Power Allocation For Outage Minimization in State Estimation Over Fading Channels

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Abstract

This paper studies the outage probability minimization problem for state estimation of linear dynamical systems using multiple sensors, where an estimation outage is defined as an event when the state estimation error exceeds a pre-determined threshold. The sensors amplify-and-forward their measurements (using uncoded analog transmission) to a remote fusion center over wireless fading channels. For stable systems, the resulting infinite horizon problem can be formulated as a constrained average cost Markov decision process (MDP) control problem. A suboptimal power allocation that is less computationally intensive is proposed, and numerical results demonstrate very close performance to the power allocation obtained from the solution of the MDP based average cost optimality equation. Motivated by practical considerations, assuming that sensors can transmit with only a finite number of power levels, optimization of the values of these levels is also considered using a stochastic approximation technique. In the case of unstable systems, a finite horizon formulation of the estimation outage minization problem is presented and solved. An extension to the problem of minimization of the expected error covariance is also studied.

Index Terms

Fading channels, Markov decision process, outage probability, power control, sensor networks, state estimation

I. Introduction

In real time applications, notions of outage are often used to quantify the time periods when the performance of a system is below what is desired. For instance, in mobile telephony, outages could correspond to times where the audio quality is very poor, and in tracking applications outages might correspond to instances where the location of a target cannot be determined to a desired accuracy.

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In communications and information theory, the notion of delay-limited or zero-outage capacity was introduced in [1]. The concepts of information outage probability and outage capacity, and the optimal power allocation to minimize the information outage probability subject to an average power constraint, were subsequently studied in [2]. Further extensions of the outage concept in communications theory include the delay-constrained outage capacity problem in [3], and the notion of service outage in [4]. In the signal processing literature, the notions of estimation outage and detection outage for the distributed estimation and detection of i.i.d. sources were introduced recently in [5] and [6] respectively. Further results on estimation outage and estimation diversity order can be found in [7], [8]. The optimal power allocation for estimation outage minimization problem, in the estimation of an i.i.d. Gaussian source, has been solved in [9] with full channel information and in [10] with quantized channel information at the sensor transmitters.

In much of this previous work, the systems that have been studied have been memoryless, so that the allocation of resources at one time instant does not affect the evolution of the system at future times. The focus of this paper is on extending the notions of estimation outage to, and solving the estimation outage minimization problem for, *dynamical systems*. In particular, we consider state estimation of linear dynamical systems using multiple sensors, where the sensors transmit their measurements to a fusion center over wireless channels using the analog amplify-and-forward technique of [11], which is a scheme that has been shown to be optimal in certain distributed estimation scenarios [12]. An outage will be defined as the event that the estimation error covariance exceeds a given threshold, and we are interested in how to optimally allocate the transmit powers of the sensors in order to minimize the probability of outage, subject to an average sum power constraint. We will use Markov decision process (MDP) and dynamic programming techniques to numerically solve these problems. Dynamic programming techniques have also been used in solving related problems such as the delay-constrained outage capacity problem in [3], and estimation error minimization problems for hidden Markov model state estimation in [13], [14].

Another area related to this paper is the analysis of the performance of Kalman filtering with packet losses, under various different notions of performance such as the expected error covariance [15], [16] and a probabilistic notion of performance [17], [18]. For continuous fading channels, the behaviour of the expected error covariance has also been studied in [19], [20]. However, the focus of these works is more on determining conditions under which the filter remains stable, and power control is not explicitly considered.

Summary of Contributions

This paper is concerned with solving the estimation outage minimization problem, in the state estimation of linear dynamical systems. In particular, we make the following contributions:

- In the case of stable systems, we formulate the outage minimization problem over an infinite horizon. This will turn out to be a constrained average cost Markov decision process (MDP) [21], which we can transform using a Lagrangian technique into an unconstrained MDP, that can then be solved numerically at the fusion center with techniques such as the relative value iteration algorithm. The optimal power allocations are then fed back to the sensors.

- In the case of unstable systems, an infinite horizon average cost problem formulation is not appropriate since increasingly large amounts of power will need to be transmitted. Instead we study a finite horizon formulation, that can be solved numerically using dynamic programming techniques.
- We propose suboptimal policies that can be more efficiently solved, especially for large numbers of sensors and/or high dimensional vector states. The suboptimal policies are motivated by the form of the optimal solution to the information outage minimization problem in communications theory [2]. For scalar systems, the power allocations can be determined analytically, but for vector systems the numerical solution of non-convex optimization problems is required at each time step. Numerical studies indicate very close performance to the optimal solutions.
- Assuming that sensors can only transmit using a fixed number of power levels, we consider the problem of optimizing the values of these powers using stochastic approximation techniques.
- We consider a related problem of minimization of the long-term average expected error covariance subject to average sum power constraints, that can be solved using similar techniques. The performance is compared with a greedy suboptimal solution studied in [22].

The organization of the paper is as follows. We will first focus on scalar linear systems in Sections II-IV, where finding optimal solutions numerically is more computationally tractable than the general vector case. Furthermore, suboptimal policies in the scalar case can be found analytically, but in the vector case will require the numerical solution of non-convex optimization problems. Stable systems are considered in Section III, where we present the outage minimization problem formulation in Section III-A. Section III-B derives some conditions on the distortion threshold that affect the solvability of the problem. The outage minimization problem is solved in Section III-C, and a sub-optimal policy is proposed in Section III-D. Optimization using a finite number of power levels is addressed in Section III-E. Unstable systems are then considered in Section IV. We present first a finite horizon formulation and suboptimal policy in Sections IV-A and IV-B respectively. Vector systems are considered in Section V, where we present a possible formulation of the outage minimization problem. We also propose a suboptimal algorithm, which however requires the numerical solution of non-convex optimization problems in general. Finally, using similar techniques studied in this paper, an extension to the problem of minimizing expected error covariance is studied in Section VI, and compared with a suboptimal greedy approach.

II. System Model

Sections II-IV will focus on scalar linear systems. Throughout this paper we will use k to denote the discrete time index, and i to denote the sensor index. We consider a discrete time scalar linear system given by

$$x_{k+1} = ax_k + w_k \tag{1}$$

where $x_k, w_k, a \in \mathbb{R}$, with x_k representing the state that we wish to estimate, and w_k is i.i.d. Gaussian noise with zero mean and variance σ_w^2. See Figure 1 for a diagram of the system model.

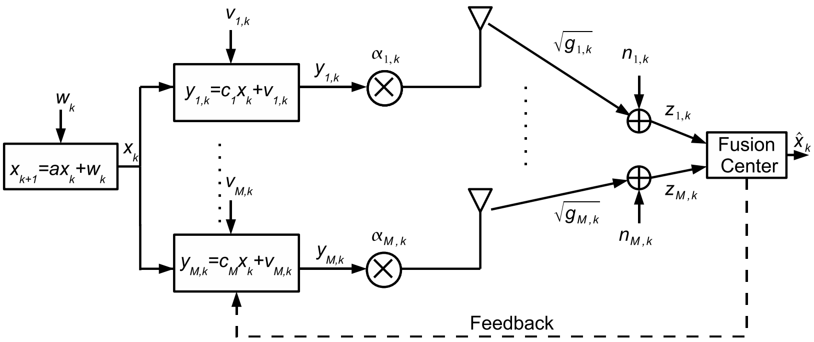

Fig. 1. System model

The system is observed by M different sensors with observations

$$y_{i,k} = c_i x_k + v_{i,k}, i = 1, \ldots, M$$

with $y_{i,k}, v_{i,k}, c_i \in \mathbb{R}$, and $v_{i,k}$ is i.i.d. Gaussian noise with zero mean and variance σ_i^2. The c_i parameters can be interpreted as a sensor's measurement gain/attenuation due to factors such as e.g. distance from the source.

The sensors then send their measurements over wireless channels to a fusion centre. We assume that the sensors use the analog amplify-and-forward technique of [11], where the sensor transmitter amplifies $y_{i,k}$ by a factor $\alpha_{i,k}$ and sends it to the fusion centre over a fading channel. The different fading channels are taken to be orthogonal, as in [5]. We remark that a non-orthogonal multi-access transmission scheme can also be considered (see [11]), and the analysis will be similar, but for brevity we will restrict ourselves to the orthogonal scheme in this paper. The received signals at the fusion centre can be written as

$$z_{i,k} = \alpha_{i,k}\sqrt{g_{i,k}}y_{i,k} + n_{i,k} = \alpha_{i,k}\sqrt{g_{i,k}}c_i x_k + \alpha_{i,k}\sqrt{g_{i,k}}v_k + n_{i,k}, \; i = 1, \ldots, M \tag{2}$$

where $g_{i,k} \geq 0$ are the random channel power gains, $n_{i,k}$ is i.i.d. Gaussian noise with zero mean and variance σ_n^2, and $\alpha_{i,k}$ are the amplification factors in the analog forwarding scheme. The channel gains $g_{i,k}, \forall i$ are assumed to be known at the receiver, while an individual sensor i has knowledge of its own channel $g_{i,k}$. The channel undergoes slow fading such that the phase of the complex channel can be estimated and compensated for at the fusion center, so that $\sqrt{g_{i,k}}$ represents the real-valued envelope of the complex channel gains. We use a block fading model, with the channel gains $g_{i,k}$ being independent and identically distributed (i.i.d.) over time, and with continuous distributions. We assume that there are noiseless feedback links from the fusion center back to the sensors, that can be used to e.g. feedback optimal values of $\alpha_{i,k}$ that are computed at the fusion center (see Section III-C). The noise and fading terms $x_0, w_k, v_{i,k}, g_{i,k}$ and $n_{i,k}$ are taken to be mutually independent. In addition, it is assumed that the fusion center has knowledge of the parameters $a, c_i, \sigma_w^2, \sigma_i^2, \sigma_n^2, \forall i$.

Call $z_k = (z_{1,k}, \dots, z_{M,k})^T, g_k = (g_{1,k}, \dots, g_{M,k})^T$, $\bar{C}_k = (\alpha_{1,k}\sqrt{g_{1,k}}c_1, \dots, \alpha_{M,k}\sqrt{g_{M,k}}c_M)^T$, $\bar{v}_k = (\alpha_{1,k}\sqrt{g_{1,k}}v_{1,k} + n_{1,k}, \dots, \alpha_{M,k}\sqrt{g_{M,k}}v_{M,k} + n_{M,k})^T$, $\bar{R}_k = diag(\alpha_{1,k}^2 g_{1,k}\sigma_1^2 + \sigma_n^2, \dots, \alpha_{M,k}^2 g_{M,k}\sigma_M^2 + \sigma_n^2)$. The equations in (2) can then be written as

$$z_k = \bar{C}_k x_k + \bar{v}_k \tag{3}$$

where $\bar{v}_k$ has the time-varying covariance matrix $\bar{R}_k$. The equations (1) and (3) form a linear time-varying system whose state x_k can be optimally estimated by a time-varying Kalman filter at the fusion centre. Define the state estimate and error covariance as[1]

$$\begin{aligned} \hat{x}_{k+1|k} &= \mathbb{E}[x_{k+1}|z_0, \dots, z_k, g_0, \dots, g_k] \\ P_{k+1|k} &= \mathbb{E}[(x_{k+1} - \hat{x}_{k+1|k})^2|z_0, \dots, z_k, g_0, \dots, g_k]. \end{aligned}$$

In the following, we will also use the short hand notation $P_{k+1} = P_{k+1|k}$.

