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Critical Evaluation and Design of Mathematics Tasks: Pre-Service Teachers

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This paper reports on a research project undertaken with a group (n=19) of Irish pre-service student teachers (PSTs) during the third year of a four-year undergraduate education course. A series of workshops were carried out on the critical evaluation and design of mathematics tasks. The research is presented as a case study using mixed methods to gather data. Through critically evaluating and designing mathematics tasks PSTs developed knowledge of cognitive demand, pedagogical design capacity and showed evidence of developing curriculum-making competences. The research highlights the need for PSTs to work together on evaluating and designing tasks.

Keywords: mathematics, tasks, design, curriculum, pedagogy

Introduction

This paper reports on a research project undertaken with a group (n=19) of Irish pre-service student teachers (PSTs) during the third year of a four-year undergraduate education course. A series of workshops were carried out on the critical evaluation and design of mathematics tasks. For the purpose of this research a 'mathematical task' is a problem or set of problems that address a specific mathematical idea, they are situated between teaching, learning and assessment (Smith & Stein, 1998). The types of tasks that students engage with have been shown to influence their development (Jonson et al., 2014), and studies have shown that students spend the majority of their time in mathematics classes working on tasks (Boston & Smith, 2009; Haggarty & Pepin, 2002). Furthermore, Smith and Stein (1998) asserted that the highest learning gains in mathematics were related to the how mathematics tasks were set up and implemented in teaching and highlighted the importance of students being engaged in high levels of cognitive thinking and reasoning (see also Swan, 2011; Boston, 2013). Many challenging questions arise from this assertion for pre-service mathematics teachers, such as, what is a good learning task? How is a good learning task set up? How is it implemented in a mathematics classroom? These questions are especially relevant in Ireland, given that a report on mathematics education found that traditional approaches to teaching and learning were widespread and recommended that students engage with more tasks which require higher order thinking skills such as problem-solving and justification (Jeffes et al., 2013).

Research Questions

In what ways did pre-service teachers' knowledge of the cognitive demands of mathematical tasks change following their participation in a module on critical evaluation and design of mathematical tasks?

How did this knowledge impact on their competences in curriculum making?

In this paper we present a review of some key research on mathematics task design. We provide an overview of the module implemented as part of the research project and the methodology used to collect data. We present the key findings from the research and discuss the salient themes emerging as they pertain to pre-service teacher education. Finally, we summarise our recommendations and conclusions.

Literature Review

The dependence of mathematics teachers on textbooks in their teaching appears to be a phenomenon in many countries (Haggarty & Pepin, 2002; Jeffes et al., 2013.). Haggarty and Pepin (2002) write about the dominance of the textbook in the mathematics classroom and conclude that without time to prepare for teaching, and, we would add, the skills to enrich the curriculum materials available, textbooks take on a prominence in "relation to teacher thinking and planning" (p.588). This is of concern as a recent review of mathematics books in Ireland found that all available books fell short of the standard needed to support mathematics teaching at that time and furthermore they especially fell short on the integration of technology, approaches to teaching for understanding and problem solving (O'Keeffe & O'Donoghue, 2011).

Synder, Bolin and Zumwalt (1992) describe three teacher curriculum approaches: the *fidelity approach*, where teachers are transmitters of the written curriculum without changing or adapting it; the *adaptation approach* where the teacher adapts the curriculum to suit their context; and the *enactment approach* where the teacher develops the curriculum in action depending on the student experience. Shawer (2010) builds on this work and identifies *curriculum-transmission strategies* where the textbook and teacher's guide are the single source of pedagogical instructions. He describes *curriculum-making strategies* as where the teacher develops their own materials in addition to those available in response to a needs assessment. *Curriculum-development strategies* on the other hand include experimentation, material writing and evaluation and involve both macro and micro level curriculum development. Within the Irish education system opportunities for curriculum-development strategies are limited since the curriculum is centrally devised with little space for school-based curriculum development, this coupled with a teaching culture that has a dependence on textbooks would point to a need for PSTs to engage with research on task development.

