Monopole-Antimonopole Solutions of the Skyrmed $SU(2)$ Yang-Mills-Higgs Model

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Abstract

Axially symmetric monopole anti-monopole dipole solutions to the second order equations of a simple SU(2) Yang-Mills-Higgs model featuring a quartic Skyrme-like term are constructed numerically. The effect of varying the Skyrme coupling constant on these solutions is studied in some detail.

1 Introduction

The SU(2) Georgi-Glashow model in the Bogomol'nyi-Prasad-Sommerfield (BPS) limit supports monopoles [1, 2] which are solutions of the first order self-duality equations [3, 4]. Away from the BPS limit, when new gauge invariant and positive definite terms are added, the resulting monopoles are described by the solutions to the second order Euler–Lagrange equations, and not to the first order self-duality equations. Once these terms are introduced to the model, the BPS topological bound cannot be saturated.

BPS and non-BPS monopoles differ in two remarkable respects. First, the BPS multimonopoles can be constructed analytically [6, 7, 8, 9] while the non-BPS monopoles, e.g. when the Higgs potential is present [1, 2], can only be constructed numerically. Secondly, and perhaps physically more interestingly, BPS monopoles do not interact while non-BPS monopoles interact. In the presence of a Higgs potential this interaction is known to be repulsive [10, 11] and has been verified to be so numerically [12], while in the presence of Skyrme like terms, higher order in both the Yang-Mills (YM) curvature and the Higgs covariant derivatives, this interaction can be both repulsive and attractive [13]. In a particularly simple such (Skyrme like) model, this interaction was found [14] to be strictly attractive, and moreover it was found [14], rather unexpectedly, that the lowest energy bound states were the axially symmetric ones and not those with Platonic symmetries. (It was unexpected since this feature contrasts with that for Skyrmion bound states [15].)

All the above monopole solutions discussed are stable relative to the topological lower bound whether they saturate this bound, as for the BPS monopoles, or not, as for non-BPS ones. There is however another class of non–selfdual solutions to the second order Euler–Lagrange equations which are not stable and represent states of monopoles and anti-monopoles in equilibrium. The existence of such solutions was first proved by Taubes [17] for the model featuring no Higgs potential (and of course no higher order terms in the curvature and covariant derivative), namely for the model which supports BPS multimonopoles. Such a non-BPS solution, namely an unstable solution of the second order equations, was first constructed for this system with $SU(3)$ gauge group and subject to spherical symmetry by Burzlaff [18]. More recently Ioannidou and Sutcliffe [19] emoplyed a harmonic map Anstaz to construct such spherically symmetric solutions to the same (BPS) system with gauge groups $SU(3)$, $SU(4)$ and $SU(N)$. Using results on sigma model instantons, these authors [19] also argued that the zero charge solutions they constructed described monopole anti-imonopole pairs.

A direct approach to constructing zero charge monopole anti-monopole pairs for the $SU(2)$ BPS model was used sometime ago by Rüber [20]. This was the numerical construction of axially symmetric solutions with suitable boundary value conditions. More recently Kleihaus and Kunz [21] constructed this zero charge solution for the full Georgi-Glashow model featuring a Higgs potential, and they studied the effect of the Higgs potential in detail. To date, no such study has been reported in the literature pertaining to the model featuring higher order Skyrme like terms. In the background of the above described scenario it is pertinent to carry out such a study.

This is the aim of the present work. We will consider the zero charge axially symmetric monopole anti-monopole solutions as in [20, 21], for the simple skyrmed Higgs model studied in [14] whose axially symmetric charge-2 monopoles are mutually attractive. This contrasts with the monopole anti-monopole solutions studied in [21] for the model whose charge-2 monopoles are mutually repulsive, which makes the comparison of our results with those of [21] interesting. In addition to constructing the vorticity-1 monopole anti-monopole solutions, as in [20] and [21], but now for the skyrmed model here, we also construct the corresponding vorticity-2 solutions.