One then has from the time-varying Kalman filter equations [23] that

$$\begin{aligned} \hat{x}_{k+1|k} &= a\hat{x}_{k|k-1} + aP_k\bar{C}_k^T(\bar{C}_kP_k\bar{C}_k^T + \bar{R}_k)^{-1}(z_k - \bar{C}_k\hat{x}_{k|k-1}) \\ P_{k+1} &= a^2P_k - a^2P_k^2\bar{C}_k^T(\bar{C}_kP_k\bar{C}_k^T + \bar{R}_k)^{-1}\bar{C}_k + \sigma_w^2. \end{aligned} \tag{4}$$

By an application of the matrix inversion lemma the recursion for the error covariance can be further simplified to:

$$P_{k+1} = \frac{a^2P_k}{1 + P_k\bar{C}_k^T\bar{R}_k^{-1}\bar{C}_k} + \sigma_w^2 = \frac{a^2P_k}{1 + P_k\sum_{i=1}^M \frac{\alpha_{i,k}^2 g_{i,k} c_i^2}{\alpha_{i,k}^2 g_{i,k}\sigma_i^2 + \sigma_n^2}} + \sigma_w^2. \tag{5}$$

The sensor transmit power $\gamma_{i,k}$ used by the i-th sensor in transmitting its measurement to the fusion centre at time k is defined as

$$\gamma_{i,k} = \alpha_{i,k}^2\mathbb{E}[y_{i,k}^2] = \alpha_{i,k}^2(c_i^2\mathbb{E}[x_k^2] + \sigma_i^2). \tag{6}$$

[1]Similarly, quantities such as $\hat{x}_{k|k}$ and $P_{k|k}$ can be defined and Kalman filtering equations for these quantities can be written, but are omitted for brevity.

III. Stable systems

A. Problem statement

In this section we will consider stable scalar linear systems, i.e. $|a| < 1$ (see Section IV for the case of unstable systems). Then as $k \to \infty$, $\{x_k\}$ becomes stationary and we have $\mathbb{E}[x_k^2] \to \sigma_w^2/(1-a^2)$, so that (6) simplifies to

$$\gamma_{i,k} = \alpha_{i,k}^2 \left(c_i^2 \frac{\sigma_w^2}{1-a^2} + \sigma_i^2 \right)$$

Let us call $\alpha_k = (\alpha_{1,k}, \ldots, \alpha_{M,k})^T$, $\gamma_k = (\gamma_{1,k}, \ldots, \gamma_{M,k})^T$. The problem we consider in this section is to choose the α_k's (and hence the γ_k's) to minimize the estimation outage probability subject to a long run average power constraint $\mathcal{P}$ on the sum of the transmitted powers. We will assume that the power allocations are causal, i.e. γ_k is a function of $(P_0, \ldots, P_k)$ and $(g_0, \ldots, g_k)$. By the Markov property, γ_k will in fact turn out to be a function of P_k and g_k.

In this paper we will declare an estimation outage event if the error covariance P_{k+1} exceeds some distortion threshold D. More formally, we want to solve over an infinite horizon the problem:

$$\begin{aligned} &\min_{\{\gamma_k\}} \limsup_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \Pr(P_{k+1} > D) = \min_{\{\gamma_k\}} \limsup_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \mathbb{E}[1_{(P_{k+1} > D)}] \\ &\text{s.t. } \limsup_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \mathbb{E}[\sum_{i=1}^{M} \gamma_{i,k}] \leq \mathcal{P} \end{aligned} \tag{7}$$

where $1_A(\cdot)$ is the indicator function, with $1_A(\omega) = 1$ if $\omega \in A$, and $1_A(\omega) = 0$ if $\omega \notin A$.

As mentioned in the introduction, the motivation for using the outage probability as the performance criterion is that we are interested in criteria that captures the short term estimation performance useful for real-time applications, in contrast to other long-term (or ergodic) performance criteria such as the average error covariance.[2] This is due to the fact that state estimates constructed from a time-varying Kalman filter based on measurements received from the sensors over randomly time-varying fading channels have prediction error covariances which are also randomly time-varying. In applications where short-term estimation performance is more critical (such as target tracking or automatic control of unstable plants), a large estimation/prediction error covariance is unacceptable and therefore minimizing the probability of estimation outage (that the prediction error covariance exceeds a certain threshold) is an appropriate performance measure in this case. This motivation is similar to the rationale behind information outage minimization in communications theory where for real-time applications such as voice or video transmissions, outage probability (that the channel capacity falls below a basic minimum rate) is adopted as an appropriate performance criterion, as opposed to ergodic capacity which is more suited to delay-insensitive data

[2]Note that the cost function in (7) is written as a long term average due to the fact that $\{P_k\}$ is not necessarily a stationary process. However the per-stage cost $\Pr(P_{k+1} > D)$, representing the outage probability at time k, can still be used to capture the short term estimation performance.

transmission scenarios. Note though that other performance criteria can be studied using similar techniques. Indeed, Section VI briefly describes how we can solve the problem of minimizing the long-term average of expected (with respect to fading channel realizations) error covariance subject to a long-term average power constraint.

B. Conditions on D

In this section we will derive some conditions on the threshold D that will affect the solvability of problem (7). Recall that the recursion for the error covariance satisfies (5). First we state some simple properties on how P_k and $\alpha_{i,k}^2$ affect P_{k+1}.

Lemma 1: Consider P_{k+1} as given by

$$P_{k+1} = \frac{a^2 P_k}{1 + P_k \sum_{i=1}^{M} \frac{\alpha_{i,k}^2 g_{i,k} c_i^2}{\alpha_{i,k}^2 g_{i,k} \sigma_i^2 + \sigma_n^2}} + \sigma_w^2.$$

(i) P_{k+1} is an increasing function of P_k.

(ii) P_{k+1} is a decreasing function of $\alpha_{i,k}^2$.

The proof of Lemma 1 is straightforward and omitted.

For an initial simple bound, note that for stable systems, if the initial error covariance P_0 satisfies $P_0 \leq \frac{\sigma_w^2}{1-a^2}$, then the following holds:

$$\sigma_w^2 < P_k \leq \frac{\sigma_w^2}{1-a^2}, \forall k.$$

To see the upper bound, suppose we set $\alpha_{i,k}^2 = 0, \forall i, k$. Then we have

$$\begin{aligned} P_k &= a^2 P_{k-1} + \sigma_w^2 = a^{2k} P_0 + (a^{2k-2} + \cdots + a^2 + 1)\sigma_w^2 \\ &= a^{2k} P_0 + \frac{(1-a^{2k})\sigma_w^2}{1-a^2} = \frac{\sigma_w^2}{1-a^2} + a^{2k}\left(P_0 - \frac{\sigma_w^2}{1-a^2}\right) \\ &\leq \frac{\sigma_w^2}{1-a^2} \text{ if } P_0 \leq \frac{\sigma_w^2}{1-a^2}. \end{aligned}$$

By Lemma 1, this then implies that $P_k \leq \frac{\sigma_w^2}{1-a^2}, \forall k$. Hence if $D \leq \sigma_w^2$, then P_{k+1} will always exceed D (i.e. we will always be in outage), and if $D > \sigma_w^2/(1-a^2)$ then we will never have any outage events.

Next, note from (5) and Lemma 1(ii) that given P_k, the error covariance at the next time instant P_{k+1} satisfies

$$\frac{a^2 P_k}{1 + P_k \sum_{i=1}^{M} c_i^2/\sigma_i^2} + \sigma_w^2 < P_{k+1} \leq a^2 P_k + \sigma_w^2, \tag{8}$$

where the lower bound comes from taking $\alpha_{i,k}^2 \to \infty, \forall i$, and the upper bound comes from taking $\alpha_{i,k}^2 = 0, \forall i$. The term $\frac{a^2 P_k}{1+P_k \sum_{i=1}^{M} c_i^2/\sigma_i^2} + \sigma_w^2$ thus can be regarded as the smallest value of P_{k+1} that can be achieved for a given value of P_k (by using an infinite amount of transmit power).

Below we present some more precise conditions. In particular we will partition the range of D such that given P_k, the condition $P_{k+1} \leq D$ can either: 1) always be achieved, 2) never be achieved, or 3) can be achieved only for P_k sufficiently small.

1) Suppose that at time k, $P_k = \sigma_w^2/(1-a^2)$, i.e. P_k is at its maximum value. Then

$$\frac{a^2 P_k}{1+P_k \sum_{i=1}^M c_i^2/\sigma_i^2} + \sigma_w^2 = \frac{a^2 \sigma_w^2}{1-a^2+\sigma_w^2 \sum_{i=1}^M c_i^2/\sigma_i^2} + \sigma_w^2 \equiv D_1.$$

Recalling that $\frac{a^2 P_k}{1+P_k \sum_{i=1}^M c_i^2/\sigma_i^2} + \sigma_w^2$ is the smallest value of P_{k+1} that can be achieved for a given value of P_k, and using Lemma 1(i), we thus have the condition that if $D \geq D_1$, then $P_{k+1} \leq D$ can be achieved in one time step for all $P_k \geq D$.

2) Consider the values of P_k such that $\frac{a^2 P_k}{1+P_k \sum_{i=1}^M c_i^2/\sigma_i^2} + \sigma_w^2 > P_k$, i.e. the values of P_k such that $P_{k+1} > P_k$, even if an infinite amount of transmit power is used. This can be easily shown to be equivalent to

$$P_k < \frac{-B + \sqrt{B^2 + 4\sigma_w^2 \sum_{i=1}^M c_i^2/\sigma_i^2}}{2\sum_{i=1}^M c_i^2/\sigma_i^2} \equiv D_2, \tag{9}$$

with $B = (1 - a^2 - \sigma_w^2 \sum_{i=1}^M c_i^2/\sigma_i^2)$. Hence we now have the condition that if $D \leq D_2$ and $P_k \geq D$, then $P_{k+1} \leq D$ cannot be achieved in one time step since $P_{k+1} > P_k \geq D$ by assumption, and therefore cannot be achieved in all subsequent time steps by Lemma 1(i). This is a tight form of the condition $D \leq \sigma_w^2$ always resulting in outage mentioned previously.

3) In the case where D satisfies $D_2 < D < D_1$, we have the situation where given P_k, the condition $P_{k+1} \leq D$ can only be achieved when

$$\frac{a^2 P_k}{1+P_k \sum_{i=1}^M c_i^2/\sigma_i^2} + \sigma_w^2 < D \text{ or } P_k < \frac{D-\sigma_w^2}{a^2 - (D-\sigma_w^2)\sum_{i=1}^M c_i^2/\sigma_i^2}, \tag{10}$$

i.e. only when P_k is sufficiently small. If (10) is not satisfied, then it will require more than one time step to bring the error covariance below the distortion threshold D. This has implications in that one cannot directly use the analogue of a scheme considered in [2] as a suboptimal policy, which will be studied in Section III-D.

C. Solution of outage minimization problem

In this section we will solve the estimation outage minimization problem (7). In communications theory, information outage minimization problems have been considered in e.g. [2], [4], and analytical solutions can be derived. However, these works consider memoryless systems, whereas in problem (7) the quantity P_k evolves dynamically over time. Furthermore, as shown in Section III-B, power allocation may need to be carried out over multiple time steps before one can move from being in outage to non-outage. Thus the techniques used in [2], [4] do not appear to be extendable to our case. Instead we will use Markov decision process (MDP) techniques to numerically solve problem (7).