Studies have previously looked at this issue as part of professional development courses for in-service teachers. For example, Boston and Smith (2011) describe a *task-centric approach* to such courses where the focus is on teachers' ability to select and implement cognitively demanding tasks. They found that after a series of workshops, where teachers analysed both the cognitive demand of tasks and the implementation of tasks, the participants increased their ability to select high-level tasks and this improvement was sustained over time. The workshops also influenced teachers to consider the impact of the tasks they selected on their students' learning (Boston, 2013). Arbaugh and Brown (2005) used a similar approach and found that introducing teachers to criteria for high-level tasks influenced their task selection, and ultimately their pedagogical content knowledge.

A number of different frameworks have been developed to classify mathematical tasks and have proved useful in research, professional development and pre-service teacher education (Boston &

Smith, 2011). In this research we used three different but complementary frameworks with the participants. The first framework is that of Smith and Stein (1998) which looks at the level of cognitive demand (LCD) of tasks. They identify two levels of LCD: Lower-level demands (with task types of *Memorization* and *Procedures without connections to meaning*), and higher-level demands (*Procedures with connections to meaning* and *Doing Mathematics*). The distinction between tasks is relevant, as the level of cognitive demand in a task provides different learning opportunities for the learner and demands a different learning environment for the development of competences required by the task. Our second framework is the mathematical reasoning framework developed by Lithner (2008). This framework can be used to classify the opportunities for different types of mathematical reasoning afforded by tasks. Lithner (2008) describes two types of reasoning: Imitative Reasoning which consists of *Memorised reasoning* and *Algorithmic reasoning*; and Creative Reasoning which involves local and global *Creative mathematically founded reasoning*. Creative reasoning tasks fulfil the criteria of novelty, plausibility and mathematical foundation. Lithner (2008) is concerned with how tasks can be used to promote creative reasoning as opposed to imitative reasoning. He contends that the teacher's task is to "arrange a suitable didactic situation in the form of a problem" (p.271) so that the learner can take responsibility for the problem solving process, and use creative reasoning.

These two frameworks can be used to classify tasks using either the degree of cognitive effort required or the type of reasoning needed. They both divide tasks into two broad categories - either high or low levels of cognitive demand in the case of Smith and Stein (1998) or imitative or creative reasoning in Lithner (2008). They have been used in professional development to alert teachers to the effects of different types of tasks (e.g. Arbaugh & Brown, 2005). In order to help the PSTs to move from classifying tasks to designing them, we introduced a third framework. Swan's framework (2008) describes five task types that encourage concept development and provides very clear design principles to inform task development and implementation. There are many examples of Swan's mathematics tasks available on-line (see, for example, Mathematics Assessment Project, n.d.). The five task types that he posits will encourage concept development are: classifying mathematical objects, interpreting multiple representations, evaluating mathematical statements, creating problems, and, analysing reasoning and solutions.

Methodology

Nineteen pre-service teachers in the second semester of year three of a four-year post-primary teacher education course took part in the research project. At the time of the research the PSTs were midway through their second school placement experience and were teaching a minimum of two hours per week. All participants were taking mathematics in their degree and one other science subject (either biology, chemistry or physics). The research is presented as a case study using mixed data collection methods looking at the group of PSTs as a whole, over a sustained period of time as they developed competences in task design (Yin, 2009). This allowed us to build on earlier research (Boston, 2013) and incorporate PSTs reflections on the design process. Jones and Pepin (2016) contend that when teachers interact with mathematical tasks, they develop knowledge; this is done individually in preparing and planning for teaching and collectively when they are afforded opportunities to develop and discuss tasks with peers. In designing curriculum materials PSTs need both *subject matter knowledge* (SMK) and *pedagogical content knowledge* (PCK) (Ball, Thames & Phelps, 2008). With this in mind we designed a module for the group of PSTs based on task evaluation and ultimately task design.

In order to investigate any gain in knowledge for the group over the course of the intervention, we administered a pre- and a post-test designed by Boston (2013). This test asked students to classify 16 tasks as either High Level or Low Level tasks, and to give a rationale for their choice. At the end of the module, the pre-service teachers were asked to complete an evaluation questionnaire which asked them: to report on a key learning moment during the module; whether their teaching had changed as a result of the module and if so, in what way; what they would change about the module; and to indicate their level of agreement with some statements about the reading from the Mathematics Education literature. 13 of the 19 students submitted the evaluation questionnaire.

