

WE NEVER DID THIS: A FRAMEWORK FOR MEASURING NOVELTY OF TASKS IN MATHEMATICS TEXTBOOKS

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Textbooks are an important resource in Irish mathematics classrooms, which can have both a positive and negative impact on teaching and learning. The Project Maths initiative is prompting teachers and students to cross boundaries and interact with mathematics in ways that had not been considered previously. Publishers have produced new texts in response to the expectations of the revised curriculum and the changed needs of the classroom. This paper presents a framework to consider the degree of novelty presented in tasks found in mathematics textbooks. Novelty is something that has been referred to, yet not addressed directly, in existing frameworks for the analysis of mathematical tasks. A particular strength of our framework is that it takes into account the experience of the solver, as opposed to just focusing on how a task has been structured. Sections of textbooks currently being used in Irish classrooms at second level have been analysed using this framework and the results indicate that while all textbooks incorporate a significant level of novelty, there is still room for more novel tasks to be included.

INTRODUCTION

Textbooks are widely accepted as a commonly used resource in mathematics classrooms. According to Jones, Fujita, Clarke and Lu (2008) “on average, internationally, over 60 per cent of teachers report using a textbook as a supplementary resource” (p. 142). Little research has been conducted into the nature of post-primary mathematics textbooks in Ireland (Conway & Sloane, 2005). However, there is some evidence that textbooks play an important role in Irish classrooms. O’Keeffe and O’Donoghue (2009) suggest that “over 75 per cent of Irish second level teachers use a textbook on a daily basis” (p. 283). Even in early-childhood mathematics classrooms, Dunphy (2009) identifies teachers as using the textbook for “structuring the programme of work and for guidance” (p. 118). A lot of the time in the classroom appears to be related to textbook usage and very often it is the only resource which students have access to during the lesson aside from the teacher, while most of the problems assigned for classwork and homework come from the textbook (Project Maths, 2012). Harbison (2009) points out that “textbooks have a role in suggesting a possible pathway for navigating through the strands and strand units of the Mathematics curriculum” (p. 131), while Moffet (2009) acknowledges that “different textbooks lead to different instruction, and different instruction leads to different learning results” (p. 265). The latter may explain why the use of textbooks can be problematic. O’Keeffe and O’Donoghue (2009) suggest that

mathematics textbooks in the Irish system promote “retention and practice, with little focus on active learning” (p. 290). Classroom inspections in Ireland have shown that teaching is highly dependent on the class textbook which has a tendency to reinforce this drill and practice style (NCCA, 2005). The Project Maths Development Team (Project Maths, 2012) has cautioned teachers in their choice of textbook: it points out that there is no single textbook which can suit the learning needs of all students and it has advised schools, when choosing a textbook, to take into account the abilities, needs and interests of their students, as well as the quality of the book. In this paper, we use our novelty framework to classify tasks on the topic of Sequences and Series from two of these textbook series written for use in Project Maths classrooms.

When considering such a choice, it would be useful to look at the differing levels of novelty offered by textbook tasks. This paper presents a framework to consider the degree of novelty presented in tasks found in mathematics textbooks. The framework has been applied to sections of textbooks currently being used in Irish classrooms at second level. A particular advantage of this framework is that it considers the previous mathematical experiences of the student within the textbook chapter and how this relates to the task posed, something that may have been neglected in existing frameworks. The reasons for considering novelty will be explored in more detail in the next section.

There has recently been a lot of change in the mathematics classroom at second level in Ireland. Project Maths is an initiative, led by the National Council for Curriculum and Assessment (NCCA), to bring about positive change in the teaching and learning of mathematics. This new curriculum introduced to schools has a greater emphasis on conceptual understanding and advocates the development of problem-solving skills. Rather than practicing routine procedures to solve predictable problems, students are encouraged to make connections between different mathematical ideas and to develop a more flexible way of thinking. One of the key objectives of the Project Maths curriculum (NCCA, 2012) is “the ability to apply their mathematical knowledge and skill to solve problems in familiar and unfamiliar contexts” (p. 6). As a result of these changes, mathematics classroom activities are now moving away from instrumental understanding ‘knowing how’ and more time is devoted to developing relational understanding ‘knowing why’.