Let us first make the following additional assumptions to problem (7).

Assumption 3.1: D satisfies the condition $D_2 < D \leq \sigma_w^2/(1-a^2)$, where D_2 is defined by (9).

Assumption 3.2: The range of $\gamma_{i,k}$ is bounded, i.e. $\gamma_{i,k} \in [0, \gamma_{max}], \forall i, k$.

Assumption 3.1 is needed for there to be non-trivial solutions to problem (7) by Section III-B. Assumption 3.2 obviously has practical purpose, and also allows us to apply existing theoretical results, e.g. [24], [25], to show the existence of solutions to associated optimality equations (see later).

The estimation outage minimization problem (7) can then be regarded as a constrained average cost MDP with $(P_k, g_k) = (P_k, g_{1,k}, \ldots, g_{M,k})$ as the composite "state" and $\gamma_k = (\gamma_{1,k}, \ldots, \gamma_{M,k})$ as the "action". More formally, under Assumptions 3.1 and 3.2, the state space $\mathcal{S} = (D_2, \frac{\sigma_w^2}{1-a^2}] \times \mathbb{R}^M$, the action space $\mathcal{A} = [0, \gamma_{max}]^M$, the set of feasible actions is $[0, \gamma_{max}]^M$ for each state, the transition laws are determined by (5) for P_k and the assumption that g_k is i.i.d., the per-stage cost is $\mathbb{E}[1_{(P_{k+1}>D)}]$, and the constraint is $\limsup_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \mathbb{E}[\sum_{i=1}^{M} \gamma_{i,k}] \leq \mathcal{P}$.

We will solve (7) using a similar approach to [14], by converting the constrained MDP into an unconstrained MDP. We first introduce the Lagrangian:

$$L^\beta = \limsup_{K\to\infty} \frac{1}{K} \left\{ \sum_{k=0}^{K-1} \mathbb{E}[1_{(P_{k+1}>D)}] + \beta \sum_{k=0}^{K-1} \mathbb{E}[\sum_{i=1}^{M} \gamma_{i,k}] \right\}$$

where $\beta \geq 0$ is a weighting parameter that takes on the role of a Lagrange multiplier, and specifies the trade-off between the relative importance of total transmit power and outage probability. Note that from (5), P_{k+1} is a function of P_k, g_k, γ_k, while γ_k is assumed to be a function of P_k and g_k. We then have the unconstrained problem

$$\min_{\{\gamma_k\}} \limsup_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \mathbb{E}[l^\beta(P_k, g_k, \gamma_k) | P_0, g_0] \tag{11}$$

where $l^\beta(P_k, g_k, \gamma_k) \equiv 1_{(P_{k+1}>D)} + \beta \sum_{i=1}^{M} \gamma_{i,k}$.

An average cost optimality inequality (ACOI) [24], [25] can be written as

$$\lambda + h(P_k, g_k) \geq \min_{\gamma_k} \left[l^\beta(P_k, g_k, \gamma_k) + \int\limits_{g_{k+1}, P_{k+1}} h(P_{k+1}, g_{k+1}) q(d(P_{k+1}, g_{k+1}) | P_k, g_k, \gamma_k) \right] \tag{12}$$

where λ represents the optimal average cost per stage, h the differential cost vector, and q is the transition law.

Lemma 2: Under Assumptions 3.1 and 3.2, there exists a solution to the average cost optimality inequality (12).

Proof: See Appendix A. ■

Remark 1: To obtain equality in (12) extra conditions such as those in Sec 5.5 of [25] will need to be satisfied, however they seem difficult to verify for our problem.

In order to obtain numerical solutions to (11) we will need to discretize the range of the quantities P_k, $g_k = (g_{1,k}, \ldots, g_{M,k})$ and $\gamma_k = (\gamma_{1,k}, \ldots, \gamma_{M,k})$. Let P_k^d, $g_k^d = (g_{1,k}^d, \ldots, g_{M,k}^d)$, and $\gamma_k^d = (\gamma_{1,k}^d, \ldots, \gamma_{M,k}^d)$ be the discretized versions of P_k, g_k, γ_k respectively. One then has the following problem (13), the solution of which will approximate the solution to (11):

$$\min_{\{\gamma_k^d\}} \limsup_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \mathbb{E}[l^\beta(P_k^d, g_k^d, \gamma_k^d) | P_0^d, g_0^d]. \tag{13}$$

The Bellman equation (average cost optimality equation (ACOE)) associated with problem (13) can then be written as follows, with λ representing the optimal average cost per stage, and h the differential cost vector:

$$\begin{aligned}
\lambda + h(P_k^d, g_k^d) &= \min_{\gamma_k^d} [l^\beta(P_k^d, g_k^d, \gamma_k^d) + \sum_{g_{k+1}^d, P_{k+1}^d} q(P_{k+1}^{rnd}, g_{k+1}^d | P_k^d, g_k^d, \gamma_k^d) h(P_{k+1}^{rnd}, g_{k+1}^d)] \\
&= \min_{\gamma_k^d} [l^\beta(P_k^d, g_k^d, \gamma_k^d) + \sum_{g_{k+1}^d, P_{k+1}^d} p(g_{k+1}^d) q(P_{k+1}^{rnd} | P_k^d, g_k^d, \gamma_k^d) h(P_{k+1}^{rnd}, g_{k+1}^d)] \\
&= \min_{\gamma_k^d} [l^\beta(P_k^d, g_k^d, \gamma_k^d) + \sum_{g_{k+1}^d} p(g_{k+1}^d) h(P_{k+1}^{rnd}, g_{k+1}^d)]
\end{aligned} \tag{14}$$

where P_{k+1}^{rnd} is the value of P_{k+1} (given P_k^d, g_k^d, γ_k^d) rounded to the nearest discretized value, such as in [13]. The last line of (14) holds because P_{k+1}^{rnd} is a deterministic function of P_k^d, g_k^d, γ_k^d, so that $p(P_{k+1}^{rnd} | P_k^d, g_k^d, \gamma_k^d)$ is either 0 or 1.

Now given any two error covariances Σ_1 and Σ_2 satisfying $D_2 \leq \Sigma_1 \leq \sigma_w^2/(1-a^2)$ and $D_2 \leq \Sigma_2 \leq \sigma_w^2/(1-a^2)$, by Assumption 3.1 one can easily construct policies that can take Σ_1 to Σ_2 in a finite number of time steps. We may then use standard results for problems with finite state and action spaces, e.g. [26], to conclude the existence of solutions to the Bellman equation (14). So for the discretized problem an average cost optimality equation will actually be satisfied. In this paper we will obtain solutions to the Bellman equation (14) numerically by using the relative value iteration algorithm, see e.g. [26, Vol I, p.391] and [27, p.373].

After running the relative value iteration algorithm at the fusion center, a “lookup table” will be constructed which will give the optimal power allocation γ_k for different values of the pairs (P_k, g_k). Note that this only has to be done once. With this lookup table constructed, the fusion center can then use knowledge of the actual channel realizations and computed error covariance to find the optimal power allocations for each sensor, which are then fed back to the sensors.

Remark 2: It should be noted that in general a discretized approximation to the original continuous state/action space MDP problem results in a sub-optimal solution. However, it is a widely accepted practice for solving continuous-state MDP problems as well as solving the average cost optimality equalities for partially observed MDP (POMDP) problems, which are converted to a fully observed problem via the information state method. One would generally expect that as the number of discretization levels increases to infinity, the solution to the discretized problem should approach the solution to the original continuous state/action space problem. However, this result is generally not easy to prove. Asymptotic convergence results for various grid based approximations have been proved in the literature both for discounted cost POMDP and average cost MDP/POMDP with various continuity conditions on the MDP cost function [28] as well as the differential cost function in the associated Bellman equation [29] (see also references therein). It remains an open problem however to prove similar asymptotic convergence results in the particular case of the problem studied in our paper as the nature of our cost function does not satisfy all the conditions required by these papers or others available in the literature.

D. Suboptimal policies

The MDP approach of Section III-C is computationally demanding, particularly as the number of sensors increases since the dimensions of g_k and γ_k will increase with each additional sensor. In this section we will consider a simpler power allocation policy, that can be easily implemented even for large numbers of sensors, and whose performance is very close to that obtained from solving the MDP.

The motivation for our suboptimal policy comes from the solution of the information outage minimization problem from communications theory studied in [2]. There, an outage is defined as the event that

$$I_M(g_k, \gamma_k) \equiv \frac{1}{M} \sum_{i=1}^{M} \log(1 + g_{i,k}\gamma_{i,k}) < R \tag{15}$$

for some rate R, where $I_M(g_k, \gamma_k)$ is defined as the instantaneous mutual information. The M in (15) refers to the number of different blocks of an M-block fading channel, rather than different sensors, though the analogies with our situation are apparent. The index k in (15) is used to denote a frame of M blocks.

The problem considered in [2] is then to allocate the power over the M blocks to minimize the outage probability subject to an average power constraint, i.e.

$$\min \Pr(I_M(g_k, \gamma_k) < R) \text{ s.t. } \mathbb{E}\left[\frac{1}{M}\sum_{i=1}^{M}\gamma_{i,k}\right] \leq \mathcal{P}.$$

For continuous fading channels, the solution to this problem involves first solving a sub-problem:

$$\min \frac{1}{M}\sum_{i=1}^{M}\gamma_{i,k} \text{ s.t. } I_M(g_k, \gamma_k) = R, \tag{16}$$

that minimizes the power usage over the M blocks $\frac{1}{M}\sum_{i=1}^{M}\gamma_{i,k}$, subject to the constraint $I_M(g_k, \gamma_k) = R$. If this minimizing sum power is less than a power threshold s^*, then the optimal power allocation is as given by the solution to the sub-problem (16). On the other hand, if the sum power required to solve the sub-problem exceeds the threshold s^*, then the optimal allocation is for transmission to be turned off. The threshold s^* is chosen to be the one that will satisfy the average sum power constraint, and can be determined either analytically in simple cases or via Monte Carlo simulations.

Motivated by this solution, the simple power allocation policy we propose for our problem (7) is the following: Given P_k and g_k, solve the sub-problem that minimizes the sum power subject to the constraint $P_{k+1} = D$. If the required sum power is less than a power threshold s^*, use this power allocation, otherwise don't transmit. The intuition behind this is that for those channel realizations where meeting the condition $P_{k+1} = D$ requires more power than s^*, not transmitting at all will be a more efficient use of the available power since here we have an average or long-term power constraint. Note however that there is a difference with the situation of [2], in that for our problem the quantity P_k is not memoryless. Thus the sub-problem is not always feasible, and it may not always be possible to satisfy $P_{k+1} = D$ in a single time step for arbitrary P_k, depending on which of the conditions of Section III-B the distortion threshold D satisfies.

For those D values satisfying the condition $D \geq D_1$ of Section III-B, the sub-problem is always feasible and the policy just outlined can be applied directly. For the condition $D_2 < D < D_1$ of Section III-B, if the value of P_k is such that $P_{k+1} = D$ cannot be achieved in one time step (i.e. does not satisfy (10)), one should arguably still transmit with some power (since not transmitting will actually cause the error covariance to increase even further) to reduce P_{k+1}, so that in future time steps, i.e. $P_{k+j} = D$ for $j > 1$ can then be achieved. The heuristic we propose in this case is to transmit with sum power equal to ηs^*, using the allocation that minimizes P_{k+1} subject to the constraint $\sum_{i=1}^{M} \gamma_{i,k} = \eta s^*$. Here s^* is the power threshold and η, where $0 \leq \eta \leq 1$, is a constant to be chosen by us. From numerical simulations, we have found that values of η around the range $1/20 - 1/5$ result in very good performance. The intuitive reason is that if η is too large then we tend to use too much power to reduce the error covariance, and if η is too small then the error covariance will not be reduced sufficiently to allow the constraint $P_{k+j} = D$ to be met at future time instances.

