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When Overconfident Traders Meet Feedback Traders

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Abstract

We develop a model in which informed overconfident market participants and informed rational speculators trade against trend-chasers. In this model positive feedback traders act as Computer Based Trading (CBT) and lead to positive feedback loops. In line with empirical findings we find a positive relationship between the volatility of prices and the size of the price reversal. The presence of positive feedback traders leads to a higher degree of trading activity by both types of informed traders. Overconfidence can lead to less price volatility and more efficient prices. Moreover, overconfident traders may be better off than their rational counterparts.

1 Introduction

The analysis of feedback trading is fundamental nowadays. Indeed, the revival of its extensive use has been blamed for the Flash Crash of the 6th of May 2010.[1] As pointed out in the Introduction of Foresight: The Future of Computer Trading in Financial Markets (2012) and also more specifically in Zygrand et al. (2012) the use of Computer Based Trading (CBT) may lead to positive feedback loops that can have profoundly damaging effects, leading to major share sell-offs. It uses very mechanical rules without any human interventions and it is estimated that CBT amounts for around 30% of the UK's equity trading volume and more than 60% for the USA. The very fact that CBT is thought to have caused major market disruptions as recently as 2010 motivates further investigation of feedback trading in a more complex environment where different types of traders interact. In particular, as overconfidence is a psychological bias present in all markets, it is interesting to know how overconfident traders behave in the presence of positive feedback traders or CBT.

In this paper we use a dynamic model where traders can be one of three types: overconfident, feedback equivalent to CBT or rational. The purpose of this paper is to analyze the interactions between the different types of traders. One question of interest, and still unanswered, is how this interaction impacts the market. In the following paper, we concentrate our analysis on one aspect of CBT namely the creation of feedback loops and we do not look at the High Frequency Trading aspect of CBT.[2]

The existence of trend-chasing behavior is a well established fact. Andreassen and Kraus (1990) and Mardyla and Wada (2009) use experiments to show its relevance. In addition, other analyses have empirically found evidences of trend-chasing behavior in financial markets. Frankel and Froot (1988) observe that in the mid-1980's the forecasting services were issuing buy recommendations while maintaining that the dollar was overpriced relative to its fundamental value. Lakonishok, Shleifer and Vishny (1994) find evidences that individual investors use positive feedback trading strategies and that this behavior can be attributed to an irrational extrapolation of past growth rates. Several studies have focused on the behavior of institutional investors (Shu (2009), Sias and Starks (1997), Sias, Starks and Titman (2001), Dennis and Strickland (2002) to name but a few). According to most of these papers, institutional investors use positive feedback strategies and therefore destabilize stock prices. Finally, Bohl and Siklos (2005) find the existence of momentum strategies during episodes of stock market crashes. Evidences of contrarian strategies are also present in financial markets. The short-term portfolio composition strategies suggested by Conrad et al. (1994), Cooper (1999), and Gervais and Odean (2001) find that US markets showed anomalous, "contrarian" behavior. Moreover, Evans and Lyons (2003)

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[1]On that day, the US equity market dropped by 600 points in 5 minutes and regained almost all the losses in 30 minutes.

[2]The interested reader can look at Ait-Sahalia and Saglam (2014) for instance.

obtain evidence of negative feedback trading, at the daily frequency.

Our paper uses the dynamic setting of De Long et al. (1990). It is also close to the spirit of Hirshleifer, Subrahmanyam and Titman (2006). The former paper establishes that rational speculation together with the presence of positive feedback traders can destabilize prices and facilitate the creation of price bubbles. However, the consequences of the interaction between feedback and overconfident traders is still unknown. We introduce the presence of informed overconfident traders in order to look at how overconfident traders exploit the presence of feedback traders.[3]

The overconfidence was first analyzed by psychologists. Kahneman and Tversky (1973), and Grether (1980) stress that people overweight salient information. This behavior is well documented in psychology for very diverse situations. Due to the essence of financial markets, overconfidence occurs among market participants. Indeed, the competition between traders implies that the most successful ones survive, leading them to overestimate their own ability. It is well documented that the presence of overconfident traders increases the volatility of prices (see Odean (1998) for instance). As a consequence, we could expect that when overconfident traders interact with positive feedback traders prices would depart even more from their fundamental values and that prices display more volatility. Indeed, positive feedback traders buy (sell) securities when prices rise (fall). In doing so, they introduce noise in the market as they lead prices to move away from fundamentals. As established by De Long et al. (1990), the presence of positive feedback traders stimulates trading by informed traders' leading to more price volatility. However, one of the main results of the paper shows that the presence of overconfident traders can diminish the volatility of prices when positive feedback traders are present in the market. Indeed, the volatility of prices decreases with the number of overconfident traders and with their level of overconfidence. This means that overconfident traders temper the destabilizing role of feedback traders and lead to a more stable market. The presence of overconfident traders leads rational traders to scale down their trading leading to a decrease of the volatility of prices caused by positive feedback traders. One interpretation of this result is that overconfident traders commit to react more to their private information. Introducing more information counterbalances the effect of positive feedback.

Our analysis, enables us to answer the following important questions. What is the main determinant of the excess price volatility? How does the link between psychological characteristics of the participants and their trading profits evolve according to different proportions of irrational traders? What is the effect of the traders' risk aversion on price stabilization? Where do the underreaction and/or overreaction to new information come from? This paper can also be seen as an attempt to understand the relationship between feedback loops and crashes (for

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[3]As in Germain et al. (2014), we consider that overconfident traders overestimate the mean of the liquidation value of the risky asset (called mean bias) but also misperceives the variance of the liquidation value.

instance Flash Crash) in a complex environment where several types of traders interact. This analysis can ultimately give us some insights regarding the regulation of CBT.

We first give our results for the case where there is no feedback trading or CBT. In that case, the trading of overconfident investors enhances the volatility of prices, and worsens the quality of prices as well as their expected utility. We obtain that the presence of overconfident implies a larger volume being traded. Statman, Thorley, and Vorkink (2006) use U.S. market level data to test this link and argue that after high returns subsequent trading volume will be higher as investment success increases the degree of overconfidence.

Our second set of results looks at the case where feedback traders are present. When there is a sufficient large number of trend-chasing speculators, overconfident traders may have higher expected utility than their rational opponents. Kyle and Wang (1997), Benos (1998) and Germain et al. (2014) find a similar result. However some other studies predict the opposite i.e. that overconfident agents trade to their disadvantage (Odean (1998), Gervais and Odean (2001), Caballé and Sákovics (2003), Biais et al. (2004) among others). Hirshleifer, Subrahmanyam and Titman (2006) show that irrational traders can earn positive expected profit in the presence of positive feedback trading if they trade early. Indeed, higher stock prices may attract customers and employees which may reduce the firm's cost of capital and provide a cheap currency for making acquisition. Also, stock prices increase may initially generate cash flow. This simple mechanism does not require irrational traders to be sophisticated enough to think of the positive feedback trading effect and to realize profits. We also find that positive feedback traders earn negative expected profits.

We obtain that price changes can be negatively or positively serially correlated at long horizons. The sign of this serial correlation depends on some of the parameters of the model. The negativity of the serial correlation is a well documented property of prices and is also found in De Long et al. (1990). We obtain that prices can be positively serially correlated when overconfident traders are not too numerous and they believe that the information of the other traders is more precise than it is. This result occurs in Odean (1998) when rational traders trade with overconfident traders who undervalue the signals of other traders. We extend that result to a situation where 3 different types of traders trade with each other. Daniel et al. (1998) consider a situation where investors are overconfident about the precision of their private signals. However, the noisy public information is correctly estimated by all market participants. Price changes exhibit a positive short-lag autocorrelations (called "overreaction phase") and a negative correlation between future returns and long-term past stock market (long-run reversals called "correction phase"). We assume that both rational and overconfident traders are risk averse with the same level of risk aversion. We find that increasing the traders' risk aversion increases the negative serial correlation of prices. The literature on feedback trading concludes that the presence of positive (negative) feedback traders leads to negative (positive) serially

correlated returns together with an increase (decrease) in volatility (see Shu (2009), Sias and Starks (1997), Sias, Starks and Titman (2001), Bohl and Siklos (2005), and Cooper (1999), for instance).

Finally, we obtain that both rational and overconfident traders trade more when feedback traders are present. This result is also present in De Long at al. (1990) for the impact of feedback trading. In addition, it also found in the literature that the presence of overconfident traders leads to a high degree of trading activity (Odean (1998), Barber and Odean (2001), Odean (1999), Glaser and Weber (2007), Statman, Thorley, and Vorkink (2006) among others).

In line with empirical findings we find a positive relationship between the volatility of prices and the size of the price reversal. However our model cannot establish any causality. Nevertheless, we obtain that this result depends on different parameters of the model such as the number of positive feedback traders, how overconfident perceive the information of others, as well as the number of informed traders present in the model. In addition we obtain that the presence of positive feedback traders leads to a higher degree of trading activity by both types of informed traders.

When we introduce negative feedback traders instead of positive feedback traders, we find that most of the results obtained with the presence of positive feedback are reversed. Price volatility is reduced whereas price efficiency is enhanced due to the presence of contrarian trading. Price volatility and price efficiency increase with both the number of overconfident traders and with their level of overconfidence. The overall volume traded by rational traders increase with the number of negative feedback whereas the volume traded by overconfident is U-shaped with respect to the number of negative feedback. We find the serial correlation of returns to be negative and to decrease with the number of negative feedback traders. Finally the expected profit of the negative feedback traders decrease with the number of negative feedback traders present in the market and is positive for low enough number of negative feedback traders.

The outline of this paper is as follows. In section 2, we introduce the general model and characterize the different types of traders. In section 3, we derive the trading equilibrium. In section 4 we analyze the effects of the overconfidence and of the positive feedback trading on some parameters of interest for financial markets. In section 5, we are interested in understanding the social standpoint. In section 6, we look at how our results are changed when we replace positive feedback traders by negative feedback traders. In section 7, we discuss our model and draw some empirical implications. Finally, we conclude in section 8. All proofs are gathered in the Appendix.

