State space model of a hydraulic power take off unit for wave energy conversion employing bondgraphs

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1. Introduction

In this work, the modeling of a Power Take-Off (PTO) unit for a point absorber wave energy converter is described. The PTO influences the energy conversion performance by its efficiency and by the damping force exerted, which affects the motion of the body. The state space model presented gives a description of the damping force and of the internal dynamics of the PTO. The aim of this work is to develop a model for the PTO as a part of a complete wave-to-wire model of a wave energy converter as in Figure 1, used for the design control techniques.

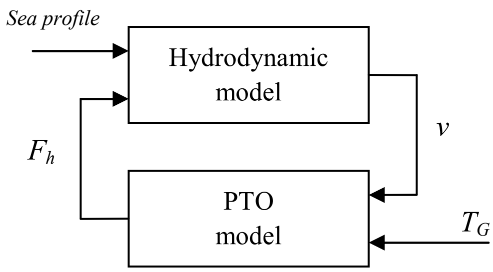

Figure 1: Wave-to-wire model structure

A bondgraph is employed to model the physical system that provides transparent and methodical means of formulating state space equations and of visualizing energy transfer throughout the system. Bondgraphs have already been shown to be a very useful tool for the modeling of PTO for wave energy converters (2). The dynamic of the mathematical model is then analyzed respect to the variation of parameters; in particular, the non-linear system obtained is linearized and its eigenvalues are calculated as function of the accumulator size and pre-charge pressure.

2. System description

The wave energy converter considered in this work is a heaving body point absorber that exploits the vertical relative motion between two bodies without any restriction. The alternating motion is converted in electricity by the mean of a hydraulic circuit (PTO) (Figure 2). The reasons for using hydraulic components are that it is an established technology and it is less expensive and more compact respect to other technologies for a given power. The oscillating body is connected to a piston that produces an alternating oil flow which is rectified by four check valves, arranged as the equivalent of an electric full wave rectifier. The flow is smoothed by a gas accumulator and then converted in rotational momentum by a hydraulic motor.

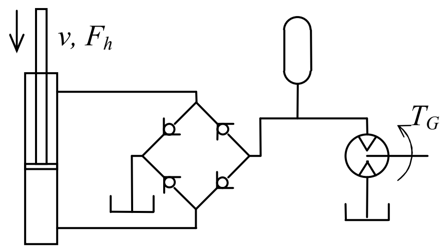

Figure 2: PTO hydraulic circuit

As the control system strategy is to control the motion of the heaving body acting on the torque of the electric generator T_G which, in turn, affects the damping force F_h, the model of the PTO has to provide a description of the damping force as a function of the applied torque. The PTO model is then connected with the hydrodynamic model (Figure 1) by the mean of the damping force (F_h) and the piston velocity (v) signals. More in detail, the hydrodynamic model supplies the value of the stroke velocity v and requires as inputs the sea profile and the damping force F_h, while the PTO model supplies the damping force F_h as function of the stroke velocity v and the applied torque on the hydraulic motor shaft T_G. In order to design a dynamic model of the system, it is necessary to characterize the main losses and energy storage parts. The losses that have been considered are due to pressure drops along pipes, oil leakages and rotational friction in the hydraulic motor, while the gas accumulator and the hydraulic motor shaft are responsible for the energy storage, in the form of pressurized oil and angular momentum.

3. Bondgraph model

The choice of bondgraph for the modeling of this system is due to its capability to provide a uniform notation for all types of physical systems based on energy and information flow (1), such as, in this case, mechanical, torsional and hydraulic systems. An important characteristic of the bondgraph, as depicted in Figure 3, is the visual description of the energy flowing through the physical system by the means of bonds, which represent bi-directional energy flows. This representation allows also a straightforward modeling of systems that produce a "back force" on the input, without introducing extra feedback loops.