2 Skyrmed $SU(2)$ Yang-Mills-Higgs Model

The static energy of the simplified Skyrme like model considered is

$$\mathcal{E} = \int \left\{ \frac{1}{2}\mathrm{Tr}\{F_{\mu\nu}F^{\mu\nu}\} + \frac{1}{4}\mathrm{Tr}\{D_\mu\Phi D^\mu\Phi\} + \frac{\kappa}{8}\mathrm{Tr}\{[D_\mu\Phi, D_\nu\Phi][D^\mu\Phi, D^\nu\Phi]\} + \frac{\lambda}{2}\mathrm{Tr}\{(\Phi^2-\eta^2)^2\} \right\} d^3r \quad (1)$$

with field strength tensor of the $su(2)$ gauge potential $A_\mu = \frac{1}{2}\tau_a A_\mu^a$,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig\,[A_\mu, A_\nu] \ , \quad (2)$$

and covariant derivative of the Higgs field $\Phi = \tau_a \phi^a$ in the adjoint representation

$$D_\mu\Phi = \partial_\mu\Phi + ig\,[A_\mu, \Phi] \ , \quad (3)$$

and g denotes the gauge coupling constant, κ the coupling strength of the quartic Skyrme like Higgs kinetic term, λ the strength of the Higgs potential and η the vacuum expectation value of the Higgs field.

The topological charge Q is the well known quantity

$$Q = \frac{1}{4\pi\eta}\varepsilon^{ijk}\int \mathrm{Tr}\left\{F_{ij}D_k\Phi\right\} d^3r \ , \tag{4}$$

corresponding to the magnetic charge $m = Q/g$, and takes integer values that equal the winding number of the Higgs field [22]. The latter is encoded with the boundary conditions which yield the value of this integer.

To construct axially symmetric solutions that describe systems of monopoles and multimonopoles, specific boundary conditions must be imposed the Higgs field at infinity. For usual multimonopoles, the Higgs field at infinity is described by the vortex number n winding the azimuthal angle φ, n times and the polar angle θ does not wind. Zero magnetic charge monopoles on the other hand, namely those we seek to construct, can be achieved by requiring that in the asymptotic Higgs field the polar angle is enhanced by another integer m. This can also be achieved automatically by incorporating this integer m in the Ansatz [20, 21] as will be done below. The integral (4) can be evaluated for a system with m zeros of the Higgs field (i.e. with m monopole and antimonopole centres), and with vorticity n, yielding

$$Q = 4\pi n\eta^3[1-(-1)^m] \ . \tag{5}$$

In this paper we will restrict to the charge zero case $m = 2$ with vorticity $n = 1$, to carry out our detailed analysis of the system, with special attention to the κ dependence of the solutions. After that, we will briefly study also the case of $n = 2$ vorticity, again with $m = 2$. These are both monopole anti-monopole solutions to the second order equations carrying $Q = 0$.

3 Static axially symmetric $Q = 0$ Ansatz

We choose the static, axially symmetric, purely magnetic Ansatz employed in [20] for the monopole-antimonopole solution and in [23, 24] for the sphaleron-antisphaleron solution of the Weinberg-Salam model. Here the gauge field is parametrized by

$$A_0 = 0 \ , \quad A_r = \frac{H_1}{2gr}\tau_\varphi^{(n)} \ , \quad A_\theta = \frac{(1-H_2)}{g}\tau_\varphi^{(n)}, \quad A_\varphi = -n\frac{\sin\theta}{g}\left(H_3\tau_r^{(2,n)} + (1-H_4)\,\tau_\theta^{(2,n)}\right) \ , \tag{6}$$

and the Higgs field by

$$\Phi = \eta\left(\Phi_1\tau_r^{(2,n)} + \Phi_2\tau_\theta^{(2,n)}\right) \ . \tag{7}$$

All functions $H_1, H_2, H_3, H_4, \Phi_1$ and Φ_2 depend on (r, θ) or equivalently on $(\rho = r\sin\theta, z = r\cos\theta)$, with the $su(2)$ matrices $\tau_r^{(2,n)}$, $\tau_\theta^{(2,n)}$ and $\tau_\varphi^{(n)}$ defined in terms of the Pauli matrices τ_1, τ_2, τ_3 as

$$\begin{aligned} \tau_r^{(2,n)} &= \sin 2\theta(\cos n\varphi\,\tau_1 + \sin n\varphi\,\tau_2) + \cos 2\theta\,\tau_3 \ , \\ \tau_\theta^{(2,n)} &= \cos 2\theta(\cos n\varphi\,\tau_1 + \sin n\varphi\,\tau_2) - \sin 2\theta\,\tau_3 \ , \\ \tau_\varphi^{(n)} &= -\sin n\varphi\,\tau_1 + \cos n\varphi\,\tau_2 \ , \end{aligned} \tag{8}$$

and for later convenience we define

$$\tau_\rho^{(n)} = \cos n\varphi\,\tau_1 + \sin n\varphi\,\tau_2 \ . \tag{9}$$

Note that the dependence on the vorticity n is encoded through $\tau_r^{(2,n)}$ and $\tau_\theta^{(2,n)}$, and of course $\tau_\rho^{(n)}$.

