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Suboptimal Causal Reactive Control of Wave Energy Converters Using a Second Order System Model

Francesco Fusco, John V. Ringwood

Center for Ocean Energy Research (COER)
National University of Ireland Maynooth
Maynooth, Co. Kildare, Ireland

ABSTRACT

Wave Energy Converters (WECs) based on oscillating bodies can achieve optimal energy absorption under certain conditions associated with reactive control. These conditions, in general, are not realisable in practice because non-causal and future values of the excitation force need to be known. In this paper, an alternative approach is presented, where the relationship between the optimal velocity and the excitation force is realised through a simple coefficient of proportionality, thus removing the problem of non-causality. From theoretical considerations and numerical simulations over a range of heaving WECs in different sea conditions, it is shown that such suboptimal and causal approximation, while significantly reducing the complexity and improving the robustness of reactive control, allows the achievement of values of energy capture very close to the ideal optimum.

KEY WORDS: Wave Energy; Reactive Control; Floating Systems Modelling.

INTRODUCTION

The efficiency of WECs that consist of oscillating systems, can be significantly increased through an automatic control that tunes its oscillations to the incident wave elevation, in such a way to improve the power transfer from the ocean to the system. The analytical optimal solution for the maximisation of the energy extraction, requires the system to be in resonance with the wave force or pressure (Falnes, 2002), and it is termed reactive control, or complex-conjugate control for its analogy in electrical systems. Alternative sub-optimal control solutions have also been proposed, where the limitations imposed by the physics of the system (e.g. amplitude of motion or velocity, applicable forces), ignored by reactive control, are also taken into account. These alternatives include latching (Babarit and Clement, 2006), where the oscillation in the system is delayed so to be in phase with the waves excitation and the power take-off (PTO) is purely passive, Model Predictive Control (MPC) (Bacelli et al., 2009; Hals et al., 2011), which well handles the use of constraints, but also constrained analytical solutions (Evans, 1981).

The effectiveness of the different real-time control strategies depends, among other issues, on the prediction of future wave elevation or wave excitation force acting on the system at least for a few seconds (Falnes, 2007). Short-term wave forecasting has been studied either with a deterministic approach (Belmont et al., 2006; Tedd and Frigaard, 2007; Van Den Boom, 2009) and as a purely stochastic univariate time series problem (Fusco and Ringwood, 2010). In the latter case it is demonstrated how accurate predictions of the swell can be achieved with autoregressive (AR) models for more than one wave period ahead.

Some control techniques, however, reactive control and MPC *in primis*, do not take into account any *a priori* information about the excitation force, particularly the fact that the range of frequencies within which the waves (and their resulting force) are contained is limited. A closer analysis of reactive control can show that, within this range of frequencies, the non-causal frequency response relating the optimal velocity to the excitation force is flat. An approximation of such relationship with a pure constant may therefore be not far from optimal over most sea conditions, while at the same time would remove the non-causality.

In the remainder of the paper, after a detailed discussion of reactive control, a methodology to determine a non-causal approximation is presented. The methodology is based on the reduction of the model of a WEC to second order. The effectiveness of the sub-optimal and causal realisation of reactive control is finally compared with the implementation of the ideal reactive control for a range of WECs consisting of heaving cylinders, over a variety of sea conditions.

REACTIVE CONTROL

The System

In this study, a floating cylinder constrained to move in the heaving direction is considered, as shown in Fig. 1. The cylinder oscillates as a result of the excitation force, $f_{ex}(t)$, due to the incident waves, and of a controllable load force, $f_u(t)$, produced by a power take-off (PTO) mechanism. The dynamics of the oscillation velocity, $v(t)$, are described

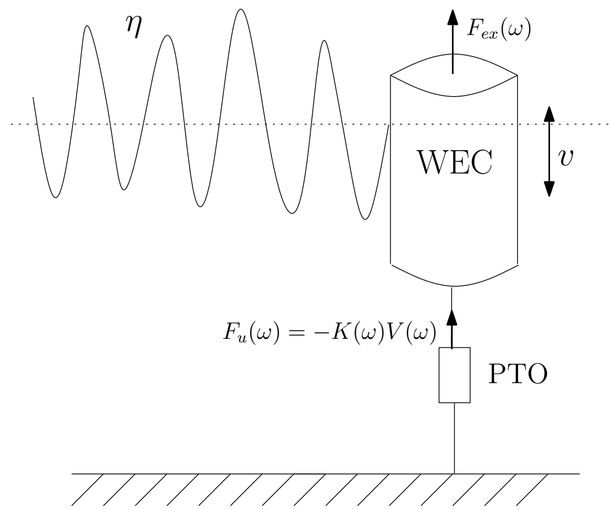

FIGURE 1 – Floating cylinder constrained to move in the heaving direction. The PTO force, f_u, is assumed to be related to the velocity through a law of the type in Eq. 6.

