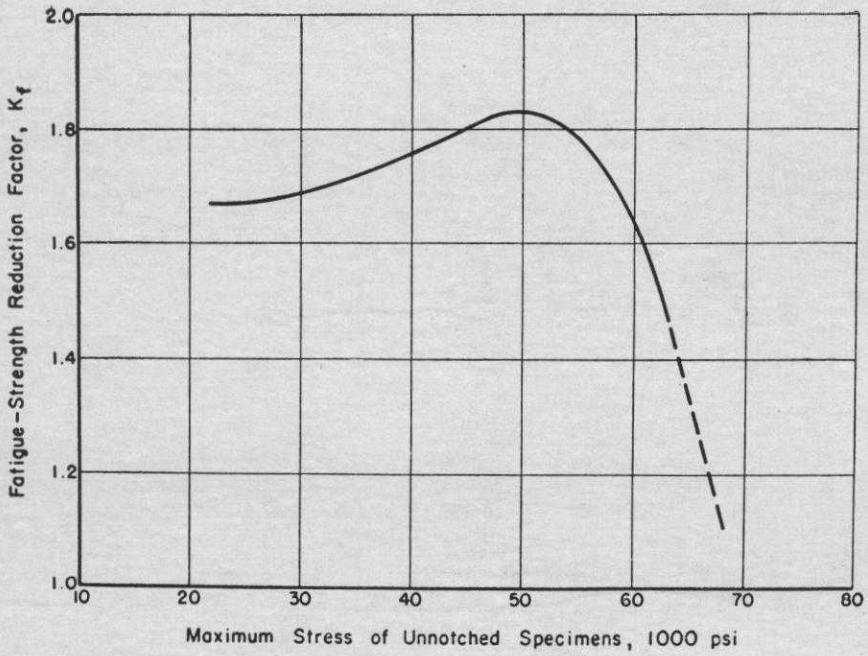


FIGURE 8 VARIATION OF FATIGUE-STRENGTH REDUCTION FACTOR WITH MAXIMUM STRESS OF UNNOTCHED SPECIMENS

One important factor is the level of repeated stress. Figure 8 shows values of K_f as a function of the stress of unnotched specimens (Reference 12). The decrease of K_f with increase of stress (and corresponding decrease of lifetime) is striking. This decrease is at least partly explainable in terms of plastic deformation and resulting alleviation of Kt as the stress level goes beyond the elastic limit. This effect of plastic deformation has been noted by several investigators (see, for example, References 13, 14, and 15).