To summarize, the proposed suboptimal power allocation policy that covers both the situations $D \geq D_1$ and $D_2 < D < D_1$ is as follows:

- Set s^* and η.
- For $k = 0, 1, \ldots,$ do the following:
- At time k, let $\hat{x}_{k|k-1}$, P_k, g_k be given.
- If $P_{k+1} = D$ can be achieved for this value of P_k (i.e. satisfies (10)), solve the following problem:

$$\min_{\alpha_k^2} \sum_{i=1}^{M} \alpha_{i,k}^2 \left(\frac{c_i^2 \sigma_w^2}{1-a^2} + \sigma_i^2 \right) \text{ s.t. } P_{k+1} = D. \tag{17}$$

  – If the minimizing sum power to problem (17) is less than the threshold s^*, then transmit using this power allocation. Update the state estimate using (4) and update the error covariance as $P_{k+1} = D$.
  – Otherwise set $\alpha_{i,k}^2 = 0, \forall i$. Update the state estimate as $\hat{x}_{k+1|k} = a\hat{x}_{k|k-1}$, and update the error covariance as $P_{k+1} = a^2 P_k + \sigma_w^2$.

- If $P_{k+1} = D$ cannot be achieved for this value of P_k, solve the following problem:

$$\min_{\alpha_k^2} P_{k+1} \text{ s.t. } \sum_{i=1}^{M} \alpha_{i,k}^2 \left(\frac{c_i^2 \sigma_w^2}{1-a^2} + \sigma_i^2 \right) = \eta s^*. \tag{18}$$

  – Transmit using the power allocation provided by the solution to (18). Update the state estimate using (4) and update the error covariance using (5).

The sub-problems (17) and (18) have previously been shown to be convex optimization problems (see [5] and [22]), and furthermore can be solved analytically for any number of sensors. In Appendix B we write down the solutions to these sub-problems.

Remark 3: Determining the threshold s^* analytically is difficult. In practice, given knowledge of the system parameters, one first runs Monte Carlo simulations of the suboptimal policy for different values of s^* to obtain corresponding average sum powers $\mathcal{P}$. By forming a plot of these pairs $(s^*, \mathcal{P})$ we can then graphically estimate the value of s^* one should use in order to achieve a given average sum power usage $\mathcal{P}$.

Remark 4: In the analytical solutions to the optimization problems (17) and (18), it turns out that even when the sensors are transmitting, there could still be some sensors which are inactive [5], due to the transmission over orthogonal channels. In the context of problems (17) and (18), the M_1 sensors (see Appendix B) that are active are the ones with the largest values of $\frac{g_{i,k}}{\sigma_w^2/(1-a^2)+\sigma_i^2/c_i^2}$, which clearly favours the sensors with better channels and higher measurement quality, see also [22].

Remark 5: Unlike the optimal solution of Section III-C, the optimization problems (17) and (18) involved in the suboptimal policy can be solved in a distributed manner. The fusion center can compute and broadcast the quantity λ_k (the quantity λ in Appendix B) to all sensors, which can then determine their optimal $\alpha_{i,k}$'s using λ_k and their local information, see [5]. Note though that due to time-varying channel gains, the quantity λ_k will vary with the time k, so the broadcasting will need to be done at every time step.

E. Outage minimization with a finite number of power levels

In this section we wish to study the outage minimization problem assuming a fixed number of power levels, which has practical significance since in practice sensors can usually only transmit using a finite number d of different powers.[3] Now for a given set of power levels, the outage minimization problem can be solved by solving the MDP problem (13) of Section III-C. Here however we also wish to optimize the values of these power levels. A similar problem of finding the optimal quantization thresholds for HMM state estimation was studied in [14]. Below we will outline the procedure for our problem.

Recall the Lagrangian L^β, and let $L^{\beta *}(\Gamma)$ be the optimal value found by solving the MDP (13), with $\Gamma \in \mathbb{R}^d$ representing the given finite set of d possible power levels. The problem we wish to solve is

$$\min_{\Gamma \in \mathbb{R}^d} L^{\beta *}(\Gamma) \tag{19}$$

i.e. we want to find the optimal set of power levels Γ.

Using the optimal power allocation given by the numerical solution to the MDP for a given set of power levels Γ, Monte Carlo simulations of $\frac{1}{K}\left\{\sum_{k=0}^{K-1}\mathbb{E}[1_{(P_{k+1}>D)}]+\beta\sum_{k=0}^{K-1}\mathbb{E}[\sum_{i=1}^{M}\gamma_{i,k}]\right\}$ can be regarded as a noisy measurement of the function $L^{\beta *}(\Gamma)$. Hence problem (19) can be viewed as a stochastic optimization problem.

[3]To keep the notations simple, we assume that all sensors use the same set of power levels.

These problems can be solved using well-known gradient-free stochastic optimization algorithms such as the Kiefer-Wolfowitz procedure [30], or more recent techniques such as the simultaneous perturbation stochastic approximation (SPSA) algorithm [31]. We will use the SPSA algorithm in our numerical studies in this paper.

F. Numerical studies

1) Single sensor: Consider first an example with $a = 0.8$, $c_1 = 1$, $\sigma_1^2 = 1$, $\sigma_w^2 = \sigma_n^2 = 1$. With these parameters the quantities D_1 and D_2 from Section III-B have values $D_1 = 1.4706$, $D_2 = 1.3700$. Also, $\sigma_w^2/(1-a^2) = 2.7778$. The fading channel is assumed to be Rayleigh, with g_k being exponentially distributed with mean 1, denoted by $g_k \sim \exp(1)$.

Figure 2 plots the outage probability and average power obtained from the MDP solution, for various D values. We use 100 discretization points for each of the quantities P_k, g_k, γ_k. We discretize P_k over the range D_2 to $\sigma_w^2/(1-a^2)$, and g_k over the range 0 to 15. The discretization range for the power γ_k is from 0 to γ_{max}, where γ_{max} varies for different average power/outage probability requirements. As a rule of thumb we took γ_{max} to be around twice the maximum power s^* used in the suboptimal policy, for a similar average power/outage probability trade-off. The relative value iteration algorithm is run for 20 iterations in solving (14) for each value of the weighting parameter β. We see from Figure 2 that smaller D values require more power to be transmitted for a given outage probability.

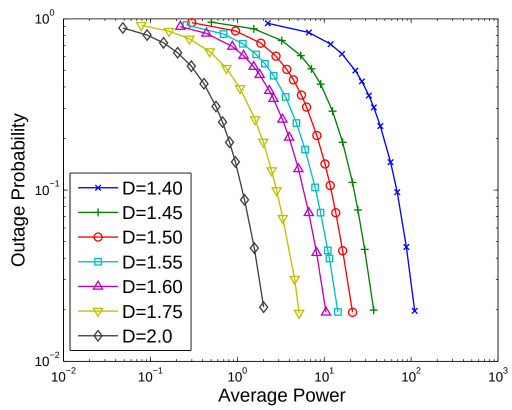

Fig. 2. Outage probability and average power for various D values.

We next compare the performance of the suboptimal policy with the MDP solution. Figure 3 plots the outage probability and average power obtained from the MDP solution and suboptimal policy, for $D = 2.0$ and $D = 1.4$. For $D = 2.0$, since $2.0 > 1.4706 = D_1$, this is the case where $P_{k+1} = D$ can always be achieved in one time step. For $D = 1.4$ we have $D_2 \leq D \leq D_1$, and we will use $\eta = 1/5$ for the suboptimal policy. In both plots it can be seen that the suboptimal policy gives very close performance to the solution obtained by solving the MDP.

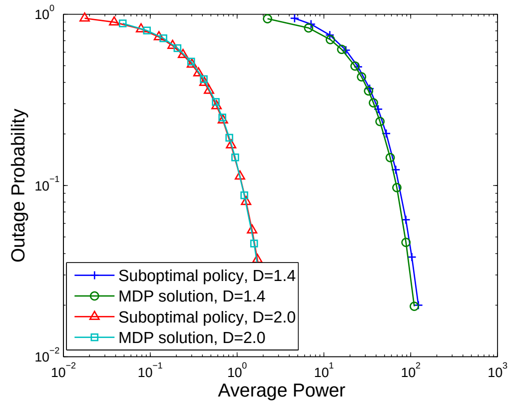

Fig. 3. Outage probability and average power for MDP and suboptimal policy, using $\eta = 1/5$

To provide some insight into why the suboptimal policy performs so well, in Figure 4 we plot for $D = 2.0$ and $P_k = 2.28$, the power allocation obtained from solving the MDP as a function of g_k, together with the corresponding value of P_{k+1} when using this power allocation. For values of g_k less than around 5, the power allocation is such

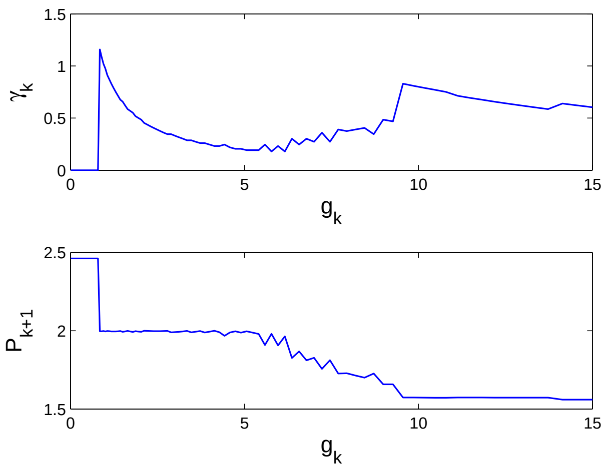

Fig. 4. Power allocations obtained from MDP solution, for a fixed P_k

that $P_{k+1} = D = 2.0$ is met provided the power required is less than some threshold, which corresponds to the behaviour of the suboptimal policy. Since $\Pr(g_k > 5) = \exp(-5) \approx 6.74 \times 10^{-3}$ is quite small, we see that most of the time the MDP solution behaves like the suboptimal policy. For values of g_k greater than 5, the power allocated is more than that required to satisfy $P_{k+1} = D$, until around values of g_k greater than 10, where the power allocated makes $P_{k+1} \approx 1.5625$. We notice that $a^2 \times 1.5625 + \sigma_w^2 = 2.0$, so the value of $P_{k+1} = 1.5625$ implies that $P_{k+2} = D$ will be achieved even without the sensor transmitting anything at time $k+1$. This qualitative behaviour in the power allocation functions obtained from the MDP solution has also been observed for other values of P_k.