The publishers of mathematics textbooks have gradually produced new texts in response to the changed needs of the classroom. O’Keeffe & O’Donoghue (2012) carried out a review of the Project Maths textbooks available for schools using a modified instrument from the 1995 Third International Mathematics and Science Study and concluded that “all textbooks included in the study fall short of the standard needed” to support the intended Project Maths curriculum effectively, while acknowledging that all textbooks examined “display a genuine attempt to match Project Maths expectations but no one textbook meets all the Project Maths expectations” (p.21). Given the importance of the textbook as a classroom resource and the need for more research on the nature of mathematics textbooks at second level in Ireland at present, it is evident that there is a need for further work to be completed on analysing textbooks.

FRAMEWORKS AND NOVELTY

Before analysing tasks in a textbook, it is necessary to be clear on what is exactly being examined. Mason and Johnston-Wilder (2006, p. 4) define a task to be “what learners are asked to do in the mathematics classroom”, while a concept is described as “a label for the flow of images, thoughts, sensitivities, connections, possible actions and so on associated with an idea”. For this paper, a task is considered to be an activity where a student interacts with a mathematical topic by attempting to solve a textbook exercise either as homework or within the classroom. We have used various existing frameworks for the classification of textbook tasks: these include the Levels of Cognitive Demand (LCD) framework of Smith and Stein (1998) and Lithner’s (2000) reasoning framework, which are outlined below.

Stein, Grover and Henningsen (1996) examine mathematical tasks in terms of how they influence the kinds of thinking processes in which students engage when solving tasks. The framework for LCD used by Stein et al. (1998), describes four levels of cognitive demand for tasks. The LCD framework has four levels: Lower level (memorisation) - these tasks involve either reproducing previously learned facts, rules, formulas or definitions or committing these to memory; Lower level (procedures without connection to meaning) - these tasks are generally algorithmic and the use of a procedure is either specifically called for or is evident from prior instruction; Higher level (procedures with connection to meaning) - these tasks focus students' attention on procedures for the purpose of developing deeper levels of understanding; Higher level (doing mathematics) - these tasks require complex and non-algorithmic thinking. Charalambous, Delaney, Hsu and Mesa (2010) use the LCD framework in their analysis of textbooks but do not take the prior experience of students into account, on the basis that the presence of a certain topic in textbooks used in earlier grades or years is not a guarantee that students understand the material.

Lithner (2008) characterises key aspects of reasoning when engaging with mathematical tasks, characterizing such reasoning as either imitative or creative reasoning. Imitative reasoning involves the use of memorization or well-rehearsed procedures while creative reasoning requires novel reasoning with arguments to back it up and attached to appropriate mathematical foundations. Lithner (2008) asserts that a novel task requires students to engage in complex thinking in order to come up with an approach or suitable method to find a solution. The student either devises a new (in the sense that it has not been encountered or utilised previously) sequence of reasoning to formulate a solution or a forgotten sequence is recreated. If a task can be solved by imitating an answer or a solution procedure then it is not seen as novel. The LCD framework also touches on the notion of the student’s familiarity by making reference to prior experience with a task but does not provide explicit criteria for judging how such experience can be gauged. Similarly, Lithner’s characterisation of reasoning refers to the presence of novelty and tasks being seen as familiar by the student. However, it does not provide a clear method for measuring familiarity in tasks. These existing frameworks attach importance to novelty but, given the absence of any method for gauging such novelty in tasks, we found it necessary to create a new framework focused specifically on novelty and measuring the degree of its presence in tasks. This was informed through our experience of classifying tasks using the LCD framework and Lithner’s reasoning framework.

Other authors have considered students' prior experience when faced with a task. Berry, Johnson, Maull and Monaghan (1998, p.15) encountered an interesting issue when attempting to characterize routine questions. In their words, they see the implication that routineness is "located in a question rather than being a psychological construct of the relation between an individual, or group and a question" as problematic and conclude that the psychological relation is likely to be a socio-psychological one. In other words what is routine for one individual is not necessarily so for another. Selden, Mason and Selden (1989, p. 45) go some way towards acknowledging this aspect of human experience by identifying two components in a problem, namely, task and solver. A problem is defined by Selden et al. as a non-routine or novel task, the solution of which consists of finding a method of solution and carrying out such a solution. Selden et al. (1989, p.45) describe a solver as "usually a person, but possibly a group of persons or a machine". Selden, Selden and Mason (1994, p.67) take the view that while some problem-solving studies do not explicitly mention the solver, it is essential to consider the solver and the skills and information that are brought to a task when examining a mathematical task. They point out that "tasks cannot be classified as problems independent of knowledge of the solver's background" (p. 67). Most importantly, Selden et al. (1989, p.45) highlight the fact that the solver "comes equipped with information and skills, perhaps misconceptions, for attempting the task". A consequence is that novel problems cannot be solved twice by the same person without a loss of novelty, as the solver would possess a method for solution the second time.