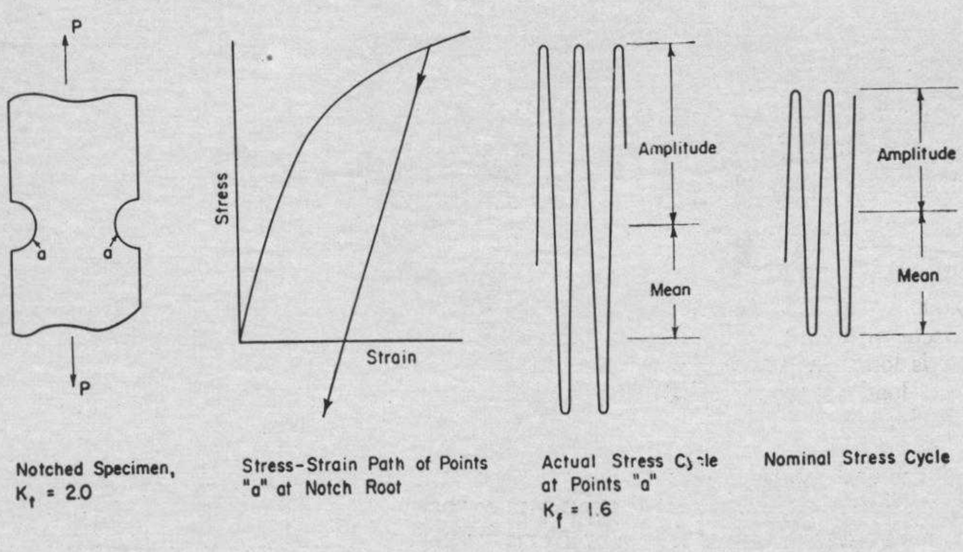


FIGURE 9. EFFECT OF PLASTIC DEFORMATION AT NOTCH ROOT

The effects of plastic deformation at the root of a notch may be complex. Figure 9 shows the results of measurements over a few cycles of slow loading. Plastic deformation on loading reduced the stress concentration factor at maximum load below that for loading in the elastic range. The resulting residual stress on unloading appreciably influenced the stress at minimum load. In some instances, it appears that the stress range determined by the first loading cycle may remain for many cycles (Reference 14); in other cases, as Orowan suggests (Reference 16), cumulative strain hardening would be expected to alter this greatly.

This complexity of the effects of plastic deformation requires the designer to use mainly empirical relationships—especially for incompletely reversed loading. One point deserves special consideration: a single overload may—by plastic deformation and the resulting residual stresses at points of stress concentration—influence the fatigue life of a member under subsequent repeated loading at lower stress levels (see, for example, Reference 15). In many instances this influence may be beneficial. This affords some justification for initial "shake-down" overloads of structural parts.

OTHER FACTORS IN DESIGN

Other factors are known to influence the fatigue behavior of components; environmental conditions (such as temperature and atmosphere), frequency of load repetition, load spectrum, and others. In some cases, available information is not extensive enough for application in design; in others, considerable work has been done to afford the designer some positive suggestions for particular problems.

Modern jet-propelled aircraft and other high-temperature applications have introduced additional problems where loads are repetitive. Not only are the mechanical properties decreased at elevated temperature, but metallurgical changes can occur during long-time, high-temperature service which may be important with regard to fatigue behavior. In addition, if loads are incompletely reversed, creep may become a complicating factor.

On the other hand, extremely low temperatures may affect the fatigue behaviour also. In this case, some mechanical properties may be increased (ultimate tensile strength, yield strength), while others (for example, ductility) may be decreased. For high load applications in notched parts, this might be detrimental since, plastic flow (or stress redistribution) would be inhibited.

Parts whose service is predominantly in corrosive atmospheres are affected more adversely by repeated loading than similar parts in neutral atmospheres. In these cases, the corrosive medium may pit the surface of the part. The stress-raising effect of these pits can be quite damaging. Alleviation of these damaging effects can be accomplished in many cases by protective surface treatments, choice of material, or neutralizing the corrosive medium.

Small laboratory fatigue specimens, subjected to load repetitions in the range of about 100 to 10,000 cycles per minute, do not appear to be affected by frequency. Some damaging effects may occur at higher stresses for lower frequency applications. Above 10,000 cycles per minute, hysteresis losses may raise the temperature of the specimen significantly. With regard to components, load frequency effects may be complicated by such factors as size, shape, and load magnitude.

Structural parts usually undergo rather complex stress histories during normal service. Thus, service-connected repeated stresses may vary considerably in magnitude. Typical S-N curves alone cannot answer design problems for parts so loaded. A number of empirical rules have been suggested for the consideration of these more complex loading situations (References 17 and 18). However, considerable experimental verification is necessary prior to their acceptance as valid design rules.

PRACTICAL DESIGN

Preliminary Design Calculations

Design calculations of the fatigue strength of machine and structural components with necessary stress raisers may be made on the basis of the data available on the fatigue properties of the unnotched material and the estimated theoretical stress-concentration factor.

For conditions of completely reversed loading, the notched fatigue strength (for a specified lifetime and nominal stress) may be computed by dividing the unnotched fatigue strength of the material (at the same life time) by the theoretical stress-concentration factor. This practice will be conservative generally, particularly when peak stresses are alleviated by local yielding. As yet, there are no safe rules to take advantage of such effects.