2 The Model

We analyze a model with four periods, at each of the first three periods trade takes place whereas consumption takes place at $t = 4$. Two assets, namely a riskless and a risky asset, are exchanged during the three trading periods. The riskless interest rate is normalized to zero. The liquidation value of the risky asset, $\tilde{v}$, is assumed to be normally distributed with $\tilde{v} \sim N(\bar{v}, h_v^{-1})$.

We consider a trading system where agents are price takers. At each auction t, the demands for the risky asset and the riskless asset are x_t and f_t respectively. Three types of investors trade the assets:

- N_1 overconfident traders. They receive information about the liquidation value of the risky asset. They believe that their private signals are more accurate than they actually are. Furthermore, they overestimate the expected liquidation value of the risky asset.

- N_2 rational traders. These traders receive private information but do not distort it.

- P feedback agents who can be either positive feedback or negative feedback traders.[4] They do not base their trading decisions on fundamental values, instead they react to stock price change. Their order size is proportional to the change in price of the asset.

We denote by P_t the price of the risky asset at time t for $t = 1, 2, 3$. At time $t = 4$, the value of the risky asset is publicly revealed, the price is then equal to the realization of $\tilde{v}$.
Trader i's wealth is $W_{ti} = f_{ti} + P_t x_{ti}$ for trading rounds $t = 1, 2, 3$ and $W_{4i} = f_{3i} + \tilde{v} x_{3i}$ for the last trading round. Let us denote $\bar{x}$ as the per capita supply of the risky asset. It is assumed to be known to all and constant over time.

No information is released before the first trading round. Before each subsequent trading round $t = 2$ and $t = 3$, each rational and overconfident trader receives one of M different signals concerning the liquidation value of the asset. Each trader receives a private signal $\tilde{y}_{ti} = \tilde{v} + \tilde{\varepsilon}_{tm}$, with $\tilde{\varepsilon}_{tm} \sim N(0, h_\varepsilon^{-1})$ and $\tilde{\varepsilon}_{t1}, \ldots, \tilde{\varepsilon}_{tm}$ for $\forall\, t = 2, 3$ being mutually independent. As in Odean (1998) we assume that $M < N_1 + N_2$, i.e. there are more traders than signals, and that both N_1 and N_2 are multiple of M, i.e. overconfident traders are, on average, equally informed as their rational counterparts. Let $\bar{Y}_t$ be the average private signal at time t, we have that:

$$\bar{Y}_t = \sum_{i=1}^{M} \frac{\tilde{y}_{ti}}{M} = \sum_{i=1}^{N_1} \frac{\tilde{y}_{ti}}{N_1} = \sum_{i=1}^{N_2} \frac{\tilde{y}_{ti}}{N_2}.$$

In other words, the informativeness of the private signals is the same for the two groups. This setup allows us to exhibit the overconfidence effect without considering informational content bias.

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[4]We mainly analyze the first case in the paper.

Overconfident market participants believe that the precision of their two signals, the one received at $t = 2$ and the one received at $t = 3$, is equal to κh_ε with $\kappa \geq 1$. They also believe that the $2M - 2$ other signals have a precision equal to γh_ε with $\gamma \leq 1$. Overconfident traders misperceive the distribution of the asset as well. Indeed, they believe that the average liquidation value equals $\bar{v} + b$ with $b > 0$ and that the precision of $\tilde{v}$ equals ηh_v ($\eta \leq 1$).[5] This framework is consistent with theoretical and empirical findings.[6] Indeed, traders tend to overestimate their own signals and to correctly evaluate (or at worst to under-weight) public information.

A rational agent correctly estimates both the mean of the liquidation value of the risky asset and her private signal. In other words, a rational investor acts as an overconfident trader with $\eta = \kappa = \gamma = 1$ and $b = 0$.

All informed agents are assumed to be risk averse. Their preferences are described by a constant absolute risk aversion (CARA) utility function of the following form

$$u(W) = -e^{-aW},$$

where a denotes the coefficient of risk-aversion and W the final wealth.

Each informed trader i chooses his order at time t, x_{ti}, so that

$$x_{ti} \in \arg\max E[-e^{-aW_{ti}}|\Phi_{ti}],$$

where Φ_{ti} denotes the available information to trader i at time t.

As in Odean (1998), De Long et al. (1990) and Brown and Jennings (1989), informed traders look one period ahead when solving for their optimal strategy i.e. they are myopic.[7]

Finally, at time $t = 2, 3$, each feedback positive agent i submits an order x_{ti}^f, with the following form $x_{ti}^f = \beta(P_{t-1} - P_{t-2})$. Feedback traders only participate to the last two rounds of trading.

3 The equilibrium

To solve their maximization programs, informed traders whether rational or overconfident assume that prices are linear functions of the average signal(s) such that:

$$P_3 = \alpha_{31} + \alpha_{32}\bar{Y}_2 + \alpha_{33}\bar{Y}_3, \tag{3.1}$$

$$P_2 = \alpha_{21} + \alpha_{22}\bar{Y}_2. \tag{3.2}$$

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[5]However, in order to keep the model simple in all the simulations we are holding $\eta = 1$ and $b = 0$.

[6]See Fabre and François-Heude (2009), for instance.

[7]As mentioned by Odean (1998), assuming myopia when traders conjecture that they do not affect prices leads to the fact the informed traders' demand do not incorporate any hedging demand. This can be seen in Brown and Jennings (1989).

At each auction, an informed agent determines his demand by considering both his private signal(s) and the price schedule(s). Each informed market participant i has access to the following information $\Phi_{2i} = [y_{2i}, P_2]^T$ and $\Phi_{3i} = [y_{2i}, y_{3i}, P_2, P_3]^T$ for date $t = 2$ and $t = 3$, respectively. Due to the presence of positive feedback trading and when deciding his demand, an informed trader takes into account that his current trade may lead the future price away from fundamentals.

Proposition 3.1 *If $a\beta P < g^*(N_1, N_2)$, in other words, when the number of feedback traders (P), the strength of feedback trading (β) or the informed traders' risk aversion is not too large[8],there exists a unique linear equilibrium in the multi-auction market characterized by:*

$$\alpha_{31} = \frac{(N_1\eta+N_2)h_v\bar{v}+N_1\eta h_v b-a(N_1+N_2+P)\bar{x}}{(N_1\eta+N_2)h_v+2(N_1(\kappa+\gamma M-\gamma)+N_2 M)h_\varepsilon} + \frac{aP\beta(\alpha_{21}-P_1)}{(N_1\eta+N_2)h_v+2(N_1(\kappa+\gamma M-\gamma)+N_2 M)h_\varepsilon},$$

$$\alpha_{32} = \alpha_{33} + \frac{a\beta P\alpha_{22}}{(N_1\eta+N_2)h_v+2(N_1(\kappa+\gamma M-\gamma)+N_2 M)h_\varepsilon},$$

$$\alpha_{33} = \frac{N_1(\kappa+\gamma M-\gamma)h_\varepsilon+N_2 M h_\varepsilon}{(N_1\eta+N_2)h_v+2(N_1(\kappa+\gamma M-\gamma)+N_2 M)h_\varepsilon},$$

where $g^*(N_1, N_2)$, α_{21}, α_{22} and the different agents' demands over time are given in the Appendix.

Proof: See Appendix.

The number of positive feedback traders has an impact on the different parameters α except on α_{33}. Indeed, at the last auction, informed agents cannot trigger feedback trading on the basis of their new information. Nevertheless, all prices are influenced and connected by the presence of trend-chasing traders. More precisely, the link between P_3 and $\bar{Y}_2$ (captured by α_{32}) depends on the link between P_2 and $\bar{Y}_2$ (i.e. α_{22}). The greater the intensity of feedback trading (β and P) the stronger this link is. Similarly, the informed traders' risk aversion, a, strengthens this link.

As both types of informed traders are aware of the presence of positive feedback traders, they take that into account when trading. Indeed, upon, for instance, receiving good news before both auctions, informed traders take larger position based on that information at $t = 2$ in order to drive prices up. This triggers even more buying later on from feedback traders which enables them to unload their position at an inflated price resulting in positive expected profit.

However, when the number of feedback traders is much larger than the number of informed traders or when the informed traders have a high level of risk aversion (a), there is not equilibrium. Indeed, the intensity of feedback trading is so strong that the prices move away too much from the fundamental value of the risky asset. And the informed market participants are reluctant to trade with such an intensity.

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[8]This condition is showed in the Appendix.

In de Long et al. (1990), they show that the condition for the existence of a stable solution is close to ours (in their article the multiplication of the intensity of feedback trading by the risk aversion coefficient is bounded and the number of feedback traders is equal to the number of informed investors).

In Hirshleifer et al. (2006), irrational traders do not anticipate the feedback effect and the rise in price causes stakeholders (for instance workers) to make greater firm-specific investments when they anticipate the growth of the firm. This in turn increases the final payoff of the risky asset. Nevertheless, risk averse traders trade less aggressively and dampen the feedback effect.

4 Volatility, Quality of Prices, Serial Correlation of prices and Trading Volume

In this section, we focus on the influence of the irrational behavior on the volatility of prices, measured as the variance of prices, on the quality of prices at t (the variance of the difference between the price and the liquidation value, $var(P_t - \tilde{v})$), the serial correlation of prices and on the different market participants' trading volume.

4.1 Volatility

Proposition 4.2 (Volatility) *For trading rounds $t = 2, 3$, the volatility of stock prices increases with the number of positive feedback traders.*

Proof: See Appendix.

Trend chasing increases volatility in the market. This works through two channels. Increasing P increases the "amount of feedback trading" which in turn also impacts how both types of informed traders trade. We can see that the overall effect is to increase volatility. De Long et al. (1990) shows that when only rational traders are present, they anticipate the presence of feedback traders leading to more trading by rational informed traders. In different setups, numerous previous studies have pointed out that the excess volatility of asset prices stems from the trading behavior of overconfident traders (Odean (1998), Caballé and Sákovics (2000), among others).

Our model predicts that the volatility is positively linked to the number of feedback traders. This implies that more CBT as measured by the number of feedback traders leads to more volatility in market.

The following result analyses, among other things, the effect of overconfident traders in our setup. This result is obtained using numerical procedures.