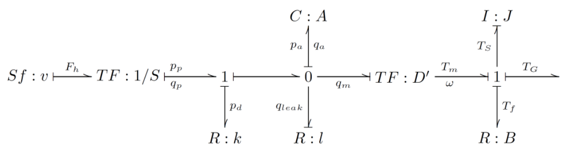

Figure 3: Bondgraph model

Each bond is associated with a flow variable f and an effort variable e; the product of these two variables is the power (P) exchanged by the two components connected by the bond:

$$P = e \cdot f$$

The half arrow on the bond indicates the conventional direction that has been chosen as the positive flow of power. The short vertical lines at one end of each bond are called causality strokes and indicate the direction of the effort; they are used to identify the independent variable of the corresponding bond. The mechanical power at the input of the PTO depicted in Figure 2 is the product between the damping force F_h exerted by the piston and the stroke velocity v; therefore the effort and flow variable on the first bond on the left hand side of Figure 3 are respectively F_h and v. The component $Sf{:}v$ represents a flow source and it indicates that the independent variable (input) is the stroke velocity v, as exposed in the previous section and indicated by the causality stroke. The mechanical power is then converted to hydraulic power by the transformer component $TF{:}1/S$; the flow and effort variables in the hydraulic circuit are respectively the oil flow (q_p) and the pressure (p_p). Losses in the circuit are described by the $R{:}k$ component that characterizes the pressure drop along pipes and by $R{:}l$ that characterizes the leakage of the hydraulic motor. The energy store by the accumulator is depicted by the $C{:}A$ component. The power conversion performed by the hydraulic motor is represented by the $TF{:}D'$ component, which takes into account also for the mechanical efficiency. The flow and effort variables for this part of the system are the angular velocity ω and the torque T_m. Energy is then stored in the form of rotational momentum ($I{:}J$), dissipated by the rotational friction ($R{:}B$) and transferred at the output with a rate of

$$P = \omega \cdot T_m$$

Integral causality has been assigned to both $C{:}A$ and $I{:}J$ components in order to chose as state variables the oil volume in the gas accumulator (V) and the angular momentum of the hydraulic motor shaft (L).

4. State space model

The state space model is formulated from the bondgraph by the means of a straightforward procedure. The objective of the modeling is to find one differential equation for each state variable (V and L) and an output equation that describes F_h as function of the state variables and the inputs, such as:

$$\begin{cases} \dot{V} = f(V, L, T_G, v) \\ \dot{L} = g(V, L, T_G, v) \end{cases}$$

$$F_h = z(V, L, T_G, v)$$

Starting from the application of the constitutive relations for the three junctions, equations (1), (2) and (3) are derived.

$$p_p = p_d + p_a \tag{1}$$

$$q_a = q_p - q_{leak} - q_m \tag{2}$$

$$T_S = T_m - T_G - T_f \tag{3}$$

Equation (1) states that the oil pressure at the outlet of the piston (p_p) is the sum of the pressure drop along the pipe (p_d) and the pressure in the accumulator (p_a). The hydraulic piston equations relate the oil pressure (p_p) and flow (q_p) with the stroke velocity (v) and the damping force (F_h) as:

$$\begin{aligned} F_h &= S \cdot p_p \\ q_p &= S \cdot v \end{aligned} \tag{4}$$

where S is the section of cylinder. The pressure drop p_d is modeled using the Haaland approximation of the Darcy equation:

$$p_d = k(q_p) = f\frac{(L_g + L_{eq})}{D_h}\frac{\rho}{2A^2} q_p |q_p| \tag{5}$$

Where

$$f=\begin{cases}K_s/Re & \text{for } Re \le Re_L\\ f_L+\dfrac{f_T-f_L}{Re_T-Re_L}(Re-Re_L) & \text{for } Re_L < Re < Re_T\\ \dfrac{1}{\left(-1.8\log_{10}\left(\dfrac{6.9}{Re}+\left(\dfrac{r/D_H}{3.7}\right)^{1.11}\right)\right)^2} & \text{for } Re \ge Re_T\end{cases}$$

$$\mathrm{Re}=\frac{qD_h}{A\nu}$$

Re	Reynolds number
Re_L	Maximum Reynolds number at laminar flow
Re_T	Minimum Reynolds number at turbulent flow
K_s	Shape factor the characterizes the pipe cross section
f_L	Friction factor at laminar border
f_T	Friction factor at turbulent border
A	Pipe cross-sectional area
D_H	Pipe hydraulic diameter
L_g	Pipe geometrical length
L_{eq}	Aggregate equivalent length of local resistance
r	Height of the roughness on the pipe internal surface
ν	Fluid kinematic viscosity

