An Explicit Criterion for Adaptive Periodic Noise Canceller Robustness Applied to Feedback Cancellation

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Abstract

This paper addresses the issue of robustness of an LMS-driven Adaptive Periodic Noise Canceller (APNC) in a closed-loop system. By adopting an analysis based on H^{∞}-theory, expressions are given under which the APNC, driven by the LMS algorithm, will exhibit robust performance properties. Simulation results are used to verify the analysis. Comparison is also made with an expression for stepsize derived for the less stringent bound of algorithm stability to demonstrate the strictness of the robustness criterion.

Keywords

Adaptive Periodic Noise Canceller, robust performance, H^{∞}-theory.

Introduction

Perhaps the greatest technical challenge in the design of a hands-free speech terminal lies in the echo suppression device that must attenuate the electro-acoustic feedback due to the far end speech signal. One possible solution has been shown to be the LMS-driven Adaptive Periodic Noise Canceller (APNC) [1]. However, it is recognised that robust performance of the Acoustic Echo Cancellation (AEC) device is essential in the context of hands-free telephony as the equipment must provide sufficiently high speech quality while dealing with sudden or rapid fluctuations in the input signal [2]. Another application that has recently been the focus of research into methods for robust feedback cancellation is that of hearing aid devices [3] [4]. However, while the work in this area has investigated the robustness properties of the overall system [4], consideration has not yet been given to the robustness of the LMS-based adaptive filtering algorithm employed. Thus, although the requirement of algorithm stability is necessary for proper operation, it may not be sufficient to guarantee a practically useful level of performance at all times given the dynamic nature of both the input speech and the operating environment. Fortunately, it is possible within the framework of H^{∞}-theory to formulate an approach to robustness analysis, and thus provide a more meaningful criterion for algorithm performance [5].

An empirical determination of the necessary conditions for robust performance of an LMS-driven adaptive filter in a one-dimensional open-loop feedback system with a white Gaussian input was given in [6], and for a one-dimensional closed-loop feedback system under similar input conditions in [7]. This paper aims to extend these analyses by superseding the empirical approach in favour of deriving an explicit expression for the upper bound on the algorithm stepsize that will ensure algorithm robustness. Following this, experimental results are presented to validate the robustness properties of this expression and to demonstrate the stringency of this maximum stepsize value compared to the maximum upper bound for algorithm stability. Lastly, suggestions for applying this expression in an arbitrary dimensional closed-loop APNC configuration are given.

Method

A model of the APNC in a closed-loop echo control configuration situation is shown in Figure 1, where $s(n)$ denotes the input and the feedback component is represented by $s_e(n)$. For analytical simplicity, the dimensionality of the problem is initially taken to be unity; the feedback environment is modelled by the transfer function $H(z) = h_1 z^{-1}$ and a single-weight APNC is assumed.

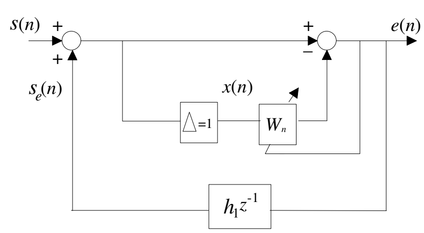

Figure 1 Block Diagram of One-Dimensional APNC Closed-Loop System

The output of the system $e(n)$ may be given

$$e(n) = s(n) + h_1 e(n-1) - W^{*} x(n) - V_n x(n) \quad (1)$$

where the filter input is given by

$$x(n) = s(n-1) + h_1 e(n-2) \qquad (2)$$

and

$$V_n = W_n - W^* \qquad (3)$$

is the difference between the filter weight and its optimum value.

Thus, the output error due to filter weight misadjustment, $\varepsilon(n)$, can be defined as

$$\varepsilon(n) = e(n)\big|_{W^*} - e(n)\big|_{W_n} \qquad (4)$$

giving

$$\varepsilon(n) = V_n\, x(n) \qquad (5)$$

From (5), it is possible to interpret the term V_n as an error gain which is a function of all previous inputs and outputs. Robust performance implies that the magnitude of this gain is minimised over time and that signal fluctuations at the system input will not disturb this minimisation procedure [1], [3]. The weight vector update equation in terms of V_n is given by

$$V_{n+1} = V_n + 2\mu e(n)x(n) \qquad (6)$$

Ideally, $\lim_{n\to\infty} V_n = 0$ but in reality there will always be an estimation error which is dependent on the disturbance terms in $x(n)$ which can disrupt the minimisation procedure. The transfer operator that maps this input disturbance $x(n)$ to the residual output error is denoted as $\boldsymbol{Z}_n(\mu)$. It is possible then to represent (6) a time-growing matrix

$$\boldsymbol{\varepsilon}(n) = \boldsymbol{Z}_n(\mu)\boldsymbol{X}(n) \qquad (7)$$

where $\boldsymbol{\varepsilon}(n)$ denotes a vector of error outputs and the vector of input disturbances $\boldsymbol{X}(n)$ is $\left\{(2\mu)^{-1/2}(V_1), \{x(n)\}_{n=1}^{\infty}\right\}$ with $(2\mu)^{-1/2}(V_1)$ being the (weighted) energy of the weight error due to the initial guess.