Result 4.1 (Volatility)

1. *For trading rounds $t = 2, 3$, the volatility of stock prices increases with β, the feedback trader's trading intensity.*

2. *Overconfidence, measured by κ or the number of overconfident traders N_1, can diminish the volatility of prices. It crucially depends on the number of feedback traders. For a large number of feedback traders, the price volatility decreases with overconfidence. When $P = 0$, the volatility of prices increases with κ and N_1.*

3. *The volatility of prices can increase with γ.*

The first point of Result 4.1 is equivalent to the result of Proposition 4.1. The other two points look at the effect of overconfidence in our setup. We find that the effect of overconfident traders on the excess volatility depends critically on the number of feedback traders in the market. When there are no positive feedback traders, we obtain the aforementioned result of Odean (1998) and Caballé and Sákovics (2000). We obtain that same result as our model is essentially the same as the two previous models. However, when positive feedback traders are present this result can be reversed. This result also contradicts the finding of Benos (1998). We find that the main source of excess volatility is due to feedback trading rather than the trading from overconfident traders. Hence, overconfident traders can alleviate the effect of the feedback traders and lead to a more stable market. This can be explained as follows, when increasing the number of overconfident traders keeping constant the number of rational traders, the following forces are at work. On the one hand increasing the number of overconfident traders stabilizes prices as it increases the risk bearing capacity of the market. On the other hand it destabilizes prices as more traders anticipate the trend-chasing behavior. However, the reaction of both rational and overconfident traders is not identical. Indeed rational investors also anticipate the impact of the presence of overconfident traders on the future price and scale down their contemporary trading as they anticipate that overconfident traders trade "too much". The overall effect is such that that for small values of P, the volatility of prices is increased by the presence of overconfident traders whereas for large values of P, the volatility of prices decreases with overconfidence. The more positive feedback traders in the market, the larger the latter effect. In other words, overconfident traders commit to trade more on their information the greater P and introducing more information counterbalances the effect of positive feedback trading or CBT.

The following two figures illustrate the effect of overconfidence on the volatility of prices as described in the previous result.

We also obtain that, provided the number of feedback traders is large enough, a market composed of overconfident traders only as opposed to a market with rational traders only can
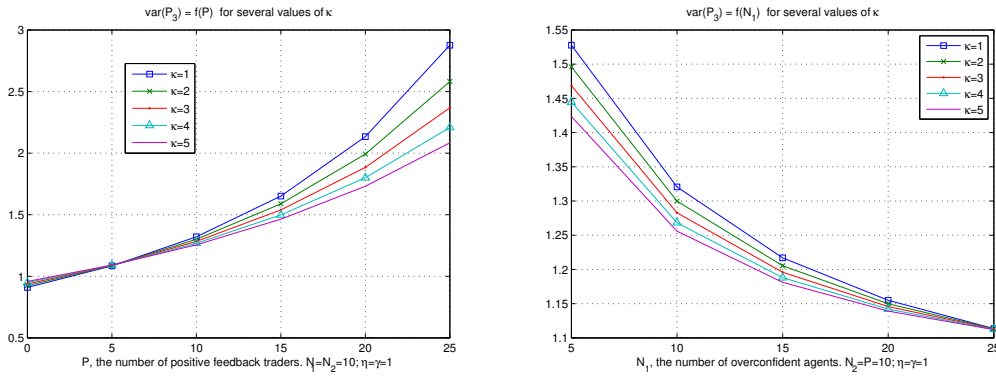
Figure 1: *The variance of prices at time $t = 3$ as a function of the number of feedback traders and of the number of overconfident traders.*
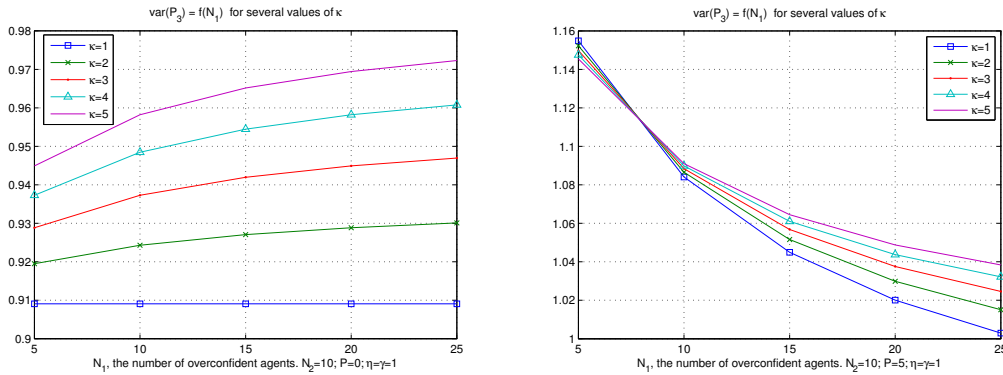

Figure 2: *The variance of prices at time $t = 3$ as a function of the number of feedback traders and of the number of overconfident traders.*

have a lower volatility.

The effect of γ on the volatility of prices also depends on the number of feedback traders. When P is small the volatility decreases with the underestimation of the precision of the other signals. This result is not consistent with Odean (1998) as he obtains that the smaller the parameter γ, the greater is the volatility of prices.

We now turn to the quality of prices.

4.2   Quality of Prices

We now examine the behavior of the quality of prices.

Proposition 4.3 (Quality of Prices) *The quality of prices declines as the number of feedback traders increases.*

Proof: See Appendix.

On the one hand, when the number of positive feedback traders increases, prices move away from fundamentals and therefore become less informative. On the other hand, as informed traders anticipate the trend-chasing strategies they trade more intensely on their private information. However as said before, as a consequence rational traders scale down their trading. As an overall, the price quality declines.

Result 4.2 (Quality of Prices)

1. *The quality of prices declines as the feedback trading intensity increases (β increases).*

2. *Depending on the number of feedback traders, the quality of prices can either increase (for large P) or decrease (for small P or even $P = 0$) with N_1. It increases with κ (for large P), decreases with it (for small P or even $P = 0$) or is non-monotonic with κ for intermediate values of P.*

In addition to responding to the trend-chasing strategies, overconfident traders alter the quality of prices due to their irrationality.[9] The impact of the overconfidence on the quality of prices depends on whether or not there are feedback traders present and on their number. If there is no trend-chasing behavior, the presence of overconfident traders moves prices away from fundamentals and diminishes market efficiency. When the number of feedback traders is large, the quality of prices improves with both the number of overconfident traders and the parameter κ. This result is in contrast with the result obtained by Odean (1998) but is in accordance with Benos (1998).

4.3   Serial Correlation of Prices

Proposition 4.4 (Serial Correlation of Prices) *When there only rational traders and positive feedback traders, the price changes increase with the number of positive feedback traders.*

Proof: See Appendix.

This result is in accordance with the intuition given previously.

Result 4.3 (Serial Correlation of Prices)

1. *The serial correlation of prices in absolute value increases with the number of overconfident*

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[9]Ko and Huang (2007) show that arrogance can be a virtue. Indeed, overconfident investors believe that they can earn extraordinary returns and will consequently invest resources in acquiring information pertaining to financial assets. In our model there is no information-seeking activity which could permit to obtain such a positive externality.

traders (for low P or even $P = 0$) and is non-monotonic (initially increasing) with N_1 for large P. It also increases with the level of overconfidence κ.

2. *When only rational traders are present, the serial correlation of prices is negative and close to zero.*

3. *The serial correlation of prices decreases with γ. In other words, the more precise the other traders' information is believed to be the smaller the price changes.*

The serial correlation of prices depends critically on the overconfidence level and on the number of positive feedback traders. In the presence of positive feedback trading, the serial correlation is generally negative. It implies that positive feedback trading destabilizes the price schedule. Informed traders cannot keep price at fundamentals or reduce the fluctuation of prices.

The term $cov(P_3 - P_2, P_2 - P_1)$ describes the correction phase in Daniel et al. (1998). They show that overconfident traders begin by overreacting to their private signals. In the second phase, irrational market participants correct their beliefs and their order as new public information arrives, this is defined as the "correction phase". In our model, agents update their beliefs concerning their private information. The informed market participants know that their earlier trades move prices away from fundamentals as they try to exploit the presence of feedback trading. The size of the departure of prices from fundamentals increases with the number of feedback traders. At date 3, informed participants correct their demands after observing their last signal which leads to price reversal. Odean (1998) finds such negative correlation by considering overconfident agents only. When rational investors are introduced to the model, he shows that the serial correlation of prices may be positive provided overconfident agents sufficiently undervalue the signals of others. We extend his former result, as we show that informed rational traders cannot prevent irrational traders (feedback agents as well overconfident traders) to destabilize prices.

Figure 3 illustrates the last point of the proposition. It shows that the price change is more important when each overconfident trader underestimates the precision of the other traders' private information. However, there is a positive momentum when each overconfident trader underestimates the other specific market participants' signals.

The empirical literature on feedback trading concludes that the presence of positive (negative) feedback traders leads to negative (positive) serially correlated returns together with an increase (decrease) in volatility (see Shu (2009), Sias and Starks (1997), Sias, Starks and Titman (2001), Bohl and Siklos (2005), Conrad et al. (1994) and Cooper (1999) to name a few). However, we find that when the market is composed with positive feedback traders and rational traders the effect of the number of feedback traders is very small and the serial correlation of prices is close to zero. The effect of P increases with the level of overconfidence. It can be seen
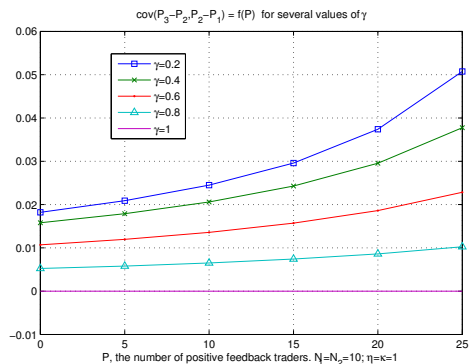
Figure 3: *The price changes as a function of the number of feedback traders for different values of γ.*

that the combination of rational investors, positive feedback traders and overconfident traders can lead to positive serial correlation of prices despite the presence of positive feedback traders. This result is in sharp contrast with the literature such as Bohl and Reitz (2006) and Daniel et al. (1998). Bohl and Reitz (2006) find a possible link between positive feedback trading and negative return auto correlation during period of high volatility in the German Neuer Market. Daniel et al. (1998) find that overconfidence and long-run reversals of returns may be linked. They also find that the momentum effect is stronger for high volume stocks.

