THE USE OF UNFAMILIAR TASKS IN FIRST YEAR CALCULUS COURSES TO AID THE TRANSITION FROM SCHOOL TO UNIVERSITY MATHEMATICS

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Research has shown that mathematics courses at university often focus more on conceptual understanding than those at secondary school (Clark & Lovric, 2008). Moreover, the literature reports that the types of tasks assigned to students affect their learning. A project was undertaken by the authors in which tasks were designed and presented to first-year undergraduate Calculus students with the aim of promoting conceptual understanding and developing mathematical thinking skills. Here we present data from interviews with five students; they reported an increased emphasis on conceptual understanding at university, and found the tasks assigned beneficial in the development of conceptual understanding. We suggest that unfamiliar tasks are useful in the transition from school to university mathematics.

INTRODUCTION

The transition from school to university has been the object of much research in recent years. Gueudet (2008) conducted a review of the literature and found four broad categories of research activity in this area. Two of these are research on thinking modes and research on the organization of knowledge and reasoning modes. In this paper, we will consider the views of students on their experiences as they deal with types of thinking and organization of knowledge new to them at the beginning of their university studies. Views of the students were collected as part of a project in which the authors designed a range of tasks for use in first year differential calculus courses. The aim of the design process was to create tasks that would promote conceptual understanding by encouraging students to develop some of the practices and 'habits of mind' of research mathematicians. According to Cuoco, Goldenberg, and Mark (1996) these include finding patterns, experimenting, conjecturing, arguing, using mathematical language, visualizing and inventing. Mason and Johnston-Wilder (2004) also mention exemplifying, generalizing, justifying, convincing and refuting as processes and actions that mathematicians employ when tackling problems. A selection of tasks was trialed in a first-year calculus module, following which five students were interviewed. The data from these interviews form the basis of this paper. We will present the students' views on the difference between mathematics at school and at university and on the effects of the tasks on their ways of working and on the promotion of understanding. We will then discuss the effect of the tasks on the students' experience of the transition process.

LITERATURE REVIEW

Clark and Lovric (2008 and 2009) have described the secondary-tertiary transition as a 'rite of passage' and detail the changes that students experience as they commence their mathematical studies at tertiary level. These include changes in the type of mathematics taught and changes in the way mathematics is taught. They contend that compared to the mathematics taught at school, mathematics at university involves an increased emphasis on conceptual understanding, multiple representations of mathematical objects, advanced mathematical thinking, proof, abstraction, and the importance of precise mathematical language. De Guzman, Hodgson, Robert, and Villani (1998) had previously reported on the difficulties that first year university students faced and had categorized them into three types: epistemological/cognitive difficulties; sociological/cultural difficulties; and didactical difficulties. The cognitive difficulties related to the transition from elementary to advanced mathematical thinking but also to students' ability to organize their mathematical knowledge and to develop connections between concepts.

Mathematical understanding has been characterized by many different authors (for example, Pirie & Kieren, 1994). Skemp (1976) spoke about instrumental understanding (or 'rules without reason') and relational understanding (knowing both what to do and why to do it). In the US, the National Research Council (2001) described 'conceptual understanding' as the "comprehension of mathematical concepts, operations and relations" (p116). Much of the literature in this area refers to the understanding of school mathematics; however, Sofronas et al. (2011) asked 24 experts the question 'What does it mean for a student to understand the first-year calculus?'. They were able to construct a set of four core goals that defined student understanding in this context. These goals were: mastery of the fundamental concepts and-or skills of the first year calculus; construction of connections and relationships between concepts and skills; the ability to use the ideas of first-year calculus; and a deep sense of the context and purpose of the calculus (Sofronas et al., 2011, p132).

Gueudet (2008) observed that many studies on transition compare the practices of students with those of mathematicians. In this regard, Gueudet discusses the work of Lithner (2000) on mathematical reasoning. He noted that first year university students often rely heavily on past experience when solving mathematical problems, while mathematicians usually display more flexibility in their thinking and reasoning. Gueudet (2008) reported that in order to deal with this issue, researchers have called for changes in the teaching methods both at school and at university. In particular, a wider range of tasks which would allow students to develop autonomy and flexibility have been proposed. Boesen et al. (2010) also contend that the types of tasks assigned to students affect their learning and the use of tasks with lower levels of cognitive demand leads to rote-learning by students and a consequent inability to solve unfamiliar problems or to transfer their mathematical knowledge to other areas competently and appropriately.