The assessment for the module consisted of the assignment outlined in Figure 1. All 19 pre-service teachers submitted this assignment and gave their consent to use it for research purposes. The tasks designed by the PSTs were analysed using the LCD and Lithner Frameworks. The classification was conducted by two researchers who were familiar with the curriculum, assessment, and textbooks relevant to the classes taught by the PSTs. The researchers used their knowledge to decide if (in the context of the PSTs' classes) the tasks should be classified as either high or low level tasks. We also looked for evidence that PSTs employed aspects of Swan's (2008) framework in their design. A general inductive approach as advocated by Thomas (2006) was taken to analyse the students' reflections on the differences between types of tasks. Analysis was guided by the research questions and a number of a priori themes (such as MKT), allowing flexibility for other themes to emerge.

Task Development

For a topic of your choice design (or significantly adapt) a series of tasks. One task/s should be suitable to be used in class while teaching, and, one for use as homework. Design an examination task/s for the topic. Present your rationale for each task based on your readings.

Reflect on the differences between classroom task, homework task and examination tasks.

Figure 1: End of module assessment

Findings

PSTs' specific learning about cognitive demands

Thirteen participants completed the Boston (2013) pre- and post-tests on levels of cognitive demand; a paired t-test was used to investigate whether the mean of the group had increased significantly over the course of the module, and found that it did (p=0.037). There was also evidence for PSTs developing knowledge about cognitive demand in their response to the question on the end of module evaluation asking what was their key learning moment.

> Realising the different reasoning and thinking about the type of question. In the textbook, where homework is usually given from, questions are repeated, low demand. In the maths exam

> students are faced with high level conceptual questions so there is a big gap there that needs to be addressed. (S14)

Here we see that S14 is *noticing* the level of demand and reasoning in the artefacts available to them in their teaching, the textbook. This text-guided Algorithmic Reasoning (AR) is supported and encouraged by the rote use of the textbook for homework (Lithner, 2008). The recognition of this by the PSTs was notable in many comments such as:

> Having completed this module, I seriously consider what I give my students as homework. Beforehand I generally gave a list of questions at the end of the chapter but now, having seen the different levels, I generally spend more time selecting and developing questions …. (S16)

The PSTs seem to be linking the levels of cognitive demand with the level of reasoning required, bringing the two theories together in their own thinking about the mathematics curriculum and assessment. The knowledge of the different frameworks is enabling the PSTs to move from a role of curriculum transmitters dependent on the text book to being curriculum makers as described by Shawer (2010).

Pedagogical design capacity and moving from curriculum transmission

The analysis of the PSTs' end of module assignments gave further evidence of them making this transition. All students showed that they were able to design or modify tasks to get high level questions. The PSTs classification of their tasks using the LCD and reasoning frameworks demonstrated that they were competent in using the frameworks for classification.

We noted the types of tasks designed by the PSTs for the three different situations of classroom tasks, homework tasks, and examination tasks. The types of tasks designed seemed to fall into two broad camps: open-ended exploratory tasks (which were mainly found in the classroom setting) and more traditional formats (which were mainly found in homework and examination tasks). The latter types of tasks mostly consisted of word problems with a real-life context; the PSTs designed a small number of other types of tasks for use as homework or examination questions, including tasks which required students to make a conjecture, provide an example, or evaluate a mathematical statement. In addition, one PST designed a homework task which involved a pre-class investigation. The majority (13 of 19) of the PSTs used card-matching designs for their classroom tasks. These tasks were based on Swan's "Interpreting Multiple Representations" (Swan, 2008, p. 3) task type. The PSTs were introduced to this idea through the Swan (2008) article and also participated in a card-matching task (on the topic of fractions) during one of the module sessions. Three of the PSTs used games (such as 'Battleships' and dice games) to devise tasks for use in the classroom, two PSTs used investigations as the basis of their task, and one designed a series of worksheets with problems of increasing difficulty.