4.4 Trading Volume

Result 4.4 (Trading Volume)

1. *The trading volume by both rational and overconfident traders increases with the number of positive feedback traders.*

2. *The trading volume by overconfident traders can be a non-monotonic function of κ and this comparative static depends on the number of feedback traders whereas the trading volume by rational traders decreases with κ.*

3. *The trading volume originating from feedback traders increases with the traders' overconfidence, κ, and with the number of feedback traders.*

When positive feedback traders are present, informed agents trade more aggressively. They anticipate that the initial price increase will stimulate buying by feedback traders at the subsequent auctions. In doing so, they drive prices up higher than fundamentals. Consequently, positive feedback traders respond by trading even more. We find that the feedback trading as well as the overconfidence enhances the trading volume. Several theoretical papers find this result as well (See Delong et al. (1990) for the effect of feedback trading and Odean (1998) for

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the effect of overconfidence). Our result is consistent with empirical findings. Glaser and Weber (2007) show that investors who think that they are above average in terms of investment skills or past performance trade more. Statman, Thorley, and Vorkink (2006) use U.S. market level data and argue that after high returns subsequent trading volume will be higher as investment success increases the degree of overconfidence. This is also confirmed by Kim and Nofsinger (2007) for Japanese traders on the Tokyo Stock Exchange. Odean (1999) analyzes the trading of 10,000 investors and find the aforementioned relationship between volume and overconfidence.

In figure 4, we compare the volume from overconfident traders and from rational traders with no feedback traders. As expected, we see that overconfident traders trade more aggressively than their rational counterparts. The volume from overconfident investors increases with κ, whereas, as explained before, the volume from rational investor's order decreases with κ. However, both expected volumes decrease with the number of overconfident traders.
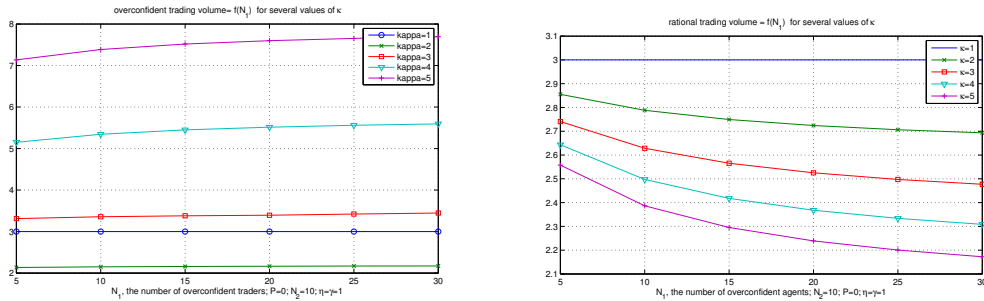

Figure 4: *The total individual overconfident trading volume as a function of the number of overconfident agents, for different values of the parameter κ.*

5   Trading Performance

We now look at expected expected utility of profits.

Result 5.5 (Expected Utility of Profits)

- *Positive feedback traders always earn negative profits. The feedback traders' losses increase with their number and the intensity of their trading. Those losses increase with γ whereas they decrease with both N_1 and κ.*

- *If both κ and the number of feedback traders are large overconfident traders may earn positive expected utility of profits. In that case overconfident traders are better off than their rational counterparts. For a moderate level of overconfidence (for instance $\kappa = 2$) and for a relatively large number of feedback traders, we also find the latter result.*

- *The expected utility of overconfident traders can be increasing, decreasing or non-monotonic with the number of positive feedback traders and with β. The effect of N_1 and κ on the profit of the overconfident traders is ambiguous: For large P, it is increasing with κ whereas decreasing with κ for small P, it decreases with N_1 for low β whereas it increases with N_1 for large β.*

- *The expected utility of profits earned by rational traders decrease with the feedback trading (P and β). They increase with the number of overconfident traders and can increase with κ for large P and low N_1 and decrease with κ for low P and large N_1.*

Proof: See Appendix for the derivation of the expressions of the expected utility of profits for the different traders.

Due to the fact that informed investors anticipate the presence of feedback trading, feedback traders either buy shares at an inflated price or sell shares at a too low a price. Such trading behavior leads these agents to lose money.

When there are only rational and overconfident traders in the market, we find that overconfident traders earn less capital gains than their rational counterparts. This result is consistent, among others, with Odean (1998), Gervais and Odean (2001). When the 3 types of traders are present (overconfident, rational and positive feedback agents) overconfident agents may outperform rational traders. This result confirms Benos (1998), Kyle and Wang (1997) and Germain et al. (2014). In these papers pure liquidity traders are present. Hirshleifer, Subrahmanyam and Titman (2006) show that irrational traders can earn positive expected profit in the presence of positive feedback trading if they trade early.

The last part of the first point in the above proposition stems from the fact that the trend-chasers' trading volume decreases with κ and increases with γ.

We dedicate the next section to contrarian trading, i.e. feedback traders selling (buying) when prices increase (decrease).

6   On Contrarian Trading

In this section we look at the case of contrarian trading. Given the setup of the model, we can investigate the effect of negative feedback trading by considering a negative β (negative feedback trading intensity). Negative feedback traders or contrarian traders sell (buy) securities when prices rise (fall).

First of all, it should be noticed that due to their trading behavior, negative feedback traders limit the movement of prices. Informed traders upon receiving some information will always fol-

low their information and trade larger quantities as they know that due to negative feedback trading subsequent prices will not fully reflect their information. This alters some of the comparative statics we found earlier for the case of positive feedback trading.

We find the obvious result that the volatility of prices decreases with the number of negative feedback and increases with the number of overconfident.

The quality of prices in both periods decreases with the number of negative feedback whereas the quality of prices in the third period increases with the number of overconfident and with κ.

The serial correlation of prices is negative and decreases with the number of negative feedback traders.

The overall volume traded by rational investors increases with the number of negative feedback and decreases with the level of overconfidence in the market, κ. As for the case of positive feedback traders, the overall volume traded by overconfident traders is non-monotonic in the number of negative feedback traders. For a low number of feedback traders, the volume traded by overconfident decreases with P whereas for a large number of feedback traders it increases with P. The range for which it decreases with P increases as the level of overconfidence, κ, increases.

Finally, we find that the expected profit of the negative feedback traders decreases with the number of negative feedback present in the market and that they can derive positive expected profits for a low number of negative feedback traders. In that case the second round gains compensate the third round losses. However, as overconfident traders become more overconfident, measured by an increase of κ, the expected profit decreases. When feedback traders are negative, they can earn positive expected profits as they sell (buy) at high (low) price due to the informed traders behavior.

7   Discussion and Empirical Implications

In this section, we are interested in understanding how prices change in a financial market. We have shown that positive feedback trading induces an increase in price volatility and worsens market efficiency more than overconfident trading. When positive feedback agents are introduced to the market, overconfident investors may obtain greater expected utility than rational speculators. Positive feedback traders can suffer important losses if there are too numerous.

In order to investigate the level of prices, we have numerically simulated the price pattern over time as a function of the number of positive feedback traders, the number of overconfident traders and the risk aversion coefficient a.

Figure 5 shows the price pattern over time as a function of the number of feedback traders

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and the number of overconfident traders. Feedback trading is based on the previous prices movement and appears as lagged trading. It can be seen that the number of both overconfident traders and feedback traders impact prices. The more feedback traders, the more the price initially increases and the more the decrease of prices between period 2 and 3. In the second graph we can see that increasing the number of overconfident do not increase the variation of prices for a relatively large number of feedback traders (here equal to 10).
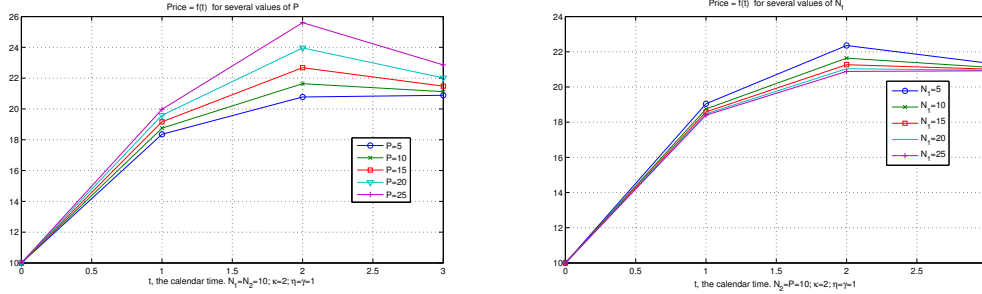

Figure 5: *Prices over time for different values of P (the number of positive feedback traders)and for different values of N_1 (the number of overconfident agents).*

We observe, in figure 6, that both the volatility of price and the price change increase with the risk aversion coefficient. Informed investors react abruptly to new information and lead prices to be more volatile. This result emphasizes that the risk aversion of the market participants can explain part of the excess volatility observed in the market.
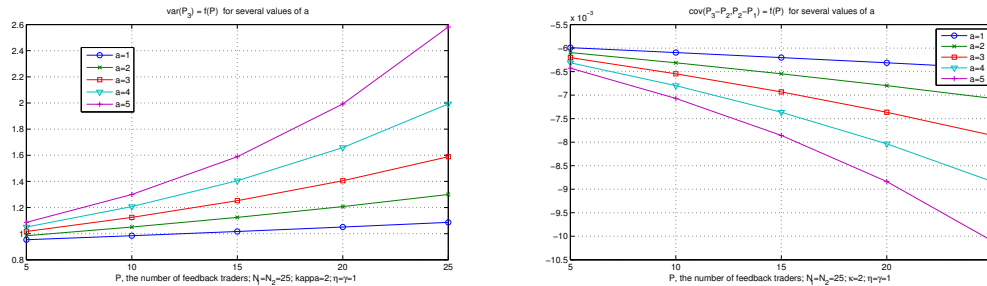