The accumulator pressure (p_a) has been expressed as a function of the oil volume (V) considering an isentropic transformation:

$$p_a=h(V)=\frac{P_{pr}}{\left(1-\dfrac{V}{V_A}\right)^k} \qquad \text{for } p_a > P_{pr} \tag{6}$$

where k is the specific heat ratio, V_A is the accumulator volume and P_{pr} is the pre-charge pressure, which is the gas pressure when the oil volume in the accumulator is zero. The oil flowing into the accumulator (q_a) is the derivative of the state variable V:

$$q_a=\frac{dV}{dt}=\dot{V} \tag{7}$$

Substituting eq. (4) (5) and (6) into eq. (1) and rearranging, the result is the output equation:

$$F_h=S\cdot h(V)+S\cdot k(S\cdot v) \tag{8}$$

The hydraulic motor model relates the accumulator pressure p_a and the flow q_m with the angular velocity ω and the torque T_m as:

$$\begin{aligned}T_m&=D\eta_m p_a=D' p_a \qquad D'=D\eta_m\\ q_m&=D\omega\end{aligned} \tag{9}$$

The parameter η_m is the mechanical efficiency of the motor and the loss due to leakage flow is determine based on the assumption that it is linearly proportional to the pressure at the input of the motor as:

$$q_{leak}=k_l\cdot p_a \tag{10}$$

where k_l is the motor leakage coefficient. Substituting (4), (9) and (10) into equation (2), the first state equation is obtained:

$$\dot{V}=-k_l\cdot h(V)-\frac{D}{J}\cdot L+S\cdot v \tag{11}$$

The flow and effort variables for the hydraulic motor shaft depicted in Figure 3 as $I{:}J$ component are characterized by equations:

$$T_S=\frac{dL}{dt}=\dot{L} \tag{12}$$

$$\omega=\frac{L}{J} \tag{13}$$

where J is the inertia momentum.
The loss due to the rotational friction of the shaft is described by:

$$T_f=B\cdot\omega \tag{14}$$

where B is the rotational friction coefficient.
The second state equation is obtained substituting equation (9), (12), (13), (14) and (5) into equation (3).
The resulting state space model is:

$$\begin{cases}\dot{V}=-k_l\cdot h(V)-\dfrac{D}{J}\cdot L+S\cdot v\\ \dot{L}=D\eta_m\cdot h(V)-\dfrac{B}{J}\cdot L-T_G\end{cases} \tag{15}$$

$$F_h=S\cdot h(V)+S\cdot k(S\cdot v) \tag{16}$$

It is composed of two non-linear state equations (15) which characterize the variation of the fluid volume (V) inside the gas accumulator and the variation of the hydraulic motor shaft angular momentum (L). The output equation (16) relates the damping force F_h to the oil volume inside the gas accumulator (V) and to the velocity of the stroke (v).

5. Analysis of the PTO dynamic

The availability of a mathematical model gives the possibility to analyze a system for the variation of its parameters. The analysis can be used either in the developing process for the components optimization or for the improvement of an existing system. In this case, the PTO dynamic is analyzed in order to characterize the response of the system respect to the variation of the accumulator volume (V_A) and the pre-charge pressure (P_{pr}). The model is first linearized around the pressure working point P_0. The linearized model of the accumulator is:

$$h(V) \approx h(V_0) + h'(V_0)(V - V_0) \tag{17}$$

where:

$$h'(V) = \frac{k \cdot P_{pr} \cdot V_A^{\ k}}{(V_A - V)^{k+1}} \qquad \text{for } p_a > P_{pr}$$

The substitution of the linearized model (17) into the equations (15) gives the system:

$$\dot{V} = -k_l h'(V_0)V - \frac{D}{J} \cdot L - k_l\big(h(V_0) - h'(V_0)V_0\big) + Sv$$

$$\dot{L} = D' h'(V_0)V - \frac{B}{J} \cdot L + D'\big(h(V_0) - h'(V_0)V_0\big) - T_G$$

which in characterized by the dynamic matrix:

$$\begin{bmatrix} -k_l h'(V_0) & -\dfrac{D}{J} \\ D\eta_m h'(V_0) & -\dfrac{B}{J} \end{bmatrix}$$