2) Multiple sensors: We now consider a two sensor example with $a = 0.8$, $c_1 = 1, c_2 = 1$, $\sigma_1^2 = 1, \sigma_2^2 = 2$, $\sigma_w^2 = \sigma_n^2 = 1$, $g_1 \sim \exp(1)$, $g_2 \sim \exp(1)$. With these parameters the quantities D_1 and D_2 now have the values $D_1 = 1.3441$, $D_2 = 1.2806$. Due to the increase in computational complexity, we now use 20 discretization points for each of the quantities P_k, $g_{1,k}$, $g_{2,k}$, $\gamma_{1,k}$, $\gamma_{2,k}$ here when solving the MDP. Figure 5 plots the outage probability and average sum power obtained from the MDP solution and suboptimal policy using $\eta = 1/5$, for a distortion $D = 1.3$. Again the two graphs are very close to each other.

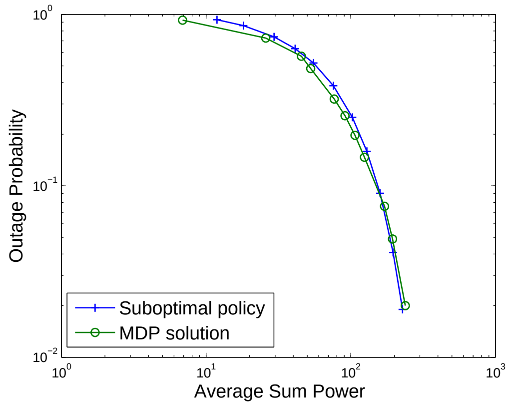

Fig. 5. Two sensor case. Outage probability and average sum power for MDP and suboptimal policy using $\eta = 1/5$, with $D = 1.3$.

We next consider the effect of increasing the number of sensors M. For simplicity we consider a "symmetric" situation with $a = 0.8$, $\sigma_w^2 = \sigma_n^2 = 1$, $c_i = 1, i = 1, \dots, M, \sigma_i^2 = 1, g_i \sim \exp(1), i = 1, \dots, M$. We use the distortion threshold $D = 1.5$. As solving the MDP is prohibitively expensive computationally for $M > 2$, we will only present the results for the sub-optimal policy, which can be easily generated. Figure 6 plots the outage probability and average sum power for this situation, where we can readily see the outage performance improvements from using multiple sensors.

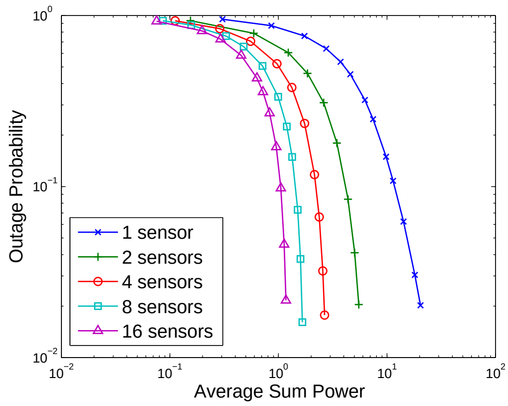

Fig. 6. Outage probability and average sum power for different numbers of sensors, using the sub-optimal policy

We will also look at how often sensors will transmit under the suboptimal policy. We again consider the symmetric situation with $a = 0.8$, $\sigma_w^2 = \sigma_n^2 = 1$, $c_i = 1, i = 1, \dots, M, \sigma_i^2 = 1, g_i \sim \exp(1), i = 1, \dots, M$. Fixing $D = 1.5$ and the outage probability to be around 0.1, in Figure 7 we plot the percentage of sensors that are active for different numbers of sensors (taking into account the periods where no sensors transmit in the suboptimal policy), where the percentage is averaged over a time horizon of 500000. We see that the percentage of active sensors decreases as M increases. This is due to the fact that with more sensors we are more likely to find sensors with good channels so that the condition $P_{k+1} = D$ can be met with a smaller percentage of sensors. Next we fix $D = 1.1$ and $M = 100$,

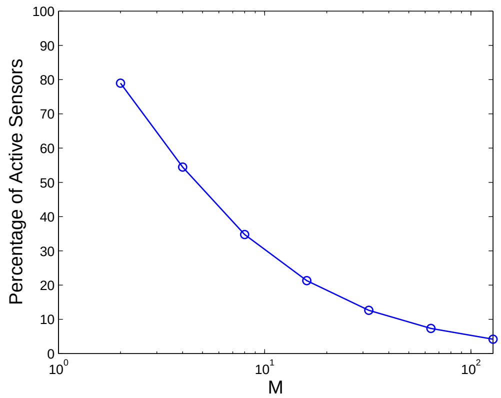

Fig. 7. Percentage of active sensors for different numbers of sensors using the sub-optimal policy

and in Figure 8 we plot the percentage of sensors that are active as the average sum power varies. As the available transmit power increases, the percentage of active sensors increases, similar to what has been observed in [5].

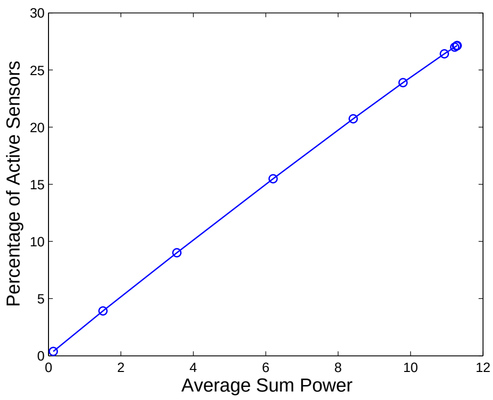

Fig. 8. Percentage of active sensors for different average sum powers using the sub-optimal policy

3) Finite number of power levels: We consider the effect of using a finite number of power levels, for the single sensor case. Figure 9 compares the performance using "continuous" power levels (though for numerical computation the range is actually discretized into 100 power levels) and different schemes using 4 power levels, which may possibly include zero. The system parameters are $a = 0.8$, $c_1 = 1$, $\sigma_1^2 = 1$, $\sigma_w^2 = \sigma_n^2 = 1$, $g_1 \sim \exp(1)$ and $D = 1.6$. We show first the performance using powers that are exponentially spaced as $\exp(i\Delta) - 1, i = 0, \dots, 3$,

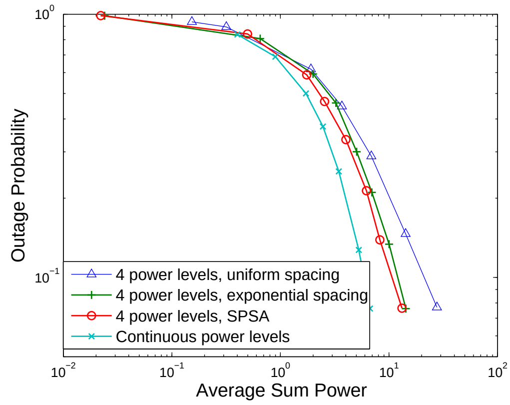

Fig. 9. Outage probability and average power using 4 power levels.

with $\Delta = \log(\gamma_{max} + 1)/3$, where γ_{max} is chosen to be around 10 times the average power in the continuous power case. We also plot the performance using powers that are uniformly spaced from 0 to γ_{max}. It can be seen that the exponential spacing appears to give better performance. Using these exponentially spaced powers as initial conditions, we then ran the simultaneous perturbation stochastic approximation (SPSA) algorithm [31] to further optimize the choice of powers. We followed the guidelines for selecting the SPSA algorithm parameters in [32]. Specifically, we chose (using the same symbols as those in [32]) $\alpha_{SA} = 0.602, \gamma_{SA} = 0.101, c_{SA} = 0.01, A_{SA} = 10$, and a_{SA} such that $a_{SA}/(A_{SA}+1)^{\alpha_{SA}} \times \hat{g}_0^{SA}(\hat{\theta}_0^{SA})$ is approximately equal to 0.02. The SPSA algorithm was then run for 1000 iterations. The performance using these optimized values is then simulated. It can be seen that there is a slight gain to be had from further optimizing the choice of powers.

IV. Unstable systems

In this section we will consider the outage minimization problem for unstable systems. There are many applications where unstable systems are used to model the behaviour of systems over a *finite* time scale such as in target tracking [33] and control theory [34]. In these cases, we will be interested in finite horizon results for unstable systems where the system states and measurements can take on large values but are still bounded.

Since for unstable systems meeting the outage constraints requires increasingly large amounts of power as the time increases, the infinite horizon problem stated by (7) is not appropriate. Instead we will present a different formulation of the outage minimization problem, namely a finite horizon version of problem (7).[4]

Instead of Assumption 3.1, for unstable systems we will make the following slightly different assumption:

Assumption 4.1: D satisfies the condition $D > D_2$, where D_2 is defined by (9).

[4]This finite horizon formulation can also be used in the case of stable systems if desired.

A. Finite horizon formulation

For the finite horizon formulation, instead of minimizing the long run averages as in problem (7), we instead are only interested in outage minimization over a finite time horizon. We can write this problem as

$$\min_{\{\gamma_k\}} \frac{1}{K}\sum_{k=0}^{K-1} \mathbb{E}[1_{(P_{k+1}>D)}] \text{ s.t. } \frac{1}{K}\sum_{k=0}^{K-1} \mathbb{E}[\sum_{i=1}^{M} \gamma_{i,k}] \leq \mathcal{P} \tag{20}$$

where K is the finite horizon over which we wish to solve the problem. The sensor transmit powers are as defined in (6), i.e. $\gamma_{i,k} = \alpha_{i,k}^2 (c_i^2 \mathbb{E}[x_k^2] + \sigma_i^2)$, except that for unstable systems $\mathbb{E}[x_k^2]$ is now time-varying, and given by

$$\mathbb{E}[x_k^2] = a^{2k} P_0 + (a^{2k-2} + \cdots + a^2 + 1)\sigma_w^2 = a^{2k} P_0 + \frac{(a^{2k}-1)\sigma_w^2}{a^2-1},\ k = 1, \ldots, K-1, \tag{21}$$

with initial covariance $\mathbb{E}[x_0^2] = P_0$. Introducing the Lagrangian

$$L^{\beta,K} = \frac{1}{K}\left\{\sum_{k=0}^{K-1} \mathbb{E}[1_{(P_{k+1}>D)}] + \beta \sum_{k=0}^{K-1} \mathbb{E}[\sum_{i=1}^{M} \gamma_{i,k}]\right\},$$

we now wish to solve the unconstrained problem

$$\min_{\{\gamma_k\}} \frac{1}{K}\sum_{k=0}^{K-1} \mathbb{E}[l_k^\beta(P_k, g_k, \gamma_k)|P_0, g_0] \tag{22}$$

where $l_k^\beta(P_k, g_k, \gamma_k) \equiv 1_{(P_{k+1}>D)} + \beta \sum_{i=1}^M \gamma_{i,k}$. The discretized version of problem (22) may then be solved numerically using the standard dynamic programming algorithm. We briefly state the algorithm below.

1) Set $J_K(P_K^d, g_K^d) = 0, \forall (P_K^d, g_K^d)$.

2) For $k = K-1, \ldots, 0$, set

$$\begin{aligned} J_k(P_k^d, g_k^d) &= \min_{\gamma_k^d} [l_k^\beta(P_k^d, g_k^d, \gamma_k^d) + \sum_{g_{k+1}^d} p(g_{k+1}^d) J_{k+1}(P_{k+1}^{rnd}, g_{k+1}^d)] \\ \gamma_k^*(P_k^d, g_k^d) &= \arg\min_{\gamma_k^d} [l_k^\beta(P_k^d, g_k^d, \gamma_k^d) + \sum_{g_{k+1}^d} p(g_{k+1}^d) J_{k+1}(P_{k+1}^{rnd}, g_{k+1}^d)] \end{aligned} \tag{23}$$

where (23) is derived in a similar manner to (14).