Figure 6: *The volatility of prices at date $t = 3$ and the $cov(P_3 - P_2, P_2 - P_1)$, for different values of the parameter a.*

Figure 7 shows that the price changes is less extreme when the market participants are less risk averse. We also show how the difference between the overconfident investors 'demand and the rational one depends on P and on the speculators' risk aversion. As the risk aversion increases and the number of feedback traders increases, rational and overconfident traders tend to trade in the same manner. The difference is extreme when there are a large number of feedback traders with a low level of risk aversion.
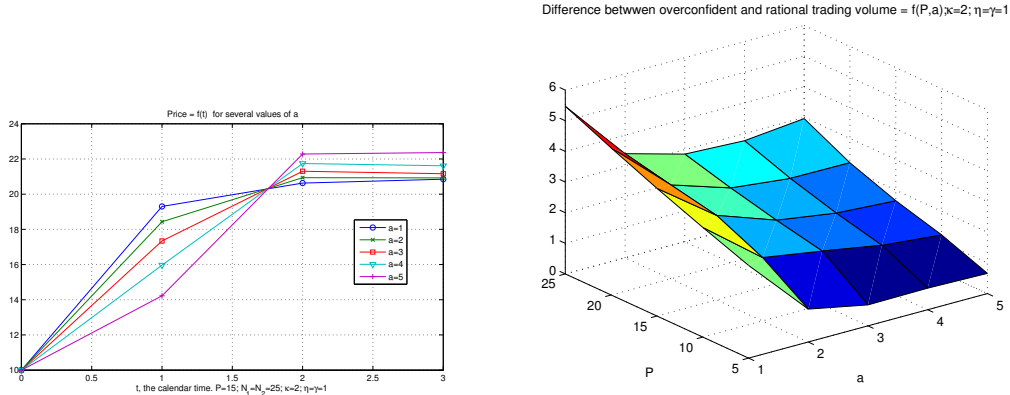
Figure 7: *Price over time for different values of a and difference demands between overconfident and rational investors.*

Empirically we find some testable implications.

Our model finds a positive relationship between the volatility of prices and the size of the price reversal. Indeed, when the price volatility is low (high), the serial correlation of prices is low (high). The causality is unclear in our model. This would have to be tested. The literature on feedback trading concludes that the presence of positive (negative) feedback traders leads to negative (positive) serially correlated returns together with an increase (decrease) in volatility. For instance Siklos and Reitz (2006) puts forward that relationship between negative serially correlated returns and high volatility when positive feedback traders are present. For further evidences, see Shu (2009), Sias and Starks (1997), Sias, Starks and Titman (2001) and Bohl and Siklos (2005), for instance.

Our model predicts that the size of the serial correlation and the volatility is linked to the number of positive feedback traders. More positive feedback traders implies more volatility and more negative serial correlation. If the two phenomena are observed in unison this could give an indication of the presence of feedback traders.

De Long et al. (1990) find that prices exhibit a positive correlation at short horizons whereas at long horizons price changes are negatively serially correlated. We show that this property depends on different parameters of the model such as the number of positive feedback traders, how overconfident perceive the information of others, as well as the number of informed traders present in the model.

However, this property can be reversed when there are no rational traders and overconfident traders are not too numerous and undervalue the information of others. In that case we find that prices exhibit a negative correlation at short horizons whereas at long horizons price changes are negatively serially correlated.

From a policy point of view our paper, as others, would recommend the limitation of Computer Based Trading (CBT) or algorithmic trading (AH) which would lead to feedback loop destabilizing markets. Our specification of feedback traders is very close to the actual behavior of computer based trading or algorithmic trading in the sense that both CBT and AH are very mechanical. Our feedback traders chase the trend in this mechanical aspect and this aspect is one major factor in the creation of asset of asset price bubbles.[10] In our model, increasing CBT, as measured by an increase of P and β, implies more volatility, greater price changes that both can possibly lead to bubbles and then crashes.

8   Conclusion

Our paper analyzes the interaction between different type of traders in a financial market. We shed some light on the result of the competition between feedback traders and two types of informed traders some being rational and others being overconfident. This enables us to revisit the well known result found by De Long et al. (1990) whereby rational speculation together with the presence of positive feedback traders can destabilize prices and facilitate the creation of price bubbles. We want to investigate the impact of introducing overconfident traders.

Positive feedback trading is one of the common mechanical strategy used by Computer Based Trading (CBT). It is estimated that CBT ranks from 30% in the UK to 60% in the USA. Our paper can be viewed as an analysis of the interaction between this type of trading and both rational and overconfident informed traders.

Positive feedback traders or CBT increase price volatility. Their trade is based on past prices which are determined by the informed trading in earlier stages. This causes a temporary miscoordination between traders. This phenomenon is amplified by the fact that both overconfident and rational traders anticipate the behavior of feedback-positive agents. The model finds some striking results. Due to the competition between overconfident traders and rational traders, we find that the presence of overconfident traders can decrease the volatility of prices and improve price quality. The presence of overconfident traders leads rational traders to scale down their trading implying a decrease of the volatility of prices caused by positive feedback traders. Overconfident traders commit to react more to their private information. Introducing more information counterbalances the effect of positive feedback.

We also find that the presence of feedback traders leads to an increase in the volume of both rational and overconfident traders. It can also be the case that the expected utility of profits from trading for overconfident traders are superior to the ones obtained by rational traders.

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[10]Feedback trading is often cited as the reason of the bubble (see Zhou and Sornette (2006, 2009), Cajueiro, Tabak and Werneck (2009), and Johansen, Ledoit, and Sornette (2003)). See also Abreu and Brunnermeier (2003).

In line with empirical findings we find a positive relationship between the volatility of prices and the size of the price reversal. However our model cannot establish any causality. Nevertheless, we show that this result depends on different parameters of the model such as the number of positive feedback traders, how overconfident perceive the information of others, as well as the number of informed traders present in the model. In addition we obtain that the presence of positive feedback traders leads to more volume being traded by both types of informed traders.

Finally, from a policy point of view our paper would recommend the limitation of Computer Based Trading (CBT) or algorithmic trading (AH) as this type of trading destabilizes markets by introducing volatility and leading prices to be less informative.

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10   Appendix

Proof of Proposition 3.1: Equilibrium

This proposition is proved by backward induction, we then start with the last period, i.e. $t = 3$.

Third round $t = 3$

At time $t = 3$, each trader, noted i, has private information Φ_{3i} which has a multivariate distribution. The information available for trader i is : $\Phi_{3i} = [y_{2i}, y_{3i}, P_2, P_3]^T$.

An overconfident trader infers the mean of this distribution, $E_b(\Phi_{3i})$, and the variance-covariance matrix, Ψ_b, as follows:

$$E_b(\Phi_{3i}) = [\bar{v} + b, \bar{v} + b, \alpha_{21} + \alpha_{22}(\bar{v} + b), \alpha_{31} + (\alpha_{32} + \alpha_{33})(\bar{v} + b)]^T,$$

$$\Psi_b = \begin{bmatrix} \frac{1}{\eta h_v} + \frac{1}{\kappa h_\varepsilon} & \frac{1}{\eta h_v} & \frac{\alpha_{22}}{\eta h_v} + \frac{\alpha_{22}}{M\kappa h_\varepsilon} & \frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{32}}{M\kappa h_\varepsilon} \\ \frac{1}{\eta h_v} & \frac{1}{\eta h_v} + \frac{1}{\kappa h_\varepsilon} & \frac{\alpha_{22}}{\eta h_v} & \frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{33}}{M\kappa h_\varepsilon} \\ \frac{\alpha_{22}}{\eta h_v} + \frac{\alpha_{22}}{M\kappa h_\varepsilon} & \frac{\alpha_{22}}{\eta h_v} & \frac{\alpha_{22}^2}{\eta h_v} + \frac{\alpha_{22}^2(\gamma+M\kappa-\kappa)}{M^2\kappa\gamma h_\varepsilon} & C_1 \\ \frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{32}}{M\kappa h_\varepsilon} & \frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{33}}{M\kappa h_\varepsilon} & C_1 & C_2 \end{bmatrix},$$

with $C_1 = \frac{\alpha_{22}\alpha_{33}}{\eta h_v} + \alpha_{22}\alpha_{32}\left(\frac{1}{\eta h_v} + \frac{\gamma+M\kappa-\kappa}{M^2\kappa\kappa\gamma h_\varepsilon}\right)$, and $C_2 = \frac{(\alpha_{32}+\alpha_{33})^2}{\eta h_v} + (\alpha_{32}+\alpha_{33})^2\left(\frac{\gamma+M\kappa-\kappa}{M^2\kappa\gamma h_\varepsilon}\right)$.

The mean, $E_r(\Phi_{3i})$, and the variance-covariance matrix Ψ_r for the rational agent are obtained by setting $b = 0$, $\eta = \kappa = \gamma = 1$ in $E_b(\Phi_{3i})$ and Ψ_b.