We change to dimensionless coordinates, Higgs field and coupling parameters by rescaling

$$r \to \frac{r}{g\eta} \ , \quad \Phi \to \eta\Phi \ , \quad \kappa \to \frac{\kappa}{g^2\eta^4} \ , \quad \lambda \to \frac{\lambda}{g^2} \ ,$$

respectively. Then this Ansatz leads to the field strength tensor

$$F_{r\theta} = -\frac{1}{2r}\left(\partial_\theta H_1 + 2r\partial_r H_2\right)\tau_\varphi^{(n)} \ ,$$

$$\begin{aligned}
F_{r\varphi} &= \frac{n}{2r}\Big\{ \left(\sin 2\theta H_1 - 2\sin\theta H_1(1-H_4) - 2\sin\theta r\partial_r H_3\right)\tau_r^{(2,n)} \\
&\quad + \left(\cos 2\theta H_1 + 2\sin\theta H_1 H_3 + 2\sin\theta r\partial_r H_4\right)\tau_\theta^{(2,n)}\Big\} \ , \\
F_{\theta\varphi} &= -\frac{n}{2}\Big\{ \left(2\sin 2\theta(H_2-1) + 2\cos\theta H_3 - 2\sin\theta H_2(1-H_4) + 2\sin\theta\partial_\theta H_3\right)\tau_r^{(2,n)} \\
&\quad + \left(2\cos 2\theta(H_2-1) + 2\cos\theta(1-H_4) + 2\sin\theta H_2 H_3 - 2\sin\theta\partial_\theta H_4\right)\tau_\theta^{(2,n)}\Big\} \ , \qquad (10)
\end{aligned}$$

and the covariant derivative of the Higgs field

$$\begin{aligned}
D_r\Phi &= \frac{1}{r}\Big\{ \left(r\partial_r\Phi_1 + H_1\Phi_2\right)\tau_r^{(2,n)} + \left(r\partial_r\Phi_2 - H_1\Phi_1\right)\tau_\theta^{(2,n)}\Big\} \ , \\
D_\theta\Phi &= \left(\partial_\theta\Phi_1 - 2H_2\Phi_2\right)\tau_r^{(2,n)} + \left(\partial_\theta\Phi_2 + 2H_2\Phi_1\right)\tau_\theta^{(2,n)} \ , \\
D_\varphi\Phi &= n\left\{\left(\sin 2\theta - 2\sin\theta(1-H_4)\right)\Phi_1 + \left(\cos 2\theta + 2\sin\theta H_3\right)\Phi_2\right\}\tau_\varphi^{(n)} \ . \qquad (11)
\end{aligned}$$

The dimensionless energy density then becomes

$$\begin{aligned}
\varepsilon &= \mathrm{Tr}\left\{\frac{1}{r^2}F_{r\theta}^2 + \frac{1}{r^2\sin^2\theta}F_{r\varphi}^2 + \frac{1}{r^4\sin^2\theta}F_{\theta\varphi}^2\right\} \\
&\quad + \frac{1}{4}\mathrm{Tr}\left\{(D_r\Phi)^2 + \frac{1}{r^2}(D_\theta\Phi)^2 + \frac{1}{\sin^2\theta r^2}(D_\varphi\Phi)^2\right\} \\
&\quad - \frac{\kappa}{4}\mathrm{Tr}\left\{\frac{1}{r^2}[D_r\Phi, D_\theta\Phi]^2 + \frac{1}{r^2\sin^2\theta}[D_r\Phi, D_\varphi\Phi]^2 + \frac{1}{r^4\sin^2\theta}[D_\theta\Phi, D_\varphi\Phi]^2\right\} + \lambda\left(\left(|\Phi|^2 - 1\right)^2 \ , \qquad (12)
\end{aligned}$$

where $|\Phi| = \sqrt{\Phi_1^2 + \Phi_2^2}$ denotes the modulus of the Higgs field.