Berry et al.'s and Selden et al.'s work suggests that a solver's previous experience is worthy of attention. It is clear that this is something that should be considered when attempting to classify tasks within a framework. In terms of textbook tasks, a certain amount of repetition is to be expected as it assists students in becoming more comfortable with a particular concept; however, it is useful to see how much novel material is introduced by authors in order to challenge the student in terms of problem solving and to promote the use of non-routine thinking and reasoning. We are also interested in examining whether the recommendations advocated by the Project Maths initiative are being implemented in textbooks.

FRAMEWORK FOR MEASURING NOVELTY

It is the aim of this paper to provide a new framework for the identification of novelty within tasks through the provision of clear criteria for three different categories described as *novel*, *somewhat novel* and *not novel*. Figure 1 gives an outline of the criteria used in this classification. This framework requires the examination of the expository material and each example in a chapter and the identification of the skills or concepts that are being demonstrated in the exemplar. For this framework, skills are taken to refer to the methods and techniques used in the solutions to tasks. Please note, it is not necessary for all characteristics in the description of the categories to apply in order for a task to be classified under a particular label. However there should be sufficient evidence to distinguish between the different categories and as many characteristics as possible should be identified before settling on a particular classification.

Novel	<p>Skills involved in finding the solution are not familiar from preceding exercises or from any previous point in the chapter being analysed.</p> <p>The mathematical concept involved is not familiar from previous exercises or examples.</p> <p>Significant adaption of the method outlined in examples and exercises must be made in order to get the required solution.</p>
Somewhat Novel	<p>The presentation of the task makes the question appear unfamiliar. However its solution requires the use of familiar skills.</p> <p>The context (perhaps the use of an unfamiliar real-world situation) makes the task appear unfamiliar but familiar skills are used in its solution.</p> <p>A new feature or aspect of a concept is encountered but the solution to the task only involves the use of familiar skills.</p> <p>A minor adaption of the method outlined in the examples has to be made in order to get the required solution. The skills required are familiar but the use or application of such skill is slightly modified.</p>
Not Novel	<p>The presentation, context and concepts of the task are familiar.</p> <p>The solution to the exercise or problem has been modelled in preceding exercises or has been encountered earlier in the same chapter.</p> <p>The skills required are very familiar to the user and the method of solution is clear due to the similarity between the exercise and preceding examples and exercises.</p>

Figure 1: Framework for classification of novelty in tasks encountered in textbooks.

DESCRIPTION OF FRAMEWORK

Novel

Something is said to be *novel* when the solver requires skills necessary for finding the solution or mathematical concepts that have not been covered in preceding exercises or at any previous point in the chapter being analysed. There cannot be any substantial similarity between the given exercise and the previous examples in this case.

The novel exercise should be original and essentially demand a new form of thinking from the solver that has not been encountered before in that chapter. This includes the situation where a task requires different methods of solution when the solver is only familiar with one method from the contents of the chapter, or when the solver has to make a significant change or alteration to the method that is familiar in order to find the required solution.

Once a novel exercise has been encountered, it diminishes the novelty for any similar exercise that follows it. While an exercise may be novel when first encountered, any exercises that follow which are of a similar likeness or approach can no longer be classified as novel.

Somewhat Novel

If there is only a superficial difference between the examples and the exercise, then it is labelled as *somewhat novel*. The presentation of the question may be different to preceding examples but when the solver goes about solving the task, it is apparent that the skills required are quite familiar from the preceding examples or earlier in the chapter. Such differences can occur when a task is presented in a different context such as a real-world scenario, which can serve to render the task unfamiliar to the solver. There can be some difficulty for the solver when determining how to solve the task but this is diminished once the familiar material is identified. Such variation necessitates the creation of an intermediate category to acknowledge that a task can have unfamiliar aspects but relies on skills that are actually quite familiar at that point.