In Ireland, research at secondary level has shown that teaching in Irish mathematics classrooms tends to be focussed on the use of algorithmic procedures, with very little emphasis on conceptual understanding, and that students appear unable to apply techniques learnt in unfamiliar contexts (for example, Lyons, Lynch, Close, Sheerin, and Boland, 2003).

In discussing how students might accomplish a successful secondary-tertiary transition in mathematics, Clark and Lovric (2009) state “what we certainly can claim is that the success depends, in great measure, on the robustness of certain parameters in secondary education (attitude, motivation, approach towards work, and, in particular, learning styles and cognitive models) that might need to be significantly modified at tertiary level" (p.759). The paragraphs below consider to what extent the series of unfamiliar tasks assigned did modify some of these parameters in the view of the five interviewed students.

THE TASK DESIGN PROJECT

The first two authors are mathematics lecturers in different third level institutions in Ireland. In the academic year 2011/12, both were teaching first year differential calculus modules. Given the procedural nature of mathematics instruction at second level in Ireland, they endeavoured to design a series of unfamiliar non-procedural tasks for their students in an effort to give students opportunities to develop their thinking skills and their conceptual understanding. (In this paper, the National Research Council’s (2001) description of conceptual understanding has been adopted.) An ‘unfamiliar task’ is one for which students have no algorithm, well-rehearsed procedure or previously demonstrated process to follow. Following Lithner’s (2000) observation that students often rely heavily on past experience when solving problems, the authors hoped, by presenting the students with unfamiliar tasks, to discourage such reliance and help them to develop the flexibility in their thinking and reasoning characteristic of mathematicians.

The tasks designed in this project required students to make use of definitions, generate examples, generalise, make conjectures, analyse reasoning, evaluate statements, or use visualisation. A sample is shown below (note that students were asked to think about this problem after they had encountered the concept of continuity but before they had seen the Intermediate Value Theorem):

[1] Do you believe the following statement is true or false?

If $f(x)$ is a continuous function on the interval $[a,b]$ and k is a number between $f(a)$ and $f(b)$, then there is at least one number c in $[a,b]$ such that $k=f(c)$.

If you think the statement is false, provide a sketch as a counterexample.

A selection of other tasks designed along with a rationale for the task framework used can be found in Breen and O'Shea (2011). Each problem set (and the final examination) contained unfamiliar non-procedural tasks as well as some more procedural tasks. For example, the following procedural task appeared on the same problem set as question 1 above:

[2] Use the definition of continuity to determine whether the function $f(x) = \frac{x^2 - 1}{x + 1}$ is continuous at *x=-1*.

In this paper we will concentrate on the module taught by the first author at St Patrick's College, Drumcondra. There were 35 students registered on this module, who had chosen to study Mathematics as part of a BEd (Primary) or BA (Humanities) degree. (BEd (Primary) students at St Patrick's College to date must choose to specialise in one Humanities subject; this subject accounts for 40% of credits awarded for the degree.) The designed tasks were assigned to the students either as homework (for students to work on independently) or as tutorial problems (for students to work on in small groups). A number of tutorial sessions were observed by the third author as part of this project. Towards the end of the module, the fourteen students who had previously attended the tutorials observed were invited to volunteer to be interviewed. (These students were selected in order to maximise later collation of the data collected.) Five students volunteered and were interviewed individually by the third author, as she was not involved in the teaching of the module nor in the design of the homework and tutorial tasks. The interviews were semi-structured and lasted between 15 and 25 minutes; they were audio-recorded and fully transcribed. The identity of the interviewees was not revealed to the first author (module lecturer) unless the interviewees chose to do so themselves. The students were assigned pseudonyms A, B, C, D, E.