For a monopole-antimonopole pair we expect a magnetic dipole field for the asymptotic gauge potential. The dipole moment $C_{\mathbf{m}}$ can be extracted from the gauge field function H_3, in the gauge where the Higgs field approaches asymototically a constant. Like in Ref. [21] we find

$$H_3 = \frac{C_{\mathbf{m}}}{r}\sin\theta \ , \qquad (13)$$

while all other gauge field functions decay faster.

4 Numerical Results

As noted in [21] the Ansatz Eqs. (6), (7) possesses a residual $U(1)$ gauge symmetry. To obtain an unique solution we use the gauge fixing condition [21]

$$G_{\mathrm{f}} = \frac{1}{r^2}\left(r\partial_r H_1 - 2\partial_\theta H_2\right) = 0 \ . \qquad (14)$$

The system of partial differential equations is solved numerically subject to the following boundary conditions, which respect finite energy and finite energy density conditions as well as regularity and symmetry requirements. These boundary conditions are at the origin

$$H_1(0,\theta) = H_3(0,\theta) = 0 \ , \qquad H_2(0,\theta) = H_4(0,\theta) = 1 \ , \qquad (15)$$

$$\sin 2\theta\Phi_1(0,\theta) + \cos 2\theta\Phi_2(0,\theta) = 0 \ , \qquad \partial_r(\cos 2\theta\Phi_1(0,\theta) - \sin 2\theta\Phi_2(0,\theta)) = 0 \ , \qquad (16)$$

at infinity

$$H_1(\infty,\theta) = H_2(\infty,\theta) = 0 \ , \qquad H_3(\infty,\theta) = \sin\theta \ , \quad (1 - H_4(\infty,\theta)) = \cos\theta \qquad (17)$$

$$\Phi_1(\infty,\theta) = 1 \ , \qquad \Phi_2(\infty,\theta) = 0 \ , \qquad (18)$$

and on the z-axis

$$H_1(r,\theta=0,\pi) = H_3(r,\theta=0,\pi) = \partial_\theta H_2(r,\theta=0,\pi) = \partial_\theta H_4(r,\theta=0,\pi) = 0 \ , \qquad (19)$$

$$\Phi_2(r,\theta=0,\pi) = \partial_\theta \Phi_1(r,\theta=0,\pi) = 0 \ . \tag{20}$$

The numerical calculations were performed with the software package CADSOL, based on the Newton-Raphson method [25]. We have carried out the main part of the numerical analysis for the case of unit vortex number $n = 1$ in (8) as in Refs. [20] and [21]. In addition we have also studied more briefly, the case of $n = 2$.

Starting with the case of vorticity $n = 1$, we have constructed monopole-antimonopole solutions for a large range of values of the coupling constant κ. For vanishing coupling constant κ the monopole-antimonopole solution corresponds to a non-Bogomol'nyi solution of the BPS system, for which our results are in good agreement with those of [21]. Our numerical analysis was carried out for the skyrmed model in the absence of the Higgs potential, namely with $\lambda = 0$ in (12). We did however check that the presence of nonvanishing λ does not change the qualitative properties of our solutions. As expected the only effect it has is in the large r asymptotic region, where the modulus of the Higgs field for example, reaches its asymptotic value faster, namely exponentially.

In Figure 1 we show the normalised energy of the solitons $E/4\pi\eta$ and the energy $E_{inf}/4\pi\eta$, of the monopole-antimonopole pair with infinite separation corresponding to twice the energy of a charge-1 monopole, as functions of the coupling constant κ. As can be seen from Figure 1 the energy of the monopole-antimonopole solution is less than the energy of a monopole-antimonopole pair with infinite separation for all values of κ.