by the following integro-differential equation:

$$m\dot{v}(t) + \int_0^t z(\tau)v(t-\tau)d\tau + K_s \int_0^t v(\tau)d\tau + K_f v(t) = f_{ex}(t) + f_u(t), \quad (1)$$

where $z(t)$ is the impulse response modelling the radiation, K_s is the buoyancy coefficient and K_f is a friction coefficient introduced to model the losses. Eq. 1 is conveniently transformed into the frequency domain:

$$\left\{\jmath\omega(m+M_\infty) + H_r(\omega) + \frac{K_s}{\jmath\omega} + K_f\right\} V(\omega) = F_{ex}(\omega) + F_u(\omega), \quad (2)$$

where the quantities with capital letter, $X(\omega)$, denote the Fourier transform of the correspondent time-domain signal or impulse response $x(t)$. Note that, in going from Eq. 1 to Eq. 2, the Fourier transform of the radiation kernel $z(t)$, namely $Z(\omega)$, has been decomposed as:

$$Z(\omega) = B(\omega) + \jmath\omega\left[M_a(\omega) + M_\infty\right] = H_r(\omega) + \jmath\omega M_\infty, \quad (3)$$

where the real part, $B(\omega)$, is the radiation damping, $M_a(\omega)$ is the added mass and M_∞ is the added mass at infinite frequency (Falnes, 2002). Based on Eq. 2, the floating cylinder in heaving mode can be modelled with a single-input single-output (SISO) transfer function, $H(\omega) = 1/Z_i(\omega)$, between the input forces (excitation from the waves and load from the PTO) and the output velocity:

$$V(\omega) = \frac{1}{Z_i(\omega)}\left[F_{ex}(\omega) + F_u(\omega)\right], \quad (4)$$

where $Z_i(\omega)$ is the well-known intrinsic mechanical impedance of the floating system, described, among others, in (Falnes, 2002):

$$Z_i(\omega) = B(\omega) + K_f + \jmath\omega\left[m + M_\infty + M_a(\omega) - \frac{K_s}{\omega^2}\right]. \quad (5)$$

Maximum Wave Energy Absorption: Reactive Control

No specifications about the power take-off (PTO) mechanism that actually provides the load force, $f_u(t)$, is made at this stage. The only assumption is that it can produce any force calculated by a control law of the form:

$$F_u(\omega) = -K(\omega)V(\omega) \quad (6)$$

Without any other constraint about the system or about the PTO (maximum amplitude of motion, maximum forces,...), and without any hypothesis about the disturbance, $f_{ex}(t)$, the only criterion for the design of the controller, $K(\omega)$, is that it allows maximum energy transfer from the excitation (disturbance) to the load, or in other words, that the wave energy absorption is maximised.
The average power absorbed at the load, P_u, is the time integral of the product of the load force and the system velocity:

$$P_u = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{+T} f_u(t)v(t)dt, \quad (7)$$

Such expression for the average power can more conveniently be expressed in the frequency domain (Falnes, 2002):

$$P_u = \frac{1}{2\pi}\int_0^{+\infty} \left[F_u(\omega)V^*(\omega) + F_u^*(\omega)V(\omega)\right] d\omega, \quad (8)$$

Maximisation of Eq. 8 with respect to $K(\omega)$ provides the optimal control law producing the ideal load force and system's velocity so that maximum wave energy is absorbed by the PTO. It can be shown (Falnes, 2002) that such a maximum is obtained, at any frequency ω, under the condition:

$$P_u(\omega) = max \quad \Leftrightarrow \quad K(\omega) = K_{opt}(\omega) = Z_i^*(\omega), \quad (9)$$

and this is *independent* of the frequency distribution of the disturbance, $F_{ex}(\omega)$. The optimal control law, Eq. 9, is termed *complex-conjugate* control, as the load impedance required in order to produce the optimal force is the complex-conjugate of the intrinsic impedance of the system:

$$F_{u,opt}(\omega) = -Z_i^*(\omega)V(\omega) \quad (10)$$

An alternative term is *reactive* control, from the fact that the imaginary part of the load impedance determines a reactive power and therefore the necessity to inject power into the system during part of the cycle. The term reactive control will be adopted throughout this paper.