B. Suboptimal policy

The suboptimal policy of Section III-D can also be applied to the finite horizon problem (22), with slight modifications due to the difference in expression for $\mathbb{E}[x_k^2]$. This is stated below.

- Set s^* and η.
- For $k = 0, 1, \ldots, K-1$, do the following:
- At time k, let $\hat{x}_{k|k-1}$, P_k, g_k be given.
- If $P_{k+1} = D$ can be achieved for this value of P_k (i.e. satisfies (10)), solve the following problem:

$$\min_{\alpha_k^2} \sum_{i=1}^M \alpha_{i,k}^2 \left(c_i^2 \left(a^{2k} P_0 + \frac{(a^{2k}-1)\sigma_w^2}{a^2-1} \right) + \sigma_i^2 \right) \text{ s.t. } P_{k+1} = D. \tag{24}$$

- If the minimizing sum power to problem (24) is less than the threshold s^*, then transmit using this power allocation. Update the state estimate using (4) and update the error covariance as $P_{k+1} = D$.
- Otherwise set $\alpha_{i,k} = 0, \forall i$. Update the state estimate as $\hat{x}_{k+1|k} = a\hat{x}_{k|k-1}$ and update the error covariance as $P_{k+1} = a^2 P_k + \sigma_w^2$.

- If $P_{k+1} = D$ cannot be achieved for this value of P_k, solve the following problem:

$$\min_{\alpha_k^2} P_{k+1} \text{ s.t. } \sum_{i=1}^{M} \alpha_{i,k}^2 \left(c_i^2 \left(a^{2k} P_0 + \frac{(a^{2k}-1)\sigma_w^2}{a^2-1} \right) + \sigma_i^2 \right) = \eta s^*. \tag{25}$$

  - Transmit using the power allocation provided by the solution to (25). Update the state estimate using (4) and update the error covariance using (5).

The optimization problems (24) and (25) can also be solved analytically, similar to problems (17) and (18), see Appendix B.

C. Outage minimization with a finite number of power levels

As in the stable system case, we can also consider the outage minimization problem using only a finite number of power levels, while also optimizing over the values of these powers. The techniques are very similar to those of Section III-E and are omitted for brevity.

D. Numerical studies

We first present numerical results for the single sensor situation. In Figure 10 we plot the outage probability and average power for various different D values, while keeping the horizon $K = 4$ fixed. We used $a = 1.1$, $\sigma_w^2 = \sigma_n^2 = \sigma_1^2 = 1$, $c_1 = 1$, $g_1 \sim \exp(1)$. The initial covariance P_0 is set to the same value of D being used. Similar to Figure 2, smaller D values will require more power to be transmitted for a given outage probability.

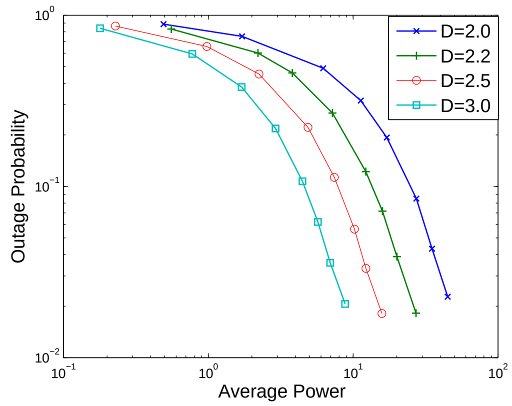

Fig. 10. Outage probability and average power for various D values, with $K = 4$.

In Figure 11 we plot the outage probability and average power for various different horizons, while keeping $D = 2.5$ fixed. We again used $a = 1.1$, $\sigma_w^2 = \sigma_n^2 = \sigma_1^2 = 1$, $c_1 = 1$, $g_1 \sim \exp(1)$ with initial covariance $P_0 = 2.5$. We can see that for a given outage probability, it will require more power (averaged over the entire horizon) to be transmitted as the horizon is increased. This agrees with the intuition that it requires increasingly large amounts of power to meet the outage requirements as time increases.

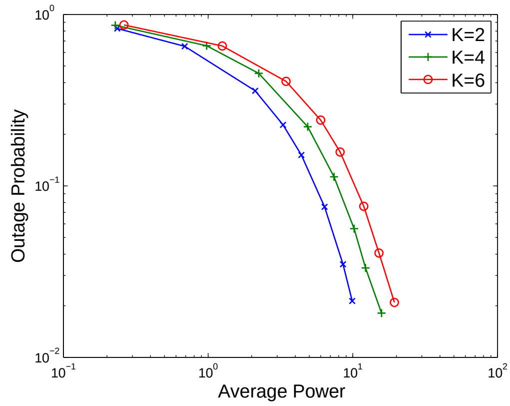

Fig. 11. Outage probability and average power for different finite horizons K, with $D = 2.5$.

In Figure 12 we compare the performance of the solution obtained by dynamic programming and the suboptimal policy. We used $a = 1.1$, $\sigma_w^2 = \sigma_n^2 = \sigma_1^2 = 1$, $c_1 = 1$, $g_1 \sim \exp(1)$ and two different values for D. For the suboptimal policy we used the value $\eta = 1/5$. As in the case of stable systems, the performance of the suboptimal policy is again very close to that of the optimal policy.

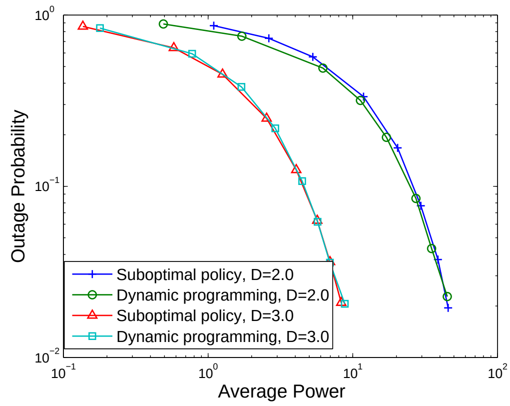

Fig. 12. Outage probability and average power for dynamic programming solution and suboptimal policy, using $\eta = 1/5$.

We next consider the effect of increasing the number of sensors M. We consider the symmetric situation with $a = 1.1$, $\sigma_w^2 = \sigma_n^2 = 1$, $c_i = 1, i = 1, \ldots, M, \sigma_i^2 = 1, g_i \sim \exp(1), i = 1, \ldots, M$. The fading channels are all taken to be Rayleigh, the finite horizon is $K = 4$, and we let the distortion threshold $D = 2.0$. Figure 13 plots the outage probability and average sum power for this situation, where the results are obtained using the sub-optimal policy, with similar interpretations as in the case of stable systems.

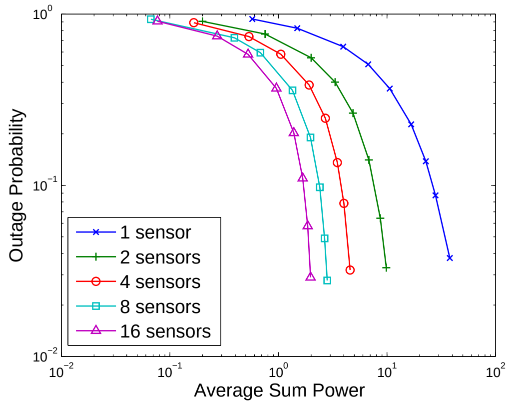

Fig. 13. Outage probability and average sum power for different numbers of sensors, using the sub-optimal policy

V. Vector systems

In this section we will describe a possible problem formulation of the outage minimization problem to vector systems. For notational simplicity, we will restrict ourselves to vector state, scalar measurement (per sensor) systems, though this can be extended to vector measurements where e.g. sensors transmit each component of their measurement vector to the fusion center separately. The linear system is now given by

$$x_{k+1} = Ax_k + w_k$$

with $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n \times n}$, and $w_k \in \mathbb{R}^n$ being i.i.d. Gaussian with zero-mean and covariance matrix Σ_w. The measurements at the sensors are taken to be scalar, so that

$$y_{i,k} = c_i x_k + v_{i,k}, i = 1, \dots, M$$

with $y_{i,k} \in \mathbb{R}$, $c_i \in \mathbb{R}^{1 \times n}$, and $v_{i,k} \in \mathbb{R}$ being i.i.d. Gaussian with zero-mean and variance σ_i^2. As in (2), under the orthogonal analog forwarding scheme the received signals at the fusion centre can be written as

$$z_{i,k} = \alpha_{i,k}\sqrt{g_{i,k}} c_i x_k + \alpha_{i,k}\sqrt{g_{i,k}} v_k + n_{i,k}, i = 1, \dots, M.$$

We define $z_k = (z_{1,k}, \dots, z_{M,k})^T, g_k = (g_{1,k}, \dots, g_{M,k})^T$, $\bar{C}_k = [\alpha_{1,k}\sqrt{g_{1,k}} c_1^T | \dots | \alpha_{M,k}\sqrt{g_{M,k}} c_M^T]^T$, $\bar{v}_k = (\alpha_{1,k}\sqrt{g_{1,k}} v_{1,k} + n_{1,k}, \dots, \alpha_{M,k}\sqrt{g_{M,k}} v_{M,k} + n_{M,k})^T$, $\bar{R}_k = diag(\alpha_{1,k}^2 g_{1,k}\sigma_1^2 + \sigma_n^2, \dots, \alpha_{M,k}^2 g_{M,k}\sigma_M^2 + \sigma_n^2)$. Then the state estimate satisfies

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k-1} + AP_{k|k-1}\bar{C}_k^T(\bar{C}_k P_k \bar{C}_k^T + \bar{R}_k)^{-1}(z_k - \bar{C}_k \hat{x}_{k|k-1}) \tag{26}$$

and the error covariance matrix satisfies

$$P_{k+1} = AP_k A^T - AP_k \bar{C}_k^T(\bar{C}_k P_k \bar{C}_k^T + \bar{R}_k)^{-1}\bar{C}_k P_k A^T + \Sigma_w. \tag{27}$$

The sensor transmit power is defined as $\gamma_{i,k} = \alpha_{i,k}^2 \mathbb{E}[y_{i,k}^2] = \alpha_{i,k}^2 (c_i \mathbb{E}[x_k x_k^T] c_i^T + \sigma_i^2)$, where $\mathbb{E}[x_k x_k^T]$ satisfies the Lyapnuov equation $\mathbb{E}[x_k x_k^T] - A\mathbb{E}[x_k x_k^T]A^T = \Sigma_w$ and can be determined numerically.

We now extend the estimation outage notion to vector systems, to be the event that $\text{Tr}(P_{k+1}) > D$, with $\text{Tr}(.)$ denoting the trace.