Not Novel

An exercise or task is *not novel* if its solution has been dealt with in preceding exercises or other examples in previous parts of the chapter. The task could be direct repetition of material covered in examples or very similar to it. If material is not covered in the examples immediately preceding a set of exercises but the solver has experience from an earlier part of the chapter then the task is not novel.

By using these criteria, we suggest that it is possible for the researcher to get a clearer picture of what a solver has been exposed to in terms of the textbook examples and previous exercises. This gives an insight into how much novelty is present when an exercise is attempted. Such information complements the goals of the other frameworks as it allows one to look not only at a task but also the background against which the task is set. This framework allows one to consider the previous experiences of the solver within a mathematics textbook chapter and how this impacts upon the solver when solving a task.

METHODOLOGY

Creation of Framework

When creating the framework, it was necessary to consider a number of elements before finalising the criteria for classifying tasks. For this analysis, the examples and information preceding a set of exercises for each textbook were examined. These included definitions, explanation of key words, exemplars demonstrating key concepts and illustration of methods of solution for problems. Before classifying tasks, each feature of an exercise was identified clearly. These included using a procedure or formula, proving a mathematical statement, constructing or interpreting a diagram, using a definition, investigating mathematical properties and more. Having considered these features of the tasks and examined the skills necessary for solution, the experience gained by the solver from the preceding examples and previous exercises was considered. Previous chapters were not examined as it is not usual in Irish schools for the teacher to follow the order of the textbook in a linear fashion and thus it is not possible for us to say which chapters a student would have seen. In both textbook series studied here, the topic of Sequences and Series is covered in a single chapter. Of course, students could have other experiences but the classification is confined to the material

contained in the textbook chapter only. Each of the three authors classified the tasks independently. Then the individual classifications were compared and elements influencing the choice of classifications were discussed. This analysis was carried out repeatedly using various topics, different chapters and textbooks in order to consider all aspects that could be encountered when classifying tasks for novelty. The criteria for classification were revised as necessary until it was felt that all aspects of novelty in a task were accounted for. The three classifications of *novel*, *somewhat novel* and *not novel* were settled on and the table in figure 1 was eventually agreed upon. It was decided that questions which formed multiple parts of an exercise would be considered as stand-alone tasks while preceding questions were taken into account when considering their impact on the next exercise. Any questions containing ambiguity or unclear elements were not considered for classification.

EXAMPLES OF CLASSIFICATION OF TASKS IN FRAMEWORK

This paper looks at applying the framework to the topic of sequences and series in two textbook series recently introduced on the Irish market and now being widely used in schools. These textbook series have been given the pseudonyms Text A and Text B. The topic of sequences and series was chosen because it was present at both Higher and Ordinary levels and allowed for the comparison of the treatment of the topic between these levels. While the series of textbooks have each been introduced at both junior and senior cycle, we decided to focus on the senior cycle material particularly because of the high-stakes examination that accompanies it. One chapter from each textbook series at each of Higher level (HL) and Ordinary level (OL), has been analysed using this framework (that is, four textbook chapters in all) and the results are presented later.

To illustrate how the framework is used and the criteria are applied, three examples showing classifications of tasks as *novel*, *somewhat novel* and *not novel* are given. In order to preserve the anonymity of the textbooks, it was decided to illustrate the framework using an older book, which was widely used in Irish second level schools during the 1990s (Roantree, 1994). An extract from the explanatory material in this chapter is given below. A definition of an arithmetic sequence, a formula for finding the general term in an arithmetic sequence, and two worked examples arising from this are provided. This is followed by a formula for finding the sum of terms in an arithmetic series, accompanied by a third worked example. Three tasks are then classified in light of this material using the criteria outlined in figure 1. An explanation for how each task was classified is provided directly after the classification.

EXAMPLES OF TASK CLASSIFICATION USING THE NOVELTY FRAMEWORK

Definition

The sequence T_1, T_2, T_3, \dots is said to be arithmetic if $T_n - T_{n-1} = \text{constant}$ for all $n > 1$.