The criterion for robust performance, then, is that the energy of the residual error must be upper bounded by the energy of the disturbances and the initial uncertainty

$$\frac{\sum_{n=1}^{\infty}\varepsilon^2(n)}{(2\mu)^{-1}V_1^2 + \sum_{n=1}^{\infty}x^2(n)} \le 1 \qquad (8)$$

This translates into ensuring that for each iteration of the algorithm the H^∞-norm of $\boldsymbol{Z}_n(\mu)$ is less than or equal to one

$$\inf_n \left\|\boldsymbol{Z}_n(\mu)\right\|_\infty \le 1 \qquad (9)$$

where the H^∞-norm of $\boldsymbol{Z}_n(\mu)$ provides a measure of the peak value of its gain and is defined as

$$\left\|\boldsymbol{Z}_n(\mu)\right\|_\infty = \sup_{x\neq 0}\frac{\|\boldsymbol{\varepsilon}\|_2}{(2\mu)^{-1/2}|V_1| + \|\boldsymbol{X}\|_2} \qquad (10)$$

Expanding (8), and substituting both (5) and the update equation for the weight-error V_n (6), an expression for the maximum allowable stepsize to ensure robustness can be found.

$$\mu \le \frac{\varepsilon^2(n) - 2\varepsilon(n)x(n+1) + x^2(n) + 2W^* x(n)\varepsilon(n)}{2e^2(n)x^2(n)} \qquad (11)$$

However, the resulting equation (11) is in terms of $x(n)$ and $e(n)$, quantities that cannot be determined until after the algorithm has been initialised. To produce a practically useful expression all instances of it can be replaced with the known quantity $s(n)$, assuming that the algorithm is stable and convergent and has a zero initial weight vector. Thus, the expression for the upper bound on the stepsize to ensure robustness is

$$\mu \le \frac{1 + D_{\min}}{2\max(s(n) + h_1 s(n))^2} \qquad (12)$$

where

$$D_{\min} = -W^{*2} + 2W^* \min\left|\left(s(n+1)\big/ s(n)\right)\right| \qquad (13)$$

where W^* is the value of the optimum weight.

Experimental evaluation of (12) was carried out, where the information signal $s(n)$ was chosen to be Gaussian, the magnitude of the feedback coefficient h_1 was varied over the range 0.1 to 1 in steps of 0.1, and the algorithm stepsize was taken as the maximum of the bound given by (12).

For this input, the optimum weight vector was calculated to be [8]

$$W^* = \frac{h_1}{1 + h_1^2} \qquad (14)$$

by making the simplifying assumptions that for a broadband input

$$E\left[s(n)e(n-k)\right] = \sigma^2, \quad k = 0$$

$$= 0, \quad \text{otherwise} \tag{15}$$

where σ^2 is the power of the white noise input, and

$$E[e(n)e(n-k)] = \sigma^2, \quad k = 0$$
$$= 0, \quad \text{otherwise} \tag{16}$$

The H^{∞}-norm is actually calculated by finding the maximum singular values $\sigma_{n\max}$ of the transfer operator $\boldsymbol{Z}_n(\mu)$ at each time instant: these are given by

$$\sigma_{n\max} = \sqrt{\max\left(\lambda_n\left(\boldsymbol{ZZ}^T\right)\right)} \tag{17}$$

where λ_n denotes the eigenvalues of the matrix $\boldsymbol{ZZ}^T$.

These maximum singular values of the transfer operator are plotted in Figure 2.

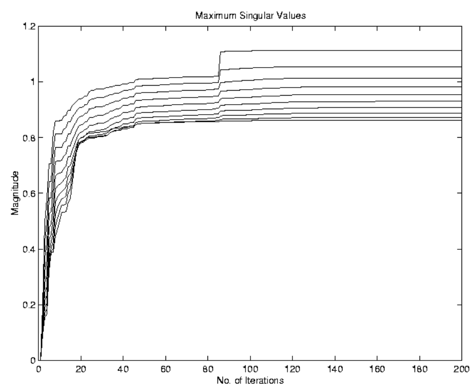

Figure 2 Maximum Singular Values for Closed Loop APNC System using (12)

From Figure 2 it can be seen that the maximum singular values of $\boldsymbol{Z}_n(\mu)$ are not less than unity in all cases demonstrating that robust performance of the APNC is not guaranteed within the stepsize bound given by (12). However, for values of feedback coefficient $h_1 \geq 0.4$, the maximum singular values of $\boldsymbol{Z}_n(\mu)$ are less than unity demonstrating the well-known property of better algorithm performance for more severe input conditions [5].