A. Stable systems

For stable systems, the outage minimization problem can then be expressed as

$$\min_{\{\gamma_k\}} \limsup_{K\to\infty} \frac{1}{K}\sum_{k=0}^{K-1} \mathbb{E}[1_{(\text{Tr}(P_{k+1})>D)}] \text{ s.t. } \limsup_{K\to\infty} \frac{1}{K}\sum_{k=0}^{K-1} \mathbb{E}[\sum_{i=1}^{M} \gamma_{i,k}] \leq \mathcal{P}. \tag{28}$$

As in the scalar case, we can use the Lagrangian technique to turn (28) into an unconstrained problem

$$\min_{\{\gamma_k\}} \limsup_{K\to\infty} \frac{1}{K}\sum_{k=0}^{K-1} \mathbb{E}[l^\beta(P_k, g_k, \gamma_k)|P_0, g_0]$$

with $l^\beta(P_k, g_k, \gamma_k) \equiv 1_{(\text{Tr}(P_{k+1})>D)} + \beta \sum_{i=1}^M \gamma_{i,k}$. The associated Bellman equation can be derived in a similar manner to (14) to be

$$\lambda + h(P_k^d, g_k^d) = \min_{\gamma_k^d} [l^\beta(P_k^d, g_k^d, \gamma_k^d) + \sum_{g_{k+1}^d} p(g_{k+1}^d) h(P_{k+1}^{rnd}, g_{k+1}^d)] \tag{29}$$

where P_{k+1}^{rnd} is the matrix P_{k+1} with each entry rounded to the nearest discretized value, while also ensuring that the positive semidefinite nature is retained. Numerical solution of (29) in the vector case will be more demanding computationally, since we now have to discretize individually the entries of P_k when it is a matrix. However since P_k is symmetric, we only need to do this for e.g. the upper triangular entries of the matrix.

An extension of the suboptimal scheme of Section III-D is as follows:

- Set s^* and η.
- For $k = 0, 1, \ldots,$ do the following:
- At time k, let $\hat{x}_{k|k-1}$, P_k, g_k be given.
- If $\text{Tr}(P_{k+1}) = D$ can be achieved for this value of P_k,[5] solve the following problem:

$$\min_{\alpha_k^2} \sum_{i=1}^{M} \alpha_{i,k}^2 (c_i \mathbb{E}[x_k x_k^T] c_i^T + \sigma_i^2) \text{ s.t. } \text{Tr}(P_{k+1}) = D. \tag{30}$$

  - If the minimizing sum power is less than the threshold s^*, then transmit using this power allocation. Update the state estimate using (26) and update the error covariance matrix using (27).
  - Otherwise set $\alpha_{i,k}^2 = 0, \forall i$. Update the state estimate as $\hat{x}_{k+1|k} = A\hat{x}_{k|k-1}$ and update the error covariance matrix as $P_{k+1} = AP_kA^T + \Sigma_w$.

[5]Define $\tilde{C} = [c_1^T | \ldots | c_M^T]$, $\tilde{R} = diag(\sigma_1^2, \ldots, \sigma_M^2)$. Then this condition in the vector case corresponds to $\text{Tr}(AP_kA^T - AP_k\tilde{C}^T(\tilde{C}P_k\tilde{C}^T + \tilde{R})^{-1}\tilde{C}P_kA^T + \Sigma_w) \leq D$.

- If $\mathrm{Tr}(P_{k+1}) = D$ cannot be achieved for this value of P_k, solve the following problem:

$$\min_{\alpha_k^2} \mathrm{Tr}(P_{k+1}) \text{ s.t. } \sum_{i=1}^{M} \alpha_{i,k}^2 (c_i \mathbb{E}[x_k x_k^T] c_i^T + \sigma_i^2) = \eta s^*. \tag{31}$$

  - Transmit using the power allocation provided by the solution to (31). Update the state estimate using (26) and update the error covariance matrix using (27).

Unlike the scalar case, problems (30) and (31) are in general non-convex [35]. In the numerical studies of Section V-C we have solved problems (30) and (31) to obtain local minima using the MATLAB routine `fmincon`.

B. Unstable systems

For unstable systems, similar problem formulations which extend those of Section IV can be studied, but will be omitted for brevity.

C. Numerical studies

We consider first a single sensor situation with $A = \begin{bmatrix} 0.8 & 0.1 \\ 0.1 & 0.8 \end{bmatrix}$, $c_1 = [\ 1 \quad 1\ ]$, $\Sigma_w = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$, $\sigma_1^2 = 1$, $\sigma_n^2 = 1$, $g_1 \sim \exp(1)$, and we set $D = 3.0$. Figure 14 compares the solution obtained by solving the MDP with the suboptimal policy using $\eta = 1/5$. The performance of the suboptimal policy is very close to the MDP solution.

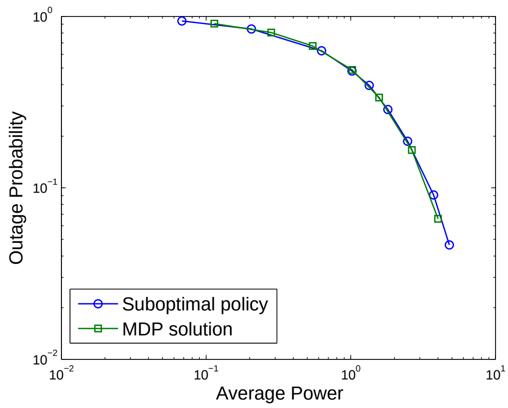

Fig. 14. Outage probability and average power comparison between MDP solution and suboptimal solution: Vector state, single sensor

Next we present in Figure 15 results for the multi-sensor situation using the suboptimal policy. The parameters are $A = \begin{bmatrix} 0.8 & 0.1 \\ 0.1 & 0.8 \end{bmatrix}$, $c_i = [\ 1 \quad 1\ ]$, $\Sigma_w = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$, $\sigma_i^2 = 1, g_i \sim \exp(1), i = 1, \ldots, M$, $\sigma_n^2 = 1$. We set $D = 3.0$ and use $\eta = 1/5$ in the suboptimal policy.

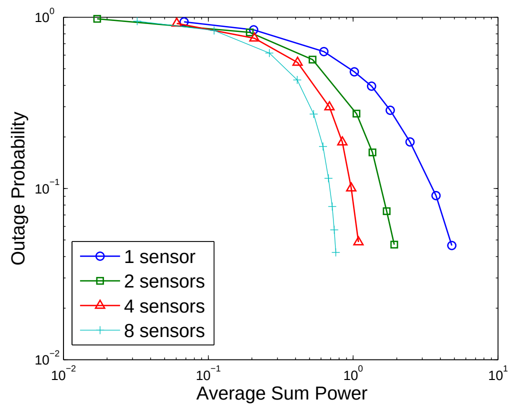

Fig. 15. Outage probabiility and average sum power for different numbers of sensors: Vector state

VI. Minimization of Expected Error Covariance

The methods we have used in this paper can be adapted to minimize other cost functions. For instance, one possibility is to minimize the expected error covariance subject to average power constraints. For simplicity, only scalar systems are considered in this section.

A. Stable systems

For the case of stable systems and an infinite horizon formulation, this problem can be written as

$$\min_{\{\gamma_k\}} \limsup_{K\to\infty} \frac{1}{K}\sum_{k=0}^{K-1} \mathbb{E}[P_{k+1}] \text{ s.t. } \limsup_{K\to\infty} \frac{1}{K}\sum_{k=0}^{K-1} \mathbb{E}[\sum_{i=1}^{M} \gamma_{i,k}] \leq \mathcal{P}. \tag{32}$$

Using the Lagrangian technique we obtain the unconstrained problem

$$\min_{\{\gamma_k\}} \limsup_{K\to\infty} \frac{1}{K}\sum_{k=0}^{K-1} \mathbb{E}[\breve{l}^\beta(P_k, g_k, \gamma_k)|P_0, g_0]$$

with $\breve{l}^\beta(P_k, g_k, \gamma_k) \equiv P_{k+1} + \beta \sum_{i=1}^M \gamma_{i,k}$. The average cost optimality equation (ACOE) can be written as

$$\lambda + h(P_k, g_k) = \min_{\gamma_k} \left[\breve{l}^\beta(P_k, g_k, \gamma_k) + \int_{g_{k+1},P_{k+1}} h(P_{k+1}, g_{k+1}) q(d(P_{k+1}, g_{k+1})|P_k, g_k, \gamma_k) \right] \tag{33}$$

which is very similar to (12) with the main difference being in the definition of $\breve{l}^\beta(P_k, g_k, \gamma_k)$. We have the following:

Lemma 3: Under Assumption 3.2, there exists a solution to the average cost optimality equation (33).

Proof: Existence of a solution to the average cost optimality inequality (with $\geq$ instead of equality in (33)) can be shown similar to Appendix A for the outage minimization problem. Furthermore, equality in (33) can also be shown, by making use of: 1) the exponential forgetting property for the initial conditions in Kalman filtering, 2) the Lipschitz continuity of the cost function $\breve{l}^\beta(\cdot,\cdot,\cdot)$, and 3) repeating the argument in the proof of Proposition 3.2 of [13]. The assumptions in Sections 5.4 and 5.5 of [25] can then be verified to conclude the existence of a solution to the ACOE. ■

As in the outage minimization problem, to obtain numerical solutions we will solve a discretized version of the ACOE (33), i.e.

$$\lambda + h(P_k^d, g_k^d) = \min_{\gamma_k^d}[\breve{l}^\beta(P_k^d, g_k^d, \gamma_k^d) + \sum_{g_{k+1}^d} p(g_{k+1}^d) h(P_{k+1}^{rnd}, g_{k+1}^d)]$$

which is derived in a similar manner to (14).

For comparison, let us also consider a simpler sub-optimal scheme for problem (32), which could be considered as a "greedy" approach that solves at each time step k the following problem:

$$\min_{\gamma_k} P_{k+1} \text{ s.t. } \sum_{i=1}^{M} \gamma_{i,k} = \mathcal{P}. \tag{34}$$

That is, at each time step we minimize the error covariance P_{k+1}, while meeting the sum power constraint with equality. This problem has been previously studied, see [22] for further details.

B. Unstable systems

In the case of unstable systems, the finite horizon formulation of the problem can be written as

$$\min_{\{\gamma_k\}} \frac{1}{K} \sum_{k=0}^{K-1} \mathbb{E}[P_{k+1}] \text{ s.t. } \frac{1}{K} \sum_{k=0}^{K-1} \mathbb{E}[\sum_{i=1}^{M} \gamma_{i,k}] \leq \mathcal{P}. \tag{35}$$

Using the Lagrangian technique, the unconstrained problem we obtain is

$$\min_{\{\gamma_k\}} \frac{1}{K} \sum_{k=0}^{K-1} \mathbb{E}[\breve{l}_k^\beta(P_k, g_k, \gamma_k) | P_0, g_0] \tag{36}$$

with $\breve{l}^\beta(P_k, g_k, \gamma_k) \equiv P_{k+1} + \beta \sum_{i=1}^{M} \gamma_{i,k}$. Similar to the outage minimization problem, a discretized version of (36) can be numerically solved with the dynamic programming algorithm.

We will also compare this solution with a greedy approach, where we now will solve (34) over the times $k = 0, \ldots, K-1$.

C. Numerical studies

In Figure 16 we plot the average error covariance and average power comparison using these two approaches, for stable systems. The parameters are $a = 0.8$, $\sigma_w^2 = \sigma_n^2 = \sigma_1^2 = 1$, $c_1 = 1$, $g_1 \sim \exp(1)$. For higher average powers, the performance of the greedy solution approaches very closely the performance of the optimal MDP solution.

In Figure 17 we plot the comparison for unstable systems. The parameters are $a = 1.2$, $\sigma_w^2 = \sigma_n^2 = \sigma_1^2 = 1$, $c_1 = 1$, $g_1 \sim \exp(1)$, $K = 5$, and initial covariance $P_0 = 3$. Similarly, the performance of the greedy approach is very close to that of the optimal dynamic programming solution for higher average powers.