Students were asked about their impressions of mathematics at university, how their experience of mathematics at school differed from that at university, how their study habits or ways of working had changed and about the tasks that they had worked on. In particular, the interview schedule produced in advance of the semi-structured interviews outlined the following questions: *What do you think of your maths course (at university)? How is it different from school mathematics?; Different types of questions were used on the problem sheets in the Calculus course, were you aware of the difference? Can you give examples?*; [Showing students two tasks on the same topic (one familiar/procedural, the other not):] *Which of these tasks is familiar to you? Why this one? How did you feel when you first saw the task? What are the differences between the tasks? What was the purpose of each of the tasks? What did each of the tasks help you learn? Did these tasks aid your development of understanding in the same way?*

Our research questions for this paper are:

1. What are the students' views on the difference between mathematics at secondary and tertiary level?
2. What are the students' views on how the designed tasks have impacted on their practices, learning and conceptual understanding in the transition to tertiary level?

The authors coded the transcripts separately in line with these research questions. Each author grouped the codes into broad categories and then all three met to discuss the codes and agreed on common categories. This paper reports on the main themes that emerged from this analysis.

RESULTS - STUDENTS' VIEWS ON DIFFERENCES BETWEEN MATHEMATICS IN SCHOOL AND UNIVERSITY

The students were first of all asked what they thought of their university mathematics course; two of them (B and E) immediately volunteered that their experiences of mathematics courses at university were very different to that of school. The other three students were asked if they had found differences between second and third level mathematics and they were all able to point to specific differences. The differences that the interviewees spoke of were categorised and two themes are reported here: a change in emphasis from procedural fluency to conceptual understanding, and a move to independent learning.

All the students interviewed described a change in emphasis from a focus on instrumental understanding or procedural fluency in school to a focus on relational or conceptual understanding at university. They all mentioned the importance of procedures in school. For example, students A and B said:

Student A: In school it was kind a lot of procedural – you just, when you saw something you knew you did this. Like there were bits you'd understand and other bits you'd just take for granted.

Student B: It's just procedure…You learn the methodologies [sic] rather than learning why you are doing what you are doing

The students also spoke about working with formulae at second level in a procedural manner.

Two students (B and D) spoke about memorisation or 'learning off' as being important at school. Two students (A and D) also felt that there was a lack of linkages between different topics in the second level curriculum:

Student D: For the [senior cycle of secondary school] they were separate questions and they didn't really tie together at all.

Student A mentioned a specific example of the disjointed nature of her mathematics course at secondary school:

Student A: It was never explained to us that limits had – dealt with functions. They were nearly kind of separate.

This was in contrast to the five students' perception of the importance of conceptual understanding and connections between topics at university. All of the interviewed students spoke about the emphasis on conceptual understanding, for example

Student C: The emphasis in the college course is about actually understanding the principles

Student D: It kind of explains why you were doing it before

The interviewees also noticed an increased emphasis being placed on connections or relations between mathematical topics and ideas in university. Both Student A and Student D asserted that links between concepts were made explicit in their university mathematics course:

Student D: It kind of ties together really everything from the [senior cycle of secondary school] … It kind of interlinks them more.

The five students also alluded to a change in teaching style between second and third level. This change seems to concern a move from a teacher-led classroom environment to one where more independent learning is required. For example:

Student B: It's hard to change from that mindset I find within a few months…Like from being given all the information to then having to find it yourself.

One student also indicated that, although unfamiliar tasks had been assigned at university, she would have been unlikely to encounter such tasks in school:

Student B: We hadn't done that in class, so we had to try to figure it out for ourselves. Whereas in school the teacher would have done that with you.

In addition, the students spoke about having to think for themselves and to schedule their own study timetables without the framework of daily homework assignments. However, the students interviewed acknowledged that the transition from school to university mathematics can be difficult and that it can be hard to make this transition in a short timeframe.

RESULTS - STUDENTS' VIEWS ON THE IMPACT OF UNFAMILIAR AND FAMILIAR TASKS

The five students were asked a series of questions concerning the tasks assigned to them during the module. We categorised the responses and report on the main themes that emerged: the students' views on the effects of the tasks on their conceptual understanding and the impact of the tasks on their mathematical thinking and ways of working.

Students' views of the effects of the tasks on their understanding

The students were asked what they gained from the tasks, and in particular which types of tasks helped them gain conceptual understanding. Four of the five students chose unfamiliar tasks designed for this study in answer to this question. Student B referred to unfamiliar tasks in general:

Student B: The ones I haven't seen before, definitely… in those ones you have to like completely understand it to get the answer.