General term

The general term in an arithmetic sequence is $T_n = a + (n - 1)d$

Example 1

Prove that the sequence $T_n = 5n - 1$ is arithmetic. Find a and d .

$$\begin{aligned} \text{Examine } T_n - T_{n-1} \quad T_n - T_{n-1} &= (5n - 1) - (5(n-1) - 1) \\ &= 5n - 1 - 5n + 6 = 5 \end{aligned}$$

As $T_n - T_{n-1}$ is constant, the sequence T_n is arithmetic.

$$\text{Hence } d = 5 \text{ and } a = T_1 = 5 - 1 = 4$$

Example 2

Two terms in an arithmetic sequence are $T_2 = -1$ and $T_5 = 14$. Find a, d and T_{10} .

$$(i) \quad T_2 = a + d = -1 \quad \text{Subtracting (i) from (ii)}$$

$$(ii) \quad T_5 = a + 4d = 14 \quad 3d = 15 \Rightarrow d = 5 \Rightarrow a = -6$$

$$\text{Then } T_{10} = a + 9d = -6 + 45 = 39$$

Formula

If $S_n = T_1 + T_2 + \dots + T_n$ is an arithmetic series, then $S_n = \frac{n}{2}[2a + (n-1)d]$

Example 3

The sum of the first 21 terms of an arithmetic series is zero. Express, in terms of a , the sum of the next 21 terms.

$$S_{21} = 0 \Rightarrow \frac{21}{2}[2a + 20d] = 0, \text{ by Formula 4}$$

$$\Rightarrow d = -\frac{a}{10}$$

(We have to find the value of $T_{22} + T_{23} + \dots + T_{42}$)

$$\begin{aligned} T_{22} + T_{23} + \dots + T_{42} &= S_{42} - S_{21} \\ &= S_{42} \quad [\because S_{21} = 0] \end{aligned}$$

$$\begin{aligned} S_{42} &= \frac{42}{2}[2a + 41d] \\ &= 21 \left[2a - \frac{41}{10}a \right] \\ &= 21 \cdot \frac{-21a}{10} = \frac{-441a}{10} \end{aligned}$$

Task 1

Is the following sequence arithmetic? $T_n = 1 - 3n$

This task is classified as *not novel* as the exercise is based on Example 1. The solver finds $T_n - T_{n-1}$ to be a constant and thus proves that it is arithmetic.

Task 2

Find a formula for the sum of the first n odd numbers.

This task is classified as *somewhat novel*. The presentation of the task, due to its wording, would make it appear unfamiliar. The student might be put off by the notion of n odd terms.

However once students focus on finding a and d for the series $1+3+5+7\dots$, similar skills to previous tasks would be used, based on the formula used in Example 3 except that it is expressed in terms of n .

Task 3

If $a^{-1}, b^{-1}, c^{-1}, d^{-1}$ are in arithmetic sequence, prove that $b = \frac{2ac}{a+c}$

This task is classified as *novel*. The skills involved in finding the solution here are not familiar from preceding exercises or from any previous point in the chapter being analysed.

RESULTS

Table 1 contains the results of our classification of the exercises on the topic of sequences and series in the two textbook series under consideration. It shows the number and percentage of exercises falling into each classification.

	Novel	Somewhat Novel	Not Novel
Text A (H)	17 (8.3%)	58 (28.4%)	129 (63.3%)
Text B (H)	30 (9.2%)	68 (20.9%)	227 (69.9%)
Text A (O)	0 (0%)	27 (11.5%)	208 (88.5%)
Text B (O)	27 (7.7%)	33 (9.4%)	291 (82.9%)

Table 1: Results of classification of sequences and series exercises in two textbook series.

Without exception, the *not novel* categorisation makes up the majority of every exercise set analysed in the textbooks within this study. The lowest occurrence of the *not novel* category within the topic of sequences and series was in Text A (Higher Level) with 63.3%, while there was one exercise set in Text B (Higher Level) where every task was found to be *not novel*. Overall *not novel* accounted for 82.9% of tasks in Text B (Ordinary Level), 69.9% in Text B (Higher Level), 88.4% in Text A (Ordinary Level). While there was a high incidence of tasks classified in the *not novel* category in all of the textbooks analysed, it appears to be more frequent in Ordinary Level textbooks than the Higher Level ones. The *somewhat novel* category has a significant difference between levels. 20.9% of tasks in Text B (Higher Level) and 28.4% in Text A (Higher Level) compared to 9.4% in Text B (Ordinary Level) and 11.5% in Text A (Ordinary Level). The *novel* tasks had a low level of occurrence within the exercises analysed. There appears to be a relatively big difference between the proportions of *novel* tasks at Higher and Ordinary Level in Text A with 8.3% of tasks in Text A (Higher Level) in the *novel* category with 0% of tasks categorised as *novel* in Text A (Ordinary Level). There is much less difference between levels in Text B where 7.7% of tasks in Ordinary Level were found to be *novel*, while 9.2% of tasks in Text B (Higher Level) were categorised as *novel*. The main difference between the distributions of the tasks in the Higher Level and Ordinary Level textbooks seems to be that there are more tasks in the *somewhat novel* category at Higher Level, with far fewer tasks at Higher Level categorised as *not novel*. The implication of these results is that students are not currently exposed to a high degree of