In the case of incompletely reversed loading, there are fewer data on which to base design rules. The most common procedure is to apply the theoretical stress-concentration factor to the alternating component.

Figure 10 shows an example of this procedure. The solid circles are taken from data in Reference 9. A line has been faired through these points which represents the 10,000,000-cycle behavior of unnotched 24S-T3 aluminum alloy. The dotted line is drawn for a notched part with stress-concentration factor Kt=2, by dividing the ordinates of the solid line by 2. The dash-dot line is faired through experimental points (triangles) for notched specimens (Kt=2) of the same material (Reference 10).

As indicated in the figure, this method of estimation of fatigue strength under incompletely reversed loading may be conservative. It should be emphasized that this is an empirical approximation whose validity has not been checked extensively. The method should be used with caution.

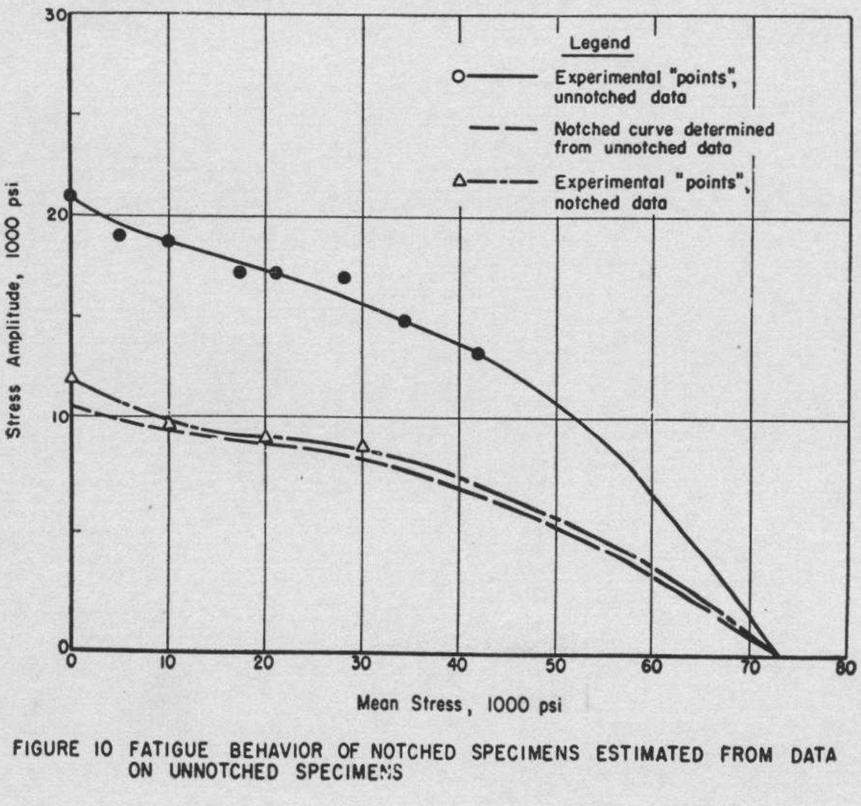


FIGURE 10 FATIGUE BEHAVIOR OF NOTCHED SPECIMENS ESTIMATED FROM DATA ON UNNOTCHED SPECIMENS

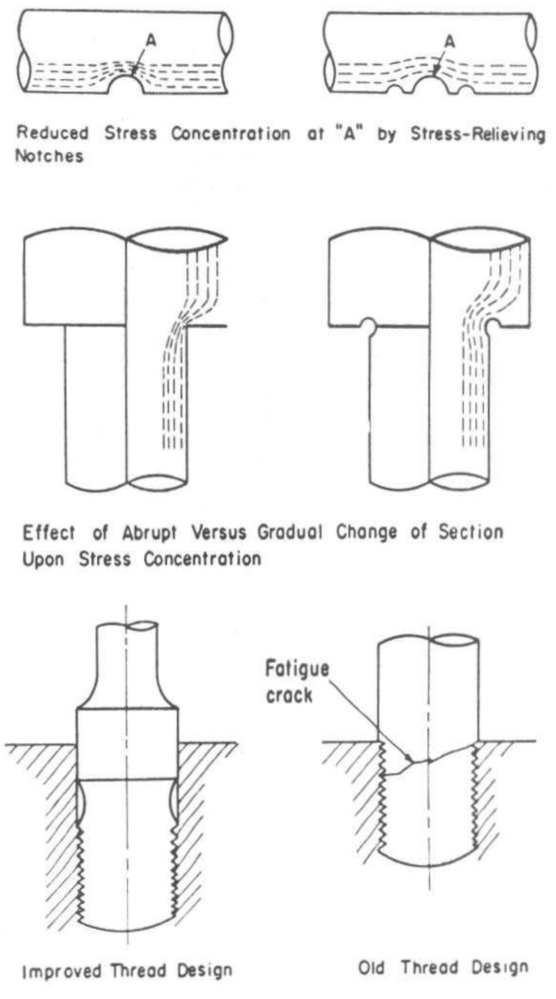
In some problems, notched fatigue data pertinent to a particular problem may exist. It is desirable to make use of such information.

Design to Minimize Stress Concentration

It is impossible to design machine parts without necessary stress raisers. However, since they are of utmost importance with respect to deleterious effects in repeated-load applications, the lowest feasible values of stress-concentration factor should be used. This is particularly the case if preliminary calculations show the part to be stressed critically.

Mitigation of stress raisers in parts generally can be accomplised by smoothing out contours at the changes in section. Large radii at fillets and grooves decrease the high stress gradients and high stresses that are undesirable under repeated loading.

Sometimes additional "stress-relieving" notches will minimize the deleterious effect of severe notches. Figure 11 shows some stress-relieving notches. The dotted lines indicate the "flow of stress" in the parts and suggest the underlying principle of such notches.



Reduced Stress Concentration at "A" by Stress-Relieving Notches