By solving the mean-variance problem, we obtain the ith insider's orders:

$$x_{3i}^b = \frac{E_b(\tilde{v}|\Phi_{3i}) - P_3}{avar_b(\tilde{v}|\Phi_{3i})},$$

$$x_{3i}^r = \frac{E_r(\tilde{v}|\Phi_{3i}) - P_3}{avar_r(\tilde{v}|\Phi_{3i})}.$$

On the other hand, we know that each feedback trader i determines her order by considering the trend of prices as follows :

$$x_{3i}^f = \beta(P_2 - P_1).$$

Using the projection theorem, we obtain the following

$$E_b(\tilde{v}|\tilde{\Phi}_{3i}) = \frac{(y_{2i} + y_{3i})(\kappa - \gamma)h_\varepsilon + (\bar{Y}_2 + \bar{Y}_3)\gamma h_\varepsilon M + \eta h_v(\bar{v} + b)}{\eta h_v + 2(\kappa + \gamma M - \gamma)h_\varepsilon},$$

$$var_b(\tilde{v}|\Phi_{3i}) = \frac{1}{\eta h_v + 2(\kappa + \gamma M - \gamma)h_\varepsilon},$$

$$E_r(\tilde{v}|\Phi_{3i}) = \frac{(\bar{Y}_2 + \bar{Y}_3)h_\varepsilon M + h_v\bar{v}}{h_v + 2Mh_\varepsilon},$$

$$var_r(\tilde{v}|\Phi_{3i}) = \frac{1}{h_v + 2Mh_\varepsilon}.$$

Replacing the above into the expressions of the different orders for the different types of traders we obtain

$$\begin{aligned} x_{3i}^b &= \tfrac{1}{q}[(y_{2i} + y_{3i})(\kappa - \gamma)h_\varepsilon + (\bar{Y}_2 + \bar{Y}_3)\gamma h_\varepsilon M + \eta h_v(\bar{v} + b) - P_3(\eta h_v + 2(\kappa + \gamma M - \gamma)h_\varepsilon)], \\ x_{3i}^r &= \tfrac{1}{a}[(\bar{Y}_2 + \bar{Y}_3)h_\varepsilon M + h_v\bar{v} - P_3(h_v + 2Mh_\varepsilon)], \\ x_{3i}^f &= \beta(P_2 - P_1). \end{aligned}$$

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In equilibrium the total demand must be equal to the exogenous total supply, this is given by

$$\sum_{i=1}^{N_1} x_{3i}^b + \sum_{i=1}^{N_2} x_{3i}^r + \sum_{i=1}^{P} x_{3i}^f = (N_1 + N_2 + P)\bar{x}. \tag{10.3}$$

From (10.3), the price P_3 can be obtained as a function of $\bar{Y}_2$, $\bar{Y}_3$, P_2 and P_1

$$P_3 = \frac{1}{\Lambda}[((N_1\gamma + N_2)Mh_\varepsilon + (\kappa - \gamma)N_1 h_\varepsilon)(\bar{Y}_2 + \bar{Y}_3) + (N_1\eta + N_2)h_v\bar{v} + N_1\eta h_v b$$
$$- a(N_1 + N_2 + P)\bar{x} + aP\beta(P_2 - P_1)],$$

with $\Lambda = (N_1\eta + N_2)h_v + 2(N_1(\kappa + \gamma M - \gamma) + N_2 M)h_\varepsilon$, $\bar{Y}_2 = \tilde{v} + \frac{\varepsilon_{2i}}{M} + \frac{1}{M}\sum_{j\neq i}\varepsilon_{2j}$ and $\bar{Y}_3 = \tilde{v} + \frac{\varepsilon_{3i}}{M} + \frac{1}{M}\sum_{j\neq i}\varepsilon_{3j}$.

Using equation (3.1) and identifying the parameters $\alpha_{i,j}$ we obtain:

$$\alpha_{31} = \frac{(N_1\eta + N_2)h_v\bar{v} + N_1\eta h_v b - a(N_1 + N_2 + P)\bar{x}}{\Lambda} + \frac{aP\beta}{\Lambda}(\alpha_{21} - P_1),$$

$$\alpha_{32} = \frac{N_1(\kappa + \gamma M - \gamma)h_\varepsilon + N_2 M h_\varepsilon}{\Lambda} + \frac{a\beta P}{\Lambda}\alpha_{22},$$

$$\alpha_{33} = \frac{N_1(\kappa + \gamma M - \gamma)h_\varepsilon + N_2 M h_\varepsilon}{\Lambda}.$$

Second round $t = 2$

Using the third round's results, we can obtain the second round's parameters.

Let us introduce the following notations where b stands for the overconfident trader whereas r stands for the rational one:

$$B_b^T = cov_b(P_3, \Phi_{2i}) = [cov_b(y_{2i}, P_3), cov_b(P_2, P_3)],$$

$$B_r^T = cov_r(P_3, \Phi_{2i}) = [cov_r(y_{2i}, P_3), cov_r(P_2, P_3)].$$

Using the projection theorem, we get

$$E_j(P_3|\Phi_{2i}) = E_j(P_3) + B_j^T var_j(\Phi_{2i})^{-1}(\Phi_{2i} - E_j(\Phi_{2i})),$$

$$var_j(\Phi_{2i}) = \begin{pmatrix} var_j(y_{2i}) & \alpha_{22}cov_j(y_{2i}, \bar{Y}_2) \\ \alpha_{22}cov_j(y_{2i}, \bar{Y}_2) & \alpha_{22}^2 var_j(\bar{Y}_2) \end{pmatrix},$$

where $j = b, r$ and E_j and var_j denote the fact that they are computed following trader j's beliefs.

We obtain

$$
\begin{aligned}
E_j(P_3|\Phi_{2i}) &= (\alpha_{32} + \alpha_{33})\bar{v}_j + \alpha_{31} + \tfrac{1}{L_j}((cov_j(y_{2i}, P_3)D_1^j + cov_j(P_2, P_3)\tfrac{D_3^j}{\alpha_{22}})(y_{2i} - \bar{v}) \\
&\quad + cov_j(y_{2i}, P_3)D_3^j + cov_j(P_2, P_3)\tfrac{D_2^j}{\alpha_{22}}(\bar{Y}_2 - \bar{v})),
\end{aligned}
$$

with

$$
\begin{aligned}
\bar{v}_j &= \begin{cases} \bar{v} \text{ if } j = r \\ \bar{v} + b \text{ if } j = b \end{cases}, \\
D_1^b &= var_b(\bar{Y}_2) = \tfrac{1}{\eta h_v} + \tfrac{(\gamma + M\kappa - \kappa)}{M^2 \gamma \kappa h_\varepsilon}, \\
D_2^b &= var_b(y_{2i}) = \tfrac{1}{\eta h_v} + \tfrac{1}{\kappa h_\varepsilon}, \\
D_3^b &= -cov_b(y_{2i}, \bar{Y}_2) = -\left(\tfrac{1}{\eta h_v} + \tfrac{1}{M\kappa h_\varepsilon}\right), \\
L_b &= var_b(y_{2i})var_b(\bar{Y}_2) - cov_b(y_{2i}, \bar{Y}_2)^2 = \tfrac{(M-1)[((M-1)\gamma + \kappa)h_\varepsilon + \eta h_v]}{M^2 \eta \kappa \gamma h_v h_\varepsilon^2}.
\end{aligned}
$$

D_1^r, D_1^r, D_3^r, and L_r can be obtained by setting $\kappa = \gamma = \eta = 1$ in D_1^b, D_1^b, D_3^b, and L_b.

At the second round, informed trader i's order is:

$$
x_{2i}^b = \frac{E_b(P_3|\Phi_{2i}) - P_2}{avar_b(P_3|\Phi_{2i})},
$$

$$
x_{2i}^r = \frac{E_r(P_3|\Phi_{2i}) - P_2}{avar_r(P_3|\Phi_{2i})}.
$$

The ith feedback agent's order is:

$$
x_{2i}^f = \beta(P_1 - P_0).
$$

By equalizing exogenous supply and demand, we have:

$$
(N_1 + N_2 + P)\bar{x} = \sum_{i=1}^{N_1} \frac{E_b(P_3|\Phi_{2i}) - P_2}{avar_b(P_3|\Phi_{2i})} + \sum_{i=1}^{N_2} \frac{E_r(P_3|\Phi_{2i}) - P_2}{avar_r(P_3|\Phi_{2i})} + P\beta(P_1 - P_0).
$$

First round $t = 1$

At the first round, none of the traders participating to the market are informed. The different agents' orders are:

$$
x_{1i}^r = \frac{E_r(P_2) - P_1}{avar_r(P_2)} = \frac{\alpha_{21} + \alpha_{22}\bar{v} - P_1}{a\alpha_{22}^2 var_r(\bar{Y}_2)},
$$

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$$x_{1i}^b = \frac{E_b(P_2) - P_1}{avar_b(P_2)} = \frac{\alpha_{21} + \alpha_{22}(\bar{v} + b) - P_1}{a\alpha_{22}^2 var_b(\bar{Y}_2)}.$$

There are no feedback traders in the first round. However, the agents who will become informed subsequently anticipate the presence of such behavior for the next two rounds.

Again, the no-excess supply equation leads to

$$(N_1 + N_2)\bar{x} = \sum_{i=1}^{N_1} x_{1i}^b + \sum_{i=1}^{N_2} x_{1i}^r.$$

The price P_1 can then be derived:

$$P_1 = \alpha_{21} + \alpha_{22}\bar{v} + \frac{-a\alpha_{22}^2\bar{x}(N_1 + N_2)var_r(\bar{Y}_2)var_b(\bar{Y}_2) + N_1\alpha_{22}bvar_r(\bar{Y}_2)}{N_1 var_r(\bar{Y}_2) + N_2 var_b(\bar{Y}_2)}.$$

From the above expression the parameters α_{21} and α_{22} can be identified.

After some computations, one can show that $\alpha_{22} = \frac{N}{ca\beta P + d}$ where N is independent of P, with

$$c = -(M-1)[N_1 var_r(P_3|\Phi_{2i})L_r t_1 + N_2 var_b(P_3|\Phi_{2i})L_b t_2],$$
$$d = \Lambda\eta h_v M^2\kappa\gamma h_\varepsilon{}^2 L_b L_r(N_1 var_r(P_3|\Phi_{2i}) + N_2 var_b(P_3|\Phi_{2i})),$$
$$N = (M-1)[N_1(\kappa + \gamma(M-1)) + MN_2](N_1 var_r(P_3|\Phi_{2i})L_r z_1 + N_2 var_b(P_3|\Phi_{2i})L_b z_2),$$

with

$$t_1 = [\kappa + \gamma(M-1)]h_\varepsilon + \eta h_v,$$
$$t_2 = \eta\kappa\gamma(Mh_\varepsilon + h_v),$$
$$z_1 = 2(\kappa + \gamma(M-1))h_\varepsilon + \eta h_v,$$
$$z_2 = 2\eta\kappa\gamma Mh_\varepsilon + \eta\kappa\gamma h_v,$$

Thus, we note that $c < 0$, $d > 0$ and $N > 0$.