novel tasks as they complete classwork and homework exercises and there is scope for textbook authors to include more novelty when designing exercises.

DISCUSSION

We have seen that the majority of tasks in both textbook series and at both levels have been classified in the *not novel* category. These results would appear to support the view outlined in O’Keeffe and O’Donoghue (2009) that mathematics textbooks in the Irish system promote “retention and practice” (p. 290). Given the large percentage of tasks classified as *not novel* in the particular chapters analysed, it would appear that a lot of exercises still promote the practising of skills. If teachers are to remain dependent on such textbooks as a source of classroom tasks then it is likely that the much criticized ‘drill and practice’ style of teaching will continue to be a feature of Irish mathematics classrooms. One of the main objectives of Project Maths is that students should be able to solve problems in unfamiliar contexts; it would appear from our results that textbooks are not currently facilitating this aim adequately.

O’Keeffe and O’Donoghue (2012) provide an analysis of specific junior cycle and senior cycle mathematics textbooks in terms of their structure, content and how they meet the expectations of the syllabus. Their research provides a variety of data such as the distribution of: narration and narration type, graphics and their purpose, exercises, worked examples, motivational factors, comprehension cues and technical aids. In relation to this paper, the analysis of the distribution of routine and non-routine problems throughout specific junior cycle and senior cycle mathematics textbooks is of particular interest. They define a routine problem to be “all problems which are ‘dressed up exercises’”, while a non-routine problem refers to “all problems which cannot be answered by a routine procedure or problems in which it not immediately obvious what one must do” (p.23). It can be seen from their analysis that routine problems outnumber the non-routine ones, something that is borne out in our results. However, there is no explanation of how problems were classified as being routine or non-routine. The novelty framework could be used to clarify this situation and to expose the influence of examples and previous exercises on new tasks. While their work provides a distribution of worked examples through the textbooks, the novelty framework presented here allows one to gauge the impact that explanatory material included in the textbook has on the student and gives an indication of the degree of experience that the solver brings to a task from the textbook chapter and its exemplars. When applied correctly, the novelty framework could provide a more informed categorization of a task in terms of its novelty, particularly with reference to the perspective of the solver.

Other authors have considered the characterizing of routine questions and encountered issues when attempting to complete the categorization. Berry et al. (1998) define routine questions as “those for which students may be expected to execute a rehearsed procedure consisting of a limited number of steps” (p. 105). They had difficulty when it came to the categorization of parts of examination questions as routine or not. They called for further investigation into refining what is meant by the term ‘routine question’. Their study relied on “debate by a group of people with considerable experience of the type of examination paper in question” (p. 108) in order to determine which parts were routine and which were non-routine.

However, when a group of students who had studied the paper were asked to categorize the

same parts of questions as routine or non-routine, it was found that there was only partial agreement between the authors' categorization and that of the students. It is hoped that use of the novelty framework would help avoid such problems as it addresses in more detail how a task can be categorized in terms of the experience that a solver brings to it.

Selden et al. (1994) point out that tasks cannot be classified independent of knowledge of the solver's background. They view 'cognitively non-trivial' problems as those where "the solver does not begin knowing a method of solution" (p. 119). While they acknowledge that sample solutions and examples can be used to make problems routine, they do not provide a definition of what is meant by 'routine'. The novelty framework can be used to give further clarification to the difference between routine and non-routine and might provide greater insight into the degree of novelty that is present in a task for any set of textbook exercises. It also takes into account some of the knowledge that a solver brings to a task, rather than just focusing on the task's structure or the demands that a particular task places on the solver. Each student will have different mathematical experiences which are not dependent on the textbook material, but our framework is not able to take these into account. This work forms part of a larger project where we consider tasks from examination papers, and old and new textbooks. We hope to use the information from our classifications to design new tasks for use in the classroom.

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