Student D referred to less procedural types of tasks:

Student D: When you weren't just procedural the whole way, when you just had to stop and be like ok what do I know about it, is it continuous, differential and all this.

The comments of some students indicate that to perform unfamiliar tasks they were forced to apply and find relationships between previously learned concepts. The quote below from Student A arose from her response when she was asked to compare two particular tasks, the first one familiar (evaluating limits) and the second one unfamiliar to the student (an example generation task). Her answer suggests that she has learned more from the unfamiliar problem.

Student A: So you're kind of bringing together what you know from other things whereas in Question 1 you kind of — you're told what you have to do. So you're literally just kind of following a procedure really...whereas for the second one you kind of actually are more thinking yourself...it is more difficult, ya, but it kind of helps you understand it better. You see the relationship between them.

Other students said that some of the designed tasks involved 'actually explaining the process involved' (Student C) and that this led to better understanding. Student B also acknowledged that she had to understand and apply previously learned knowledge to perform unfamiliar tasks:

Student B: you had to go back on what continuity means and then try and apply it to the different ones. So it's making us see like when something is continuous and when it wasn't.

Two students also spoke of the benefit of unfamiliar tasks for assessing their own understanding of concepts. For example, Student E when referring to an example generation task, stated she found it beneficial because

Student E: it kind of proves you understand it more.

Students' views of the impact of the tasks on their mathematical thinking and ways of working

The interviews give some insight into how the tasks assigned encourage or stimulate thinking practices and ways of working. Some interviewees described certain tasks, which they had identified as unfamiliar or non-procedural, as encouraging habits such as thinking, analysing, questioning or exploring patterns. Four of the five students interviewed (A, C, D, E) asserted that the unfamiliar tasks made them think more or think for themselves. Student D claimed that, whereas for familiar tasks she would "just like write" and "plough through", the unfamiliar tasks "really make [her] think".

Considering specific types of tasks, Student C said about a conjecturing task:

Student C: You have to think about it and then, it's not actually a procedure, it's about you analysing the pattern and stuff.

This student also commented on a task which involves evaluating a statement:

> Student C: about whether is the statement true or false, analysing it, ya it gets you thinking more than just actually using the knowledge that you learned like.

Student B described how performing an analysing reasoning task prompted her to ask herself questions:

> Student B: Ahm, because it makes you analyse the proof and ask yourself questions like why you do things like that. Whereas if you were just given the proof, the correct one, you just take it for granted that that was correct.

A number of the students (A, B, C) also mentioned that, when they encounter an unfamiliar task, they refer to the basic definitions or theorems on the course and think about what they know (in relation to the task) and how they can apply it. One student (A) explained that she approached an example generation exercise by breaking it down or taking it step-by-step but also "drawing on other things" that she knew and bringing them together. Student C described using the following techniques (sometimes in combination) when confronted with various unfamiliar tasks: sketching a graph, examining different cases, generating examples, generalising, working backwards. When speaking more generally about their study habits in relation to the Calculus module, Students C and D mentioned focussing on understanding the concepts and Student B claimed a lot of independent work is required.

DISCUSSION

Research literature on the transition from second to third level mathematics seems to agree on the requirement at university level for more relational and conceptual understanding and more flexibility in thinking or approaching mathematical problems in comparison to second level mathematics (for example, Clark & Lovric, 2008; De Guzman et al., 1998). The responses of the students interviewed show that they are aware of the different requirements and, moreover, most of them welcome the change in focus. Various authors have made recommendations about the type of tasks that should be assigned to undergraduate students in order to gain the required relational or conceptual understanding and flexibility (for example, Geuedet, 2008; Boesen et al., 2010). Though the students interviewed in the study reported here struggle with the unfamiliar tasks assigned to them, they find such tasks beneficial regarding the development of conceptual understanding and their learning more generally. All of the interviewees stated that some of the unfamiliar tasks encouraged them to think more, or to analyse or question the information given. Some interviewees explicitly described how unfamiliar tasks they encountered led them to connect ideas met previously. Furthermore, the comments of the students interviewed indicate that unfamiliar tasks may not only have the effect of stimulating the development of conceptual understanding, but also may have some potential to raise an awareness that more than instrumental understanding is required at university, thereby easing their transition to university practices.