Effect of Abrupt Versus Gradual Change of Section Upon Stress Concentration

Improved Thread Design

Old Thread Design

FIGURE 11. ILLUSTRATION OF STRESS-RELIEVING NOTCHES

Minimization of Notch Effects by Fabrication Procedures

Once a part has been designed, a number of things can be done to minimize notch effects, such as: (1) attention to the fabrication of the part (2) mechanical working of the surface near the stress raiser, and (3) heat treatment in the neighborhood of the notch.

The first item requires ascertaining that the fabricator machines the part correctly. Sometimes reasonably designed parts have failed as a result of poor workmanship at necessary stress raisers.

If a residual compressive stress exists on the surface of a part, the part usually can withstand higher repeated stresses without fatigue failure. This is one reason for shot peening machine parts. Other fabricating procedures which produce compressive surface stresses include: thread rolling, rolling of grooves and fillets, reaming drilled holes, and burnishing surfaces. Materials which have a high capacity for strain hardening are usually more suitable for such fabrication techniques.

Other types of surface treatment also increase the fatigue strength of parts. Heat treatment, for instance, may be beneficial in this regard by : (1) increasing the strength of the surface material, and (2) developing favorable residual stresses. Flame and induction hardening are particularly valuable in this resepect (Reference 19). Carburizing and nitriding material in the vicinity of a stress raiser are also helpful.

SUMMARY

Some factors in the design of parts containing necessary stress raisers to resist repeated loads have been discussed in this paper. These factors include the theoretical stress-concentration factor, effects of anisotropic and inhomogeneous material in regions of high stress gradients, effects of plastic flow, and others. Some suggestions have been advanced to explain these effects. Other suggestions include some practical considerations for minimizing notch effects. Although these suggestions may be helpful, they are not intended to relieve the designer of the responsibility of critically examining all the information pertinent to each design problem.

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