We obtain the condition of equilibrium by considering that the parameter before the mean of the signals α_{22}

On the other hand, after some computations we have obtained the expression of α_{21} :

$$\alpha_{21} = \frac{A_1/B_1 + B_2}{D}$$

with

$$D = 1 - \frac{avar_b(P_3|\Phi_{2i})(2Mh_\varepsilon + h_v)\alpha_{33}^2\beta P}{N_1(2Mh_\varepsilon + h_v)\alpha_{33}^2 + N_2 Mh_\varepsilon(Mh_\varepsilon + h_v)var_b(P_3|\Phi_{2i})}$$

$$B_1 = \frac{N_1(2Mh_\varepsilon + h_v)\alpha_{33}^2}{Mh_\varepsilon(Mh_\varepsilon + h_v)} + N_2 var_b(P_3|\Phi_{2i})$$

$$B_2 = \frac{avar_b(P_3|\Phi_{2i})(2Mh_\varepsilon + h_v)\alpha_{33}^2\beta P\alpha_{22}}{N_1(2Mh_\varepsilon + h_v)\alpha_{33}^2 + N_2 var_b(P_3|\Phi_{2i})Mh_\varepsilon(Mh_{varepsilon} + h_v)}(v + E)$$

with

$$E = \frac{-ax\alpha_{22}(N_1 + N_2)\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right)\left(\frac{1}{\eta h_v} + \frac{\gamma + M\kappa - \kappa}{M^2\kappa\gamma h_\varepsilon}\right) + N_1 b\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right)}{N_1\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right) + N_2\left(\frac{1}{\eta h_v} + \frac{1}{M^2\kappa\gamma h_\varepsilon}\right)}$$

$$A_1 = N_1\frac{(2Mh_\varepsilon + h_v)}{Mh_\varepsilon(Mh_\varepsilon + h_v)}\alpha_{33}^2(\alpha_{31} + F(v + b)) + N_2 var_b(P_3|\Phi_{2i})G + H$$

$$F = \left((\alpha_{32} + \alpha_{33}) - \frac{\left(\frac{\alpha_{32} + \alpha_{33}}{\eta h_v} + \frac{\alpha_{32}}{M\kappa h_\varepsilon}\right)\left(\frac{\gamma + M\kappa - \kappa}{M^2\kappa\gamma h_\varepsilon} - \frac{1}{M\kappa h_\varepsilon}\right)\left(\frac{\alpha_{33}}{\eta h_v} + \alpha_{33}\left(\frac{1}{\eta h_v} + \frac{\gamma + M\kappa - \kappa}{M^2\kappa\gamma h_\varepsilon}\right)\right)\left(\frac{1}{\kappa h_\varepsilon} - \frac{1}{M\kappa h_\varepsilon}\right)}{\left(\frac{1}{\eta h_v} + \frac{1}{\kappa h_\varepsilon}\right)\left(\frac{1}{\eta h_v} + \frac{\gamma + M\kappa - \kappa}{M^2\kappa\gamma h_\varepsilon}\right) - \left(\frac{1}{\eta h_v} + \frac{1}{M\kappa h_\varepsilon}\right)^2}\right)$$

$$G = \alpha_{31} + (\alpha_{32} + \alpha_{31})v - \frac{\left(\frac{\alpha_{33}}{h_v} + \alpha_{32}\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right)\right)\left(\frac{1}{h_\varepsilon} - \frac{1}{Mh_\varepsilon}\right)v}{\left(\frac{1}{h_v} + \frac{1}{h_\varepsilon}\right)\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right) - \left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right)^2}$$

$$H = \frac{-a(2Mh_\varepsilon + h_v)\alpha_{33}^2 var_b(P_3|\Phi_{2i})((N_1 + N_2 + P)x + \beta P P_0)}{Mh_\varepsilon(Mh_\varepsilon + h_v)}$$

Therefore there exists an equilibrium when $\begin{cases} \alpha_{22} > 0 \\ D > 0 \end{cases}$

In other words when,

$$\begin{cases} a\beta P < \frac{-d}{c} \\ a\beta P < \frac{N_1}{var_b(P_3|\Phi_{2i})} + \frac{N_2 Mh_\varepsilon(Mh_\varepsilon + h_v)}{\alpha_{33}^2(2Mh_\varepsilon + h_v)} \end{cases}$$

We have pointed out the condition of the equilibrium that maintains prices positive (and the coefficient $\alpha_{22} > 0$). Thus we exhibit a linear equilibrium when $a\beta P < g^*(N_1, N_2)$; with $g^*(N_1, N_2) = min\left(\frac{-d}{c}, \frac{N_1}{var_b(P_3|\Phi_{2i})} + \frac{N_2 Mh_\varepsilon(Mh_\varepsilon + h_v)}{\alpha_{33}^2(2Mh_\varepsilon + h_v)}\right)$

Proof of Proposition 4.2: Volatility

We now derive the variance of prices with respect to P.

- **t = 2**

The variance is given by $var(P_2) = \alpha_{22}^2 var(\bar{Y}_2)$ with $\alpha_{22} = \frac{N}{ca\beta P + d}$.

The derivative of $var(P_2)$ with respect to P is then equal to

$$\frac{\partial var(P_2)}{\partial P} = \frac{\partial \alpha_{22}^2}{\partial P} var(\bar{Y}_2) = 2\alpha_{22} \frac{\partial \alpha_{22}}{\partial P} var(\bar{Y}_2) = 2\alpha_{22} \left(-ca\beta \frac{N}{(ca\beta P + d)^2} \right) var(\bar{Y}_2)$$

.

Given that $var(\bar{Y}_2) > 0$, $\alpha_{22} > 0$ and $-ca\beta \frac{N}{(ca\beta P + d)^2} > 0$ it can be established that

$$\frac{\partial var(P_2)}{\partial P} > 0$$

.

- **t = 3**

The variance for $t = 3$ prices is given by

$$var(P_3) = \alpha_{32}^2 var(\bar{Y}_2) + \alpha_{33}^2 var(\bar{Y}_3) + 2\alpha_{32}\alpha_{33} cov\left(\bar{Y}_2, \bar{Y}_3\right)$$
$$= (\alpha_{32}^2 + \alpha_{33}^2)(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}) + \frac{2\alpha_{32}\alpha_{33}}{h_v}.$$

Using the fact that $\alpha_{32} = \alpha_{33} + \frac{a\beta P}{\Lambda}\alpha_{22}$, we can rewrite the variance of prices as follows

$$var(P_3) = \left(\alpha_{33} + \frac{aP\beta}{\Lambda}\alpha_{22} \right) \left[\left(\alpha_{33} + \frac{aP\beta}{\Lambda}\alpha_{22} \right) \left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon} \right) + \frac{2\alpha_{33}}{h_v} \right] + \alpha_{33}^2 \left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon} \right).$$

The derivative is then given by the following expression

$$\frac{\partial var(P_3)}{\partial P} = 2 \left(\frac{a\beta P}{\Lambda} \frac{\partial \alpha_{22}}{\partial P} + \frac{a\beta P\alpha_{22}}{\Lambda} \right) \left[\left(\alpha_{33} + \frac{a\beta P\alpha_{22}}{\Lambda} \right) \left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon} \right) + \frac{\alpha_{33}}{h_v} \right].$$

Knowing that the condition of the equilibrium gives that $\alpha_{22} > 0$ and $\frac{\partial \alpha_{22}}{\partial P} > 0$ we directly deduce that $\frac{\partial var(P_3)}{\partial P} > 0$

Proof of Proposition 4.3: Quality of Prices

We now look at the quality of prices. We first start with $t = 2$ and then turn to $t = 3$.

The quality of prices for $t = 2$ is given by $var(P_2 - \tilde{v})$ which is given by the following expression

$$var(P_2 - \tilde{v}) = \frac{\alpha_{22}^2}{h_v} + \frac{\alpha_{22}^2}{Mh_\varepsilon} + \frac{1}{h_v} - 2\frac{\alpha_{22}}{h_v} = \frac{(\alpha_{22} - 1)^2}{h_v} + \frac{\alpha_{22}^2}{Mh_\varepsilon}$$

The derivative with respect to P is positive and given by

$$\frac{\partial var(P_2 - \tilde{v})}{\partial P} = \frac{2(\alpha_{22} - 1)}{h_v}\frac{\partial \alpha_{22}}{\partial P} + \frac{2\alpha_{22}}{Mh_\varepsilon}\frac{\partial \alpha_{22}}{\partial P} = 2\frac{\partial \alpha_{22}}{\partial P}[\frac{\alpha_{22} - 1}{h_v} + \frac{\alpha_{22}}{Mh_\varepsilon}].$$

For the quality of prices for $t = 3$, we proceed as for $t = 2$. The quality of prices is equal to

$$var(P_3 - \tilde{v}) = \frac{(\alpha_{32} + \alpha_{33} - 1)^2}{h_v} + \frac{\alpha_{32}^2 + \alpha_{33}^2}{Mh_\varepsilon}.$$

The derivative is then

$$\frac{\partial var(P_3 - \tilde{v})}{\partial P} = \frac{\partial \alpha_{32}}{\partial P}\left(\frac{2(\alpha_{32} + \alpha_{33} - 1)}{h_v} + \frac{2\alpha_{32}}{Mh_\varepsilon}\right)$$

.

Since $\alpha_{22} > \frac{N}{d} > 1$, we conclude that $\frac{\partial var(P_2 - \tilde{v})}{\partial P} > 0$ and $\frac{\partial var(P_3 - \tilde{v})}{\partial P} > 0$

Proof of Proposition 4.4: Serial Correlation of Prices

We now look at the serial correlation of prices. It is given by $cov(P_3 - P_2, P_2 - P_1)$.

We have

$$cov(P_3 - P_2, P_2 - P_1) = cov(P_3 - P_2, P_2) = cov(P_3, P_2) - cov(P_2, P_2)$$

.