While all the interviewees reported using the practices or 'habits of mind' (generating examples, generalising, visualising etc.) of mathematicians when specifically called on to do so by a particular task (an example generation or conjecturing task, say), one student, Student C, described drawing on such practices in a broader sense when confronted with an unfamiliar task. Given the assertion by Cuoco et al. (1996) that it is these practices which 'give students the tools they will need in order to use, understand and even make the mathematics that does not yet exist' (p.376), this is a very positive development.

However, it should be noted that some of the interviewed students also described significant advantages of familiar and/or procedural tasks on their learning and confidence. In practice, the sets of problems assigned to students in this project also contained procedural-type questions, to aid the development of procedural fluency (the "ability to carry out procedures flexibly, accurately, efficiently and appropriately" as described by the National Research Council, 2001, p116). In combination, these two types of tasks helped to address the first three end-goals of a first-year calculus course as described by Sofronas et al. (2011): the mastery of fundamental concepts and-or skills; the construction of connections and relationships between and among concepts and skills; the ability to use the ideas of calculus.

The data analysed here consists of self-reported views of what the students gained from the tasks assigned. The next step in this project is to conduct a series of task-based interviews to explore the practices engaged in by students as they attempt the tasks and to investigate progress in their understanding of particular mathematical concepts related to the tasks.

While developing conceptual understanding requires a suitable teaching and learning environment as well as attention to task design, our findings to date suggest that the inclusion of unfamiliar tasks is beneficial in helping students negotiate the secondary-tertiary transition in terms of the changes in the type of mathematics taught and the way in which mathematics is taught. Clark and Lovric (2009) mentioned that certain parameters, such as a student's attitude, approach towards work and learning style, may have to be modified to successfully accomplish the transition. We suggest that the tasks assigned to students can be used as a vehicle through which these parameters might be changed.

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REFERENCES

Boesen, J., Lithner, J, & Palm, T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educational Studies in Mathematics*, *75*, 89-105.

Breen, S., & O'Shea, A. (2011). Designing rich sets of tasks for undergraduate calculus courses. In Dooley, T., Corcoran, D. & Ryan, M. (Eds.), *Proceedings of the Fourth Conference on Research in Mathematics Education MEI4*, Dublin, 82-92.

Clark, M., & Lovric, M. (2008). Suggestion for a theoretical model for secondary-tertiary transition in mathematics. *Mathematics Education Research Journal, 20*(2), 25-37.

Clark, M., & Lovric, M. (2009). Understanding secondary-tertiary transition in mathematics. *International Journal of Mathematical Education in Science and Technology, 40*(6), 755-776.

Cuoco, A., Goldenberg, E.P., & Mark, J. (1996). Habits of mind: an organising principle for mathematics curricula. *Journal of Mathematical Behaviour*, *15*, 375-402.

De Guzman, M., Hodgson, B. R., Robert, A., & Villani, V. (1998). Difficulties in the passage from secondary to tertiary education, *Proceedings of the International Congress of Mathematicians*, Berlin, Documenta mathematica, extra volume ICM 1998, 747-762.

Gueudet, G. (2008). Investigating the secondary-tertiary transition. *Educational Studies in Mathematics, 67*, 237-254.

Lithner, J. (2000). Mathematical reasoning in task solving. *Educational Studies in Mathematics, 41*, 165-190.

Lyons, M., Lynch, K., Close, S., Sheerin, E., & Boland, S. (2003). *Inside classrooms: the teaching and learning of mathematics in social context*. Dublin: IPA.

Mason, J., & Johnston-Wilder, S. (2004). D*esigning and using mathematical tasks*, St Alban's: Tarquin Press.

National Research Council. (2001). Adding it up: Helping children learn mathematics. In Kilpatick, J., Swafford, J. & Findell B., (Eds.), Washington DC: National Academy Press.

Pirie, S.E., & Kieren, T. (1994). Growth in mathematical understanding: how can we characterise it and how can we represent it? *Educational Studies in Mathematics, 26*, 165-190.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, *77*, 20-26.

Sofronas, K.S., De Franco, T.C., Vinsonhaler, C., Gorgievski, N., Schroeder, L., & Hamelin, C. (2011). What does it mean for a student to understand the first-year calculus? Perspectives of 24 experts. *The Journal of Mathematical Behavior*, *30*, 131-148.