From the results it was concluded that a modification to this expression was required, as robustness was not demonstrated for small-values of h_1. Thus, (12) was modified to become

$$\mu \leq \frac{1 + D_{\min}}{2\max(2s(n))^2} \tag{18}$$

Figure 3 shows the resultant maximum singular values of the Transfer operator matrix for this system under the same experimental conditions as specified above with the stepsize taken as the maximum of the bound given by (18). As can be seen in the figure, (18) ensures that the maximum singular values are less than unity in all cases, thus demonstrating algorithm robustness. Additionally, better performance, in terms of both algorithm convergence and lower error gain, is obtained for the worst case input conditions.

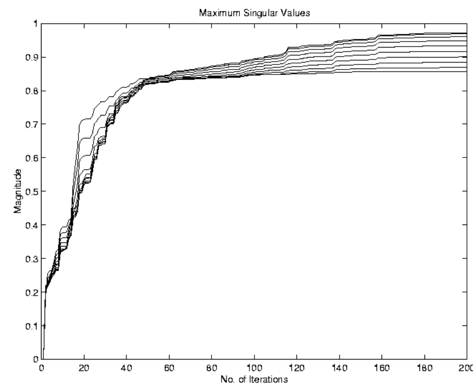

Figure 3 Maximum Singular Values with Modified Algorithm Stepsize

It is worthwhile to compare the maximum stepsize for algorithm robustness as given by (18) with the bound proposed for algorithm stability in [9]. This is calculated under the assumption that the filter input $x(n)$ is a Gaussian process. [13] presented a derivation of an exact expression for the maximum stepsize for algorithm stability based on a second moment analysis of the filter, demonstrating that it was second moment stable provided that

$$\mu < \frac{1}{2\lambda_i} \quad i = 0, \ldots, N-1 \tag{19}$$

and

$$\eta(\mu) = \sum_{i=0}^{N-1} \frac{\mu\lambda_i}{1 - 2\mu\lambda_i} < 1 \tag{20}$$

where μ is the stepsize and λ_i are the N eigenvalues of the filter data autocorrelation matrix. Thus, assuming that the input $x(n)$ to the APNC can be regarded as a Gaussian process and with $N = 1$, there exists only a single eigenvalue of the filter input autocorrelation matrix

$$\lambda = \left(1 + h_1^2\right)\sigma^2 \tag{21}$$

Substituting (19) into condition (20) produces

$$\eta(\mu) = \frac{\mu\left(1 + h_1^2\right)\sigma^2}{1 - 2\mu\left(1 + h_1^2\right)\sigma^2} < 1 \qquad (22)$$

and then by rearranging (22) the maximum upper limit on the algorithm stepsize for stability is

$$\mu < \frac{1}{\left(1 + h_1^2\right)\sigma^2} \qquad (22)$$

where σ is the power of the input.

Figure 4 shows a plot of the resulting stepsizes calculated using equations (18) and (22) for the same input conditions, where the values (18) is drawn with a solid line and those of (21) with a dashed line. As anticipated, the stepsize values for robust performance are smaller in magnitude demonstrating the greater strictness of this requirement over algorithm stability.

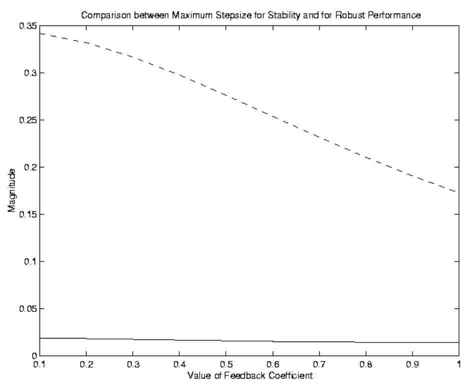

Figure 4 Maximum Stepsize Values for Stability and Robust Performance

Conclusion

Two analytical expressions for the upper bound on the stepsize to ensure robustness of the APNC in a closed-loop configuration were determined. Simulation results showed that the second expression, a modified version of the first, ensured robust performance of the APNC in a closed loop configuration. It is most probable that the necessity for the modification lies with the simplifying assumptions used to form (12). The comparison between the expressions on the upper bound on the stepsize for robustness and for algorithm stability revealed that a more stringent limit is placed on the value of the stepsize by demanding that algorithm performance be robust. The benefits of the robustness criterion are that it provides a practically useful quantitative measure of algorithm performance.

It can be shown that (12) is a valid upper bound on the stepsize in all cases where the number of weights match the degree of the feedback path. However, for over- and under-determined systems, i.e. when the number of filter weights is mismatched with the underlying parameters of the interference, re-evaluation of the expression would be required. The immediate intention for future work is to evaluate system performance for a speech input signal [10].

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