After some manipulations, it can be rewritten as

$$cov(P_3 - P_2, P_2 - P_1) = \alpha_{22}\left(var(\bar{Y}_2)(\alpha_{32} - \alpha_{22}) + \alpha_{32}cov\left(\bar{Y}_2, \bar{Y}_3\right)\right).$$

Using the fact that $\alpha_{32} = \alpha_{33} + \frac{a\beta P}{\Lambda}\alpha_{22}$, $cov\left(\bar{Y}_2, \bar{Y}_3\right) = \frac{1}{h_v}$ and that $var(\bar{Y}_2) = \frac{1}{h_v + Mh_\varepsilon}$, we obtain

$$cov(P_3 - P_2, P_2 - P_1) = \alpha_{22}\left[\left(\alpha_{33} + \alpha_{22}\left(\frac{a\beta P}{\Lambda} - 1\right)\right)\frac{1}{h_v + Mh_\varepsilon} + \frac{\alpha_{33}}{h_v}\right].$$

The derivative with respect to P is then given by

$$\frac{\partial cov(P_3 - P_2, P_2 - P_1)}{\partial P} = \left[\frac{a\beta}{\Lambda}\alpha_{22}^2 + \frac{\partial \alpha_{22}}{\partial P}\left(\alpha_{33} + 2\alpha_{22}\left(\frac{a\beta P}{\Lambda} - 1\right)\right)\right]\left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right) + \alpha_{33}\frac{\partial \alpha_{22}}{\partial P}\frac{1}{h_v}.$$

We now consider the case where $N_1 = 0$, after developing the different coefficients, we find:

$$\frac{\partial cov(P_3 - P_2, P_2 - P_1)}{\partial P} \frac{(ca\beta P + d)^3}{-ca\beta N N_2(M-1)var_b(P_3|\Phi_{2i})} = (-a\beta P + N_2(2Mh_\varepsilon + h_v)) Z$$

with

$$Z = \left[1 + \left(\frac{Mh_\varepsilon + h_v}{2Mh_\varepsilon + h_v}h_\varepsilon\right)\left(\frac{2}{h_v} + \frac{1}{Mh_\varepsilon}\right) - \left(\frac{1}{h_v} + \frac{1}{Mh_\varepsilon}\right)\right]$$

This ends the proof. When $N_1 \neq 0$, we have obtained the results by simulations.

Proof of Proposition 5.6: The expected utility of profits

We now compute the expected utility profits for all type of traders for each trading round.

Third round for the different expected utility of profits

The overconfident traders' expected utility of profits, Π_{3i}^b, is given by

$$\Pi_{3i}^b = E(x_{3i}^b(\tilde{v} - P_3)) = E(x_{3i}^b \tilde{v}) - E(x_{3i}^b P_3),$$

where the demand, x_{3i}^b, is

$$x_{3i}^b = \alpha^*(y_{2i} + y_{3i}) + \beta^*(Y_2 + Y_3) - \gamma^* P_3 + \delta^*,$$

with $\alpha^* = \frac{(\kappa - \gamma)h_\varepsilon}{a}$, $\beta^* = \frac{\gamma Mh_\varepsilon}{a}$, $\gamma^* = \frac{\eta h_v + 2(\kappa + \gamma(M-1))h_\varepsilon}{a}$ and $\delta^* = \frac{\eta h_v(\bar{v} + b)}{a}$.

After having computed the two elements of the expected profit and after some simplifications we obtain

$$\Pi_{3i}^b = (\alpha^* + \beta^*)[\frac{2}{h_v} + 2\bar{v}^2 - 2\alpha_{31}\bar{v} - (\alpha_{32} + \alpha_{33})(\frac{2}{h_v} + 2\bar{v}^2 + \frac{1}{Mh_\varepsilon})]$$
$$- \gamma^*[\alpha_{31}(1 - 2(\alpha_{32} + \alpha_{33}))\bar{v} - \alpha_{31}^2 + (\alpha_{32} + \alpha_{33})(1 - (\alpha_{32} + \alpha_{33}))(\frac{1}{h_v} + \bar{v}^2) - \frac{(\alpha_{32}^2 + \alpha_{33}^2)}{Mh_\varepsilon} - \alpha_{31}^2]$$
$$+ \delta^*[\bar{v}(1 - (\alpha_{32} + \alpha_{33})) - \alpha_{31}].$$

We now focus on the rational traders' expected profit, Π_{3i}^r. In order to compute Π_{3i}^r the same steps as for calculating the expected profit of the overconfident traders can be followed. The

demand for the rational trader, x_{3i}^r, has the same linear form as the demand for the overconfident trader. The coefficients are given by $\alpha^* = 0$, $\beta^* = \frac{Mh_\varepsilon}{a}$, $\gamma^* = \frac{h_v + 2Mh_\varepsilon}{a}$ and $\delta^* = \frac{h_v \bar{v}}{a}$.

The rational traders' expected profit, Π_{3i}^r, can be obtained from (??) by replacing $\kappa = \gamma = \eta = 1$ and $b = 0$.

The feedback traders' expected profit Π_{3i}^f depends on the first and second round prices P_1 and P_2 through their demand $x_{3i}^f = \beta(P_2 - P_1)$. It can be calculated by computing the following expression

$$\Pi_{3i}^f = E(x_{3i}^f(\tilde{v} - P_3)) = E(x_{3i}^f \tilde{v}) - E(x_{3i}^f P_3).$$

After some computations and simplifications, it is given

$$
\begin{aligned}
\Pi_{3i}^f &= \beta\left[(\alpha_{21} - P_1)(\alpha_{31} + (1 - (\alpha_{32} + \alpha_{33}))\bar{v}) - \alpha_{22}\alpha_{31}\bar{v}\right] \\
&+ \beta\left[(\tfrac{1}{h_v} + \bar{v}^2)[\alpha_{22}(1 - \alpha_{32} - \alpha_{33})] - \alpha_{22}\alpha_{32}\tfrac{1}{Mh_\varepsilon}\right].
\end{aligned}
$$

Second round profit

We now determine the overconfident traders' expected profit Π_{2i}^b. As before the expected profit is given by

$$\Pi_{2i}^b = E(x_{2i}^b(\tilde{v} - P_2)) = E(x_{2i}^b \tilde{v}) - E(x_{2i}^b P_2),$$

where $x_{2i}^b = \frac{E_b[P_3/\Phi_{2i}] - P_2}{avar_b[P_3/\Phi_{2i}]}$.

After some computations, we obtain

$$\Pi_{2i}^b = \frac{1}{avar_b[P_3/\Phi_{2i}]}[(A_b - \alpha_{21})(-\alpha_{21} + (1 - \alpha_{22})\bar{v})(S_b + T_b - \alpha_{22})(\frac{1}{h_v} + \bar{v}^2 - \alpha_{21}\bar{v} - \alpha_{22}(\frac{1}{h_v} + \bar{v}^2 + \frac{1}{Mh_\varepsilon}))],$$

with

$$A_b = \alpha_{31} + (\alpha_{33} + \alpha_{22})(\bar{v} + b) - (cov_b(\tilde{y}_{2i}, P_3))(D_1^b + D_3^b) + (\frac{cov_b(P_2, P_3)(D_2^b + D_3^b)}{\alpha_{22}})\frac{\bar{v} + b}{L_b},$$

$$S_b = \frac{cov_b(\tilde{y}_{2i}, P_3)}{L_b}D_1^b + \frac{cov_b(P_2, P_3)}{\alpha_{22}L_b}D_3^b,$$

$$T_b = \frac{cov_b(\tilde{y}_{2i}, P_3)}{L_b}D_3^b + \frac{cov_b(P_2, P_3)}{\alpha_{22}L_b}D_2^b.$$

The expected profit of the rational traders can be computed following the same steps, we then obtain that

$$\Pi_{2i}^r = \frac{1}{avar_r[P_3/\Phi_{2i}]}[(A_r - \alpha_{21})(-\alpha_{21} + (1 - \alpha_{22})\bar{v})(S_r + T_r - \alpha_{22})(\frac{1}{h_v} + \bar{v}^2 - \alpha_{21}\bar{v} - \alpha_{22}(\frac{1}{h_v} + \bar{v}^2 + \frac{1}{Mh_\varepsilon}))],$$

with

$$A_r = \alpha_{31} + (\alpha_{33} + \alpha_{22})(\bar{v}) - (cov_r(\tilde{y_{2i}}, P_3))(D_1^r + D_3^r) + (\frac{cov_r(P_2, P_3)(D_2^r + D_3^r)}{\alpha_{22}})\frac{\bar{v}}{L_r},$$

$$S_r = \frac{cov_r(\tilde{y_{2i}}, P_3)}{L_r}D_1^r + \frac{cov_r(P_2, P_3)}{\alpha_{22}L_r}D_3^r,$$

$$T_r = \frac{cov_r(\tilde{y_{2i}}, P_3)}{L_r}D_3^r + \frac{cov_b(P_2, P_3)}{\alpha_{22}L_b}D_2^r.$$

The expected profit of the feedback traders is given by:

$$\Pi_{2i}^f = E(x_{2i}^f(\tilde{v} - P_2)) = E(x_{2i}^f\tilde{v}) - E(x_{2i}^f P_2),$$

with $x_{2i}^f = \beta(P_1 - P_0)$.

The prices P_1 and P_0 are known before trading, the expected profit can be written as

$$\Pi_{2i}^f = \beta(P_1 - P_0)(\bar{v}(1 - \alpha_{22}) - \alpha_{21}).$$

First Round profit

At the first round only the rational and the overconfident agents are trading. However, they are not informed at $t = 1$. As a consequence, the price at the first round P_1 is known before any trade is done.

The expected profit for the overconfident traders is given by

$$\Pi_{1i}^b = E[x_{1i}^b(\tilde{v} - P_1)],$$

with $x_{1i}^b = \frac{\alpha_{21} + \alpha_{22}(\bar{v}+b) - P_1}{a\alpha_{22}^2 var_b(\bar{Y}_2)}$. It is straightforward to find that

$$\Pi_{1i}^b = \frac{\alpha_{21} + \alpha_{22}(\bar{v}+b) - P_1}{a\alpha_{22}^2 var_b(\bar{Y}_2)}(\bar{v} - P_1).$$

The expected profit for the rational traders' profit is given by

$$\Pi_{1i}^r = \frac{\alpha_{21} + \alpha_{22}\bar{v} - P_1}{a\alpha_{22}^2 var_r(\bar{Y}_2)}(\bar{v} - P_1).$$

Finally, we have obtained all results about the different profits by numerical simulations.