The resulting eigenvalues are complex conjugate and the magnitude is plotted as function of the accumulator size, pre-charge pressure and pressure working point. It is evident from Figure 4, Figure 5 and Figure 6 that the system responds faster (bigger magnitude of the eigenvalues) for smaller accumulator volume and lower pre-charge pressure. Therefore, if the objective is to keep the pressure inside the hydraulic circuit constant, it's preferable to have a bigger accumulator with a high pre-charge pressure and allow the system to work around a low pressure set point (P_0). Otherwise, if the objective is to regulate the damping force in real time acting on the torque of the hydraulic motor, it is preferable to use a small accumulator with a low pre-charge pressure.

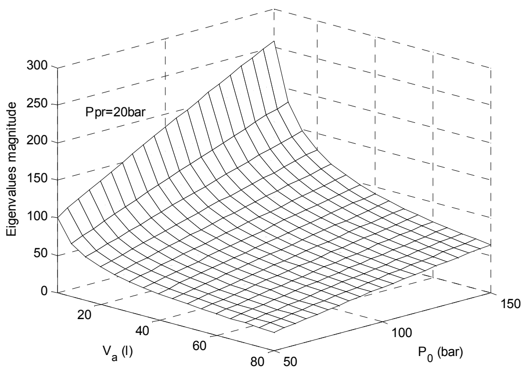

Figure 4: Eigenvalues magnitude for accumulator pre-charge pressure of 20bar

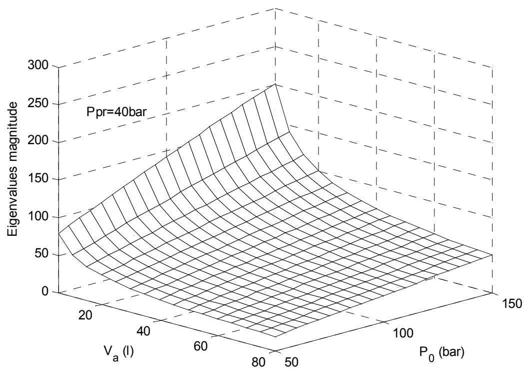

Figure 5: Eigenvalues magnitude for accumulator pre-charge pressure of 40bar

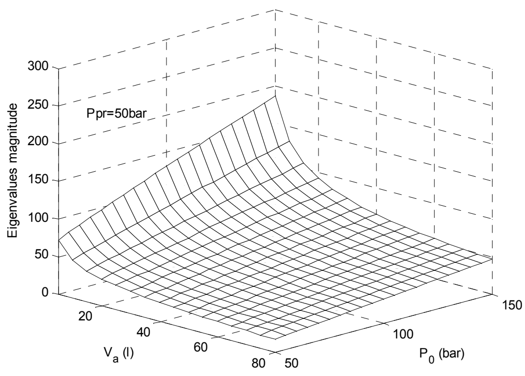

Figure 6: Eigenvalues magnitude for accumulator pre-charge pressure of 50bar

6. Conclusions

The model provides a non-linear dynamic description of the state space variables (V,L) and the output (F_h) as functions of inputs (T_G,v). The input and output variables have been selected to combine this model with a hydrodynamic model of the device, in order to obtain a complete wave-to-wire model (Figure 1). Bondgraph significantly supported the development of the model and its analysis showed the influence of the accumulator parameters on the dynamic of the system. In particular, the analysis showed that the system exhibits a faster dynamic for a smaller volume of the accumulator, for a smaller pre-charge pressure and for a higher working pressure.

Reference:

1. Karnopp, D. C.; Margolis, D. L.; Rosenberg, R. C.; "System dynamics", Wiley Interscience, 2000
2. Engja, H. & Hals, J., "Modelling and Simulation of Sea Wave Power Conversion Systems" 7th European Wave and Tidal Energy Conference, EWTEC, 2007