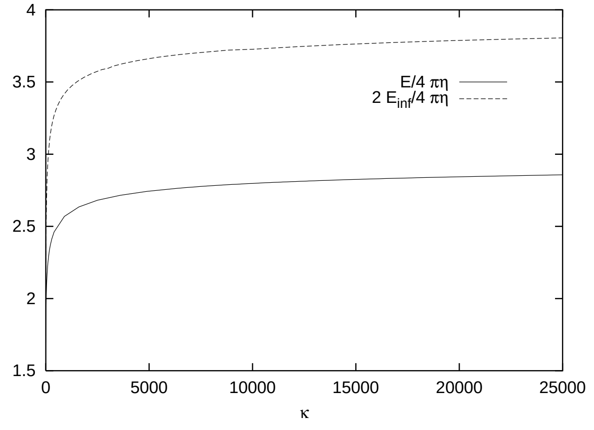

Figure 1: The energy of the monopole-antimonopole solution (solid line) and the energy of a monopole-antimonopole pair with infinite separation (dashed line), for $n = 1$.

In Figure 2 we exhibit the modulus of the Higgs field $|\Phi(\rho,z)|$ as a function of the coordinates $\rho = \sqrt{x^2+y^2}$ and z for $\kappa = 0$ and $\kappa = 100$. The zeros of $|\Phi(\rho,z)|$ are located on the positive and negative z-axis at $\pm z_0 \approx 2.1$ for $\kappa = 0$ and at $\pm z_0 \approx 1.5$ for $\kappa = 100$. The distance d of the two zeros of the Higgs field decreases monotonically with increasing κ.

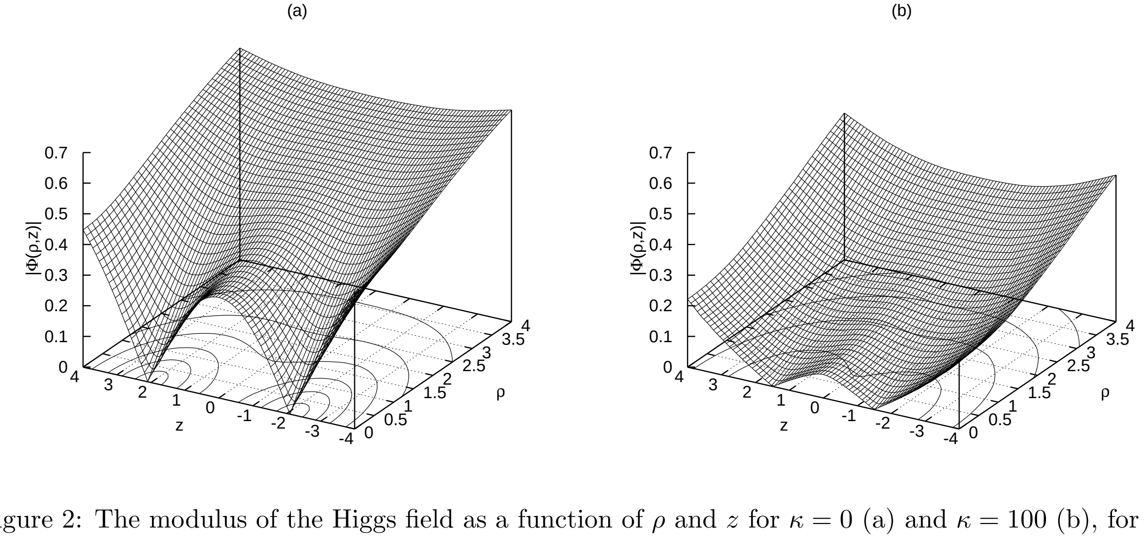

Figure 2: The modulus of the Higgs field as a function of ρ and z for $\kappa = 0$ (a) and $\kappa = 100$ (b), for $n = 1$

Asymptotically $|\Phi(\rho, z)|$ approaches the value 1. But at the origin the value of the modulus of the Higgs field decreases monotonically with increasing κ (see Figure 3). In the limit $\kappa \to \infty$ $|\phi_0| \approx 0.015$, and we expect the modulus of the Higgs field to be very small for $|z| \leq 4$.