The PSTs showed creativity and an appreciation for tasks with high levels of cognitive demand. However, an analysis of their designed tasks showed that the design process was not without difficulty for the group. Some of the questions were not always clear due to missing or confusing instructions, and sometimes the context made the question ambiguous (this has also been a problem in state examinations in Ireland). Occasionally it seemed as if the PSTs did not have a clear understanding of the underlying mathematics themselves, possibly owing to their level of SMK, and sometimes their use of mathematical language caused difficulty (such as using the term ‘equation’ instead of ‘expression’ for something like $2x+1$). The learning trajectories for the tasks or sets of tasks were not always clear - sometimes it was not clear what understanding and what concept the PSTs were trying to develop.

Pre-service teachers’ pedagogical content knowledge

The PSTs’ knowledge of levels of cognitive demands challenged their deeply held view of how best to teach mathematics. Previous research with a similar co-hort of PSTs found that they focused on content when planning for teaching and placed little emphasis on the learner or prior learning (Nolan, Dempsey, Lovatt & O’Shea, 2015). Most of the respondents said the module impacted on how they taught mathematics with the majority citing a change in how they asked questions, placing more emphasis on higher level of cognitive demand in questions. Prior to this module, these PSTs would have completed a module which included a significant input on questioning skills for teaching; they seem to have needed the knowledge of cognitive demand in order to have changed their questioning practices. It must be noted that this was reported but may not have been the reality when one takes into account the examples provided by the PSTs in their end of module assessment. However, an increased emphasis on discussing mathematics problems appears to be evident with comments such as

> I try to think more about pushing my students to reason more when completing tasks. I try to ask questions, give tasks to my students with much less information, and I want my students to rely less on me giving them the answer. (S11)

This PST also spoke about the effect of the intervention on her teaching:

> I never really thought much into the differences between the tasks that I give during class, homework or exams or the impact it could have on my students’ development in a subject. Having studied and researched the classification of math’s tasks and implementing my own selection/adaption of tasks into my class, I now feel that I have gained a deeper understanding into the effect my choice of tasks can have on the progression and learning... (S11)

The PSTs who implemented their tasks in their teaching placements, realised the effect that the teacher or set-up can have on the cognitive level of the task and this led to them thinking about different types of tasks or redesigning their original tasks:

> The students struggled very much with it at the beginning and due to my own fault I went through an algorithm with them and then the task immediately became a lower demand one, just requiring the students to reproduce an algorithm each time. If I were to redesign the tasks, I would change tasks E and F [card-matching tasks] to tasks where the students have to spot a mistake in a question/statement and justify their reasoning and how they would alter the question/statement … in order to encourage them to develop critical thinking skills. (S12)

This reflection would suggest that the PST is developing her thinking on organising pedagogical content, adapting materials to suit students and adopting curriculum planning and making strategies (Shawer, 2010).

Discussion

Increasing the PSTs awareness of different levels of tasks and giving them an opportunity to design and modify tasks would appear to have allowed them to develop skills such as the ability to classify tasks and design tasks at different levels. They also seem to have developed knowledge especially PCK which linked to their knowledge of cognitive demand has enabled them to adapt their practice especially around questioning. We note though that the evidence we have presented in this regard is based on self-reported data. The importance of applying frameworks in order to increase awareness of concepts such as levels of cognitive demand is significant for PST education; awareness may be a crucial first step in knowledge acquisition. Similar to findings from Boston (2013) and Swan (2007) who worked with practicing teachers, our research appears to demonstrate the need for an awareness of cognitive demand in order for mathematics educators to be able to select and develop rich and engaging tasks. This increase in knowledge and skills seems to be crucial in order to make the transition from curriculum transmitters to curriculum makers (Shawer, 2010). The space in our intervention for discussing textbook questions and State Examinations materials was cited as being the most impactful for the PSTs. They suggested that the module could be enhanced with more time devoted to this kind of peer interaction in task evaluation and design. This need for space for curriculum making and professional learning, and, the challenges therein has not been fully explored within PST education.

This research has highlighted a gap in the PSTs' education on task design in the case study institution, and, as such will be used to make changes to the module design and implementation. PSTs' practices and beliefs around tasks for homework merits further exploration. We intend to carry out more analysis on our data such as on the tasks assigned by PSTs (pre- and post-intervention) and of their reflections on tasks linking back to Lithner's (2008) concept of sociocultural milieu.

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