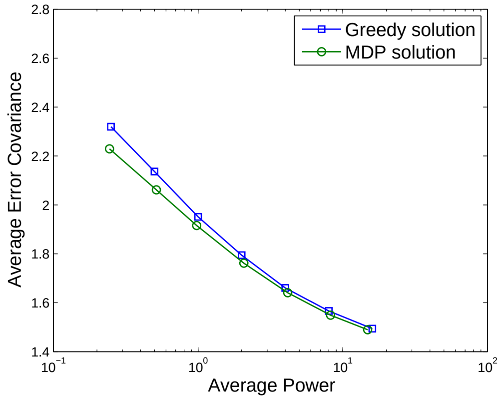

Fig. 16. Average error covariance and average power comparison between MDP solution and suboptimal greedy solution: Stable systems

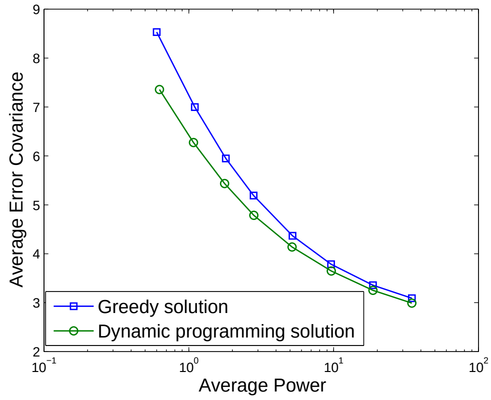

Fig. 17. Average error covariance and average power comparison between MDP solution and suboptimal greedy solution: Unstable systems

VII. Conclusions and extensions

We have considered the estimation outage minimization problem for state estimation of linear systems. For stable systems we used an infinite horizon problem formulation and for unstable systems we used a finite horizon formulation. Suboptimal policies were presented which gave very close to optimal performance, and optimization of powers assuming a finite number of power levels was also studied.

Extensions of the outage concept to control problems over fading channels will be a topic of future investigation, as will be more detailed investigation into the vector case. In addition, typical information theoretic notions such as the diversity order of the outage probability (how fast does the outage probability decay with the number of sensors) will be investigated for the estimation outage scenario in future work.

Appendix

A. Proof of Lemma 2

We will use the conditions (W) and (B) of [24] that will guarantee the existence of a solution to the ACOI (similar conditions can also be found in [25]). Call the state space $\mathcal{S}$ and action space $\mathcal{A}$, i.e. $(P_k, g_k) \in \mathcal{S}, \gamma_k \in \mathcal{A}$. We first give condition (W) of [24], which says that:

(0) The state space $\mathcal{S}$ is locally compact.

(1) Let $U(.)$ be the mapping that assigns to each (P_k, g_k) the nonempty set of available actions. Then $U(P_k, g_k)$ lies in a compact subset of $\mathcal{A}$ and $U(.)$ is upper semicontinuous.

(2) The transition probabilities are weakly continuous.

(3) l^β is lower semicontinuous.

By Assumption 3.2, (0) and (1) of (W) can be easily verified. For (2) note that P_{k+1} is a continuous function of (P_k, g_k, γ_k), which then shows weak continuity by p.177 of [25]. For (3), it can be easily shown that $1_{(P_{k+1}>D)}$ is lower semicontinuous, and $\beta \sum_{i=1}^M \gamma_{i,k}$ is clearly a continuous function, so that $l^\beta(P_k, g_k, \gamma_k) = 1_{(P_{k+1}>D)} + \beta \sum_{i=1}^M \gamma_{i,k}$ is lower semicontinuous.

It then remains to verify condition (B), which in the notation of this paper says that

$$\sup_{\delta<1} w_\delta(P_0, g_0) < \infty, \forall (P_0, g_0)$$

where $w_\delta(P_0, g_0) = v_\delta(P_0, g_0) - m_\delta$, $v_\delta(P_0, g_0) = \inf_{\{\gamma_k\}} \mathbb{E}[\sum_{k=0}^\infty \delta^k l^\beta(P_k, g_k, \gamma_k)|P_0, g_0]$, and $m_\delta = \inf_{(P_0,g_0)} v_\delta(P_0, g_0)$.

Following Sec. 4 of [24], define the stopping time $\tau = \inf\{k \geq 0 : v_\delta(P_k, g_k) \leq m_\delta + \varsigma\}$ for some $\varsigma \geq 0$. Given $\varsigma > 0$ and an arbitrary (P_0, g_0), consider a suboptimal power allocation policy where all sensors transmit with power γ_{max}, until $v_\delta(P_N, g_N) \leq m_\delta + \varsigma$ is satisfied at some time N. By Assumption 3.2, we have $N < \infty$ with probability 1 and $\mathbb{E}[N] < \infty$. Since $\tau \leq N$, we have $\mathbb{E}[\tau] < \infty$. Then by Lemma 4.1 of [24],

$$w_\delta(P_0, g_0) \leq \varsigma + \inf_{\{\gamma_k\}} \mathbb{E}\left[\sum_{k=0}^{\tau-1} l^\beta(P_k, g_k, \gamma_k)|P_0, g_0\right] \leq \varsigma + \mathbb{E}[\tau] \times (1 + \beta M \gamma_{max}) < \infty \quad (37)$$

where the second inequality uses Wald's equation. Hence condition (B) of [24] is satisfied and a solution to the ACOI exists.

B. Analytical solutions to sub-problems (17) and (18)

Here we will state the analytical solutions to the optimization problems (17) and (18). Derivations can be found in [5] and [22].

1) Solution to sub-problem (17): The constraint $P_{k+1} = D$ can be shown using (5) to be equivalent to the constraint

$$\sum_{i=1}^{M} \frac{\alpha_{i,k}^2 g_{i,k} c_i^2}{\alpha_{i,k}^2 g_{i,k} \sigma_i^2 + \sigma_n^2} = \frac{a^2 P_k + \sigma_w^2 - D}{P_k(D - \sigma_w^2)}.$$

We then have the following optimization problem, which is a slightly more general version of sub-problem (17), that also covers sub-problem (24) in the unstable case.

$$\min_{\alpha_1^2,\ldots,\alpha_M^2} \sum_{i=1}^{M} \alpha_i^2 \kappa_i \text{ s.t. } \sum_{i=1}^{M} \frac{\alpha_i^2 \rho_i^2}{\alpha_i^2 \tau_i + \sigma_n^2} = \frac{x}{y} \tag{38}$$

where $x > 0, y > 0, \kappa_i > 0, \rho_i \in \mathbb{R}, \tau_i > 0, i = 1, \ldots, M$ are constants.

Assume that the sensors are ordered such that $\frac{\rho_1^2}{\kappa_1} \geq \cdots \geq \frac{\rho_M^2}{\kappa_M}$. Then the optimal values of α_i^2 can be expressed as

$$\alpha_i^{*2} = \begin{cases} \frac{1}{\tau_i}\left(\sqrt{\frac{\lambda \rho_i^2 \sigma_n^2}{\kappa_i}} - \sigma_n^2\right) & , \quad i \leq M_1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

$$\text{where } \sqrt{\lambda} = \frac{\sum_{i=1}^{M_1} \frac{|\rho_i|}{\tau_i} \sqrt{\kappa_i \sigma_n^2}}{\sum_{i=1}^{M_1} \frac{\rho_i^2}{\tau_i} - \frac{x}{y}}$$

and the number of sensors which are active, M_1, satisfies

$$\sum_{i=1}^{M_1} \frac{\rho_i^2}{\tau_i} - \frac{x}{y} \geq 0, \frac{\sum_{i=1}^{M_1} \frac{|\rho_i|}{\tau_i} \sqrt{\kappa_i \sigma_n^2}}{\sum_{i=1}^{M_1} \frac{\rho_i^2}{\tau_i} - \frac{x}{y}} \sqrt{\frac{\rho_{M_1}^2 \sigma_n^2}{\kappa_{M_1}}} - \sigma_n^2 > 0 \text{ and } \frac{\sum_{i=1}^{M_1+1} \frac{|\rho_i|}{\tau_i} \sqrt{\kappa_i \sigma_n^2}}{\sum_{i=1}^{M_1+1} \frac{\rho_i^2}{\tau_i} - \frac{x}{y}} \sqrt{\frac{\rho_{M_1+1}^2 \sigma_n^2}{\kappa_{M_1+1}}} - \sigma_n^2 \leq 0.$$

2) Solution to sub-problem (18): In [22] it is shown that minimizing P_{k+1} is equivalent to minimizing

$$-\sum_{i=1}^{M} \frac{\alpha_{i,k}^2 g_{i,k} c_i^2}{\alpha_{i,k}^2 g_{i,k} \sigma_i^2 + \sigma_n^2}.$$

We then have the following optimization problem, which is a slightly more general version of sub-problem (18), that also covers sub-problem (25) in the unstable case.

$$\min_{\alpha_1^2,\ldots,\alpha_M^2} -\sum_{i=1}^{M} \frac{\alpha_i^2 \rho_i^2}{\alpha_i^2 \tau_i + \sigma_n^2} \text{ s.t. } \sum_{i=1}^{M} \alpha_i^2 \kappa_i = \gamma_{total} \tag{39}$$

where $x > 0, y > 0, \kappa_i > 0, \rho_i \in \mathbb{R}, \tau_i > 0, i = 1, \ldots, M$ are constants. Assuming that the sensors are ordered so that $\frac{\rho_1^2}{\kappa_1} \geq \cdots \geq \frac{\rho_M^2}{\kappa_M}$, the optimal values of α_i^2 to problem (39) can be expressed as

$$\alpha_i^{*2} = \begin{cases} \frac{1}{\tau_i}\left(\sqrt{\frac{\rho_i^2 \sigma_n^2}{\lambda \kappa_i}} - \sigma_n^2\right) & , \quad i \leq M_1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

$$\text{where } \frac{1}{\sqrt{\lambda}} = \frac{\gamma_{total} + \sum_{i=1}^{M_1} \frac{\kappa_i}{\tau_i} \sigma_n^2}{\sum_{i=1}^{M_1} \frac{|\rho_i|}{\tau_i} \sqrt{\kappa_i \sigma_n^2}}$$

and the number of sensors which are active, M_1, satisfies

$$\frac{\gamma_{total} + \sum_{i=1}^{M_1} \frac{\kappa_i}{\tau_i} \sigma_n^2}{\sum_{i=1}^{M_1} \frac{|\rho_i|}{\tau_i} \sqrt{\kappa_i \sigma_n^2}} \sqrt{\frac{\rho_{M_1}^2 \sigma_n^2}{\kappa_{M_1}}} - \sigma_n^2 > 0 \text{ and } \frac{\gamma_{total} + \sum_{i=1}^{M_1+1} \frac{\kappa_i}{\tau_i} \sigma_n^2}{\sum_{i=1}^{M_1+1} \frac{|\rho_i|}{\tau_i} \sqrt{\kappa_i \sigma_n^2}} \sqrt{\frac{\rho_{M_1+1}^2 \sigma_n^2}{\kappa_{M_1+1}}} - \sigma_n^2 \leq 0.$$

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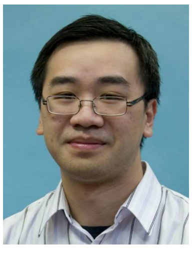

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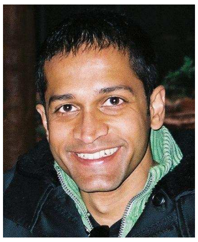

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