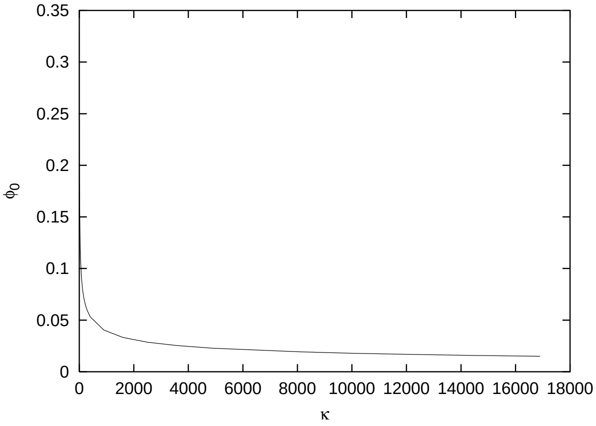

Figure 3: The modulus of Higgs fields at the origin as a function of κ, for $n = 1$

In Figure 4 we show the energy density of the monopole-antimonopole solution as a function of the coordinates $\rho = \sqrt{x^2 + y^2}$ and z for $\kappa = 0$ and $\kappa = 100$. At the locations of the Higgs field the energy density posesses maxima.

For small values of coupling constant κ the equal energy density surfaces near the locations of the zeros of the Higgs field assume a shape close to a sphere, centered at the location of the respective zero (see Figure 4 (a)). This presents further support for the conlcusion, that at the two zeros of the Higgs field a monopole and an antimonopole are located, which can be clearly distinguished from each other, and which together form a bound state.

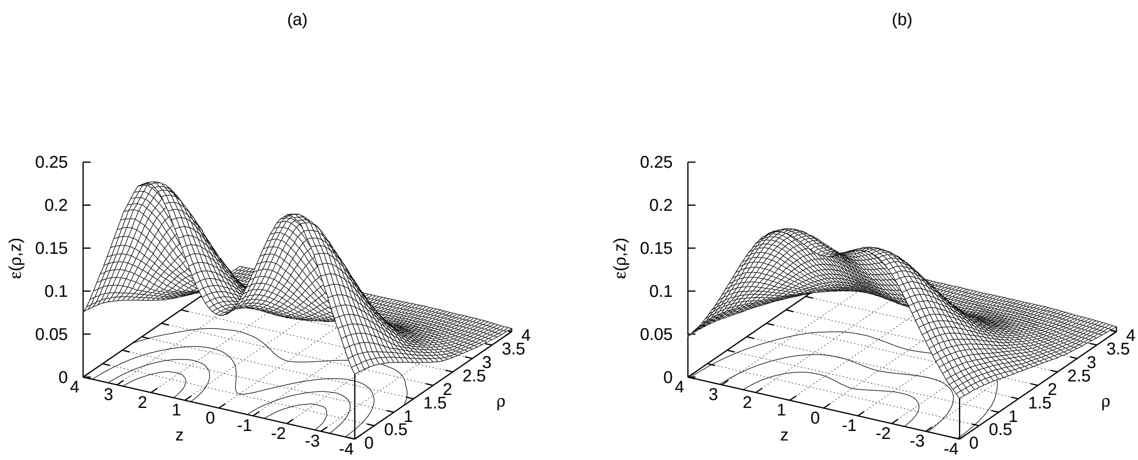

Figure 4: The dimensionless energy density as a function of ρ and z for $\kappa = 0$ (a) and $\kappa = 100$ (b), for $n = 1$

With increasing κ the distance d between the monopole anti-monopole centres becomes smaller tending to a limit as $\kappa \to \infty$. At the same time the spherical equal energy surfaces in Figure 4(b) become larger, and the equal energy density surfaces assume a shape that looks like the intersection of two spheres (see Figure 4 (b)), thus making it more difficult to distinguish the monopole from the anti-monopole. The dependence of the separation length d is given in Table 1 below as a function of κ.

Having exibited the qualitative properties of our *dipole* solutions, we give the values of the dipole moment that we calculated as a function of the coupling constant κ, again in the Table 1. As expected, with decreasing d the dipole moment $C_{\mathbf{m}}$ also decreases.

κ	0	9	16	25	36	49	64	100	8100	10000
d	4.19	3.64	3.49	3.38	3.29	3.21	3.16	3.06	2.54	2.53
$C_{\mathbf{m}}$	2.36	2.27	2.23	2.19	2.15	2.11	2.07	2.02	1.66	1.65

Table 1: Monopole anti-monopole separation d and dipole moment $C_{\mathbf{m}}$ as a functions of κ

Finally we constructed solutions for the case of vorticity $n = 2$. Most of the qualitative properties of these solutions do not differ from those of the $n = 1$ case just described. The most noticable quantitative difference concerns the value of the modulus of the Higgs field at the origin, analogous with Figures 1(a) and 1(b). We do not exhibit here these analogous figures, but simply note that the the moduli of the Higgs fields at the origin are *smaller* than those in Figures 1(a) and 1(b) for the same values of the coupling constant κ.

Another difference, qualitative though expected, is that the surfaces of equal energy are not spheres centred on the z-axis but describe rings or tori around it. This is exhibited in Figures 5(a) and 5(b), analogously with Figures 4(a) and 4(b).

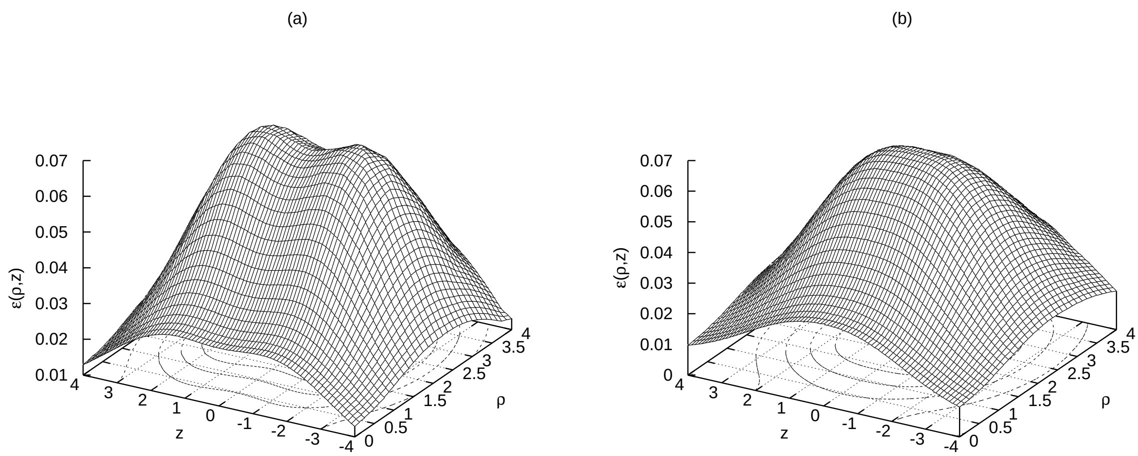

Figure 5: The dimensionless energy density as a function of ρ and z for $\kappa = 0$ (a) and $\kappa = 100$ (b), for $n = 2$

Again, as κ grows, the distiction between the monopole and anti-monopole rings gets blurred.

5 Summary

We have contructed axially symmetric solutions to a simple $SU(2)$ skyrmed YM-Higgs model, with such boundary conditions that result in the description of a monopole anti-monopole pair with zero magnetic charge. These solutions have lower mass than two infinitely separated charge-1 monopoles, and since they are characterised by zero magnetic charge, are not topologically stable.

When the usual boundary conditions are imposed, the skyrmed $SU(2)$ YM-Higgs model employed here supports mutually attractive monopoles, including axially symmetric charge-2 monopoles. This is in contrast to the Georgi-Glashow model studied in [21] where due to the Higgs potential the monopoles are mutually repulsive [12]. Nevertheless, the qualitative features of the monopole anti-monopole solutions in the two models are similar. Increasing the Skyrme coupling constant κ in the present model results in the approaching of the monopole and the anti-monopole centres down to a limiting value 2.53 as $\kappa \to \infty$, just as it does to the limiting value 3.0 as $\lambda \to \infty$ in the Georgi-Glashow model λ being the Higgs coupling constant. (Our results are for $\lambda = 0$.)

Another parallel property in the two models is the changing dipole moment with respect to the change in the Skyrme coupling constant κ and the Higgs coupling constant λ, in the two models respectively. Specifically in the present model the magnetic moment decreases with increasing κ, with limiting value 1.64, while in the Georgi-Glashow model it decreases with increasing λ, with limiting value 1.55, in the same units.

Finally, we studied also the case of a zero charge monopole which has vortex number $n = 2$ rather than $n = 1$. The qualitative properties again stay unchanged. The most noticable quantitative difference of the $n = 2$ soltion is that the value of the modulus of the Higgs field at the origin is smaller than that of the $n = 1$ solution, for the same value of κ, and, the distance between the two centres is also smaller. For example at $\kappa = 25$ the distance $d = 3.38$ for the $n = 1$ solutions while that for the $n = 2$ is $d = 1.33$.

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