Real-time Implementation and Causal Approximation

The optimal controller in Eq. 10 is not realisable because it is non-causal (Falnes, 2002), that is current values of the optimal load force depend on future values of the velocity. A suboptimal implementation of such a control law can be realised using predictions of the system's velocity (Korde, 2000), but the problem is complicated by the fact that the variable to predict, the velocity, is the controlled variable so that forecasting and control cannot be dealt with separately.
It would simplify the control design, in the authors opinion, if the control law were expressed as a function of the excitation force, which is only related to the undisturbed incident wave field and to the fundamental properties of the system, which are unaffected by the system's motion.

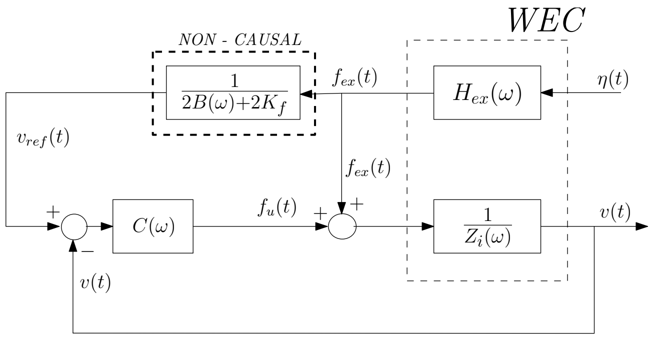

FIGURE 2 – Real-time implementation of reactive control as velocity-following. In general, the reference generation is *non-causal*.

In particular, from Eqs. 9∼10, the optimal velocity and the optimal PTO force can be expressed as:

$$V_{opt}(\omega) = \frac{1}{2B(\omega)+2K_f} F_{ex}(\omega) \tag{11}$$

$$F_{u,opt}(\omega) = -\frac{Z_i^*(\omega)}{2B(\omega)+2Kf} F_{ex}(\omega). \tag{12}$$

Eqs. 11∼12 can both be utilised as reference-generation logic for a state of the system, velocity or load force, that shall then be imposed through a lower level feedback control loop (Fusco et al., 2010). The problem of non-causality would still remain, but the implementation would rely on the prediction of a quantity, the excitation force, unaffected by the controller and system dynamics. Prediction and control can be therefore treated as two separate problems. Note that some wave forecasting algorithms have been shown to provide accurate predictions for more than 1 wave period into the future (Fusco and Ringwood, 2010).

In the present study, reactive control is implemented through calculation of the optimal velocity from Eq. 11 and through a velocity-following lower level loop, imposing the desired velocity on the system. The approach is illustrated in Fig. 2, where $C(\omega)$ represents the dynamics of the power take-off plus the lower level control. As will become clearer in the following, the choice of Eq. 11, as opposed to Eq. 12, will allow for a simpler and more intuitive causal approximation of the control problem, which is the purpose of this paper.

Central to this study is the non-causal reference-generation logic, that is the transfer function between the excitation force and the optimal oscillation velocity, defined as:

$$H_{opt}(\omega) \triangleq \frac{1}{2B(\omega)+2K_f}. \tag{13}$$

In Fig. 3, an example of $H_{opt}(\omega)$ is shown, along with its inverse Fourier transform $h_{opt}(t)$. Based on properties of the Fourier transform and on physical properties of the system, as well as on qualitative observation, as from Fig. 3, a few important considerations on the nature of such a non-causal transfer function may be drawn:

- $H_{opt}(\omega)$ is real and even (radiation resistance real and even) and does not introduce any phase shift, which means that the optimal velocity is always in phase with the excitation force (Falnes, 2002).
- $H_{opt}(\omega)$ has a band-stop like behavior (although being always > 0). From Eq. 13, it is minimum and quite flat within the frequency-band where the radiation resistance is significantly different from zero, while it is maximum and constant for lower and higher frequencies, the constant being the reciprocal of the friction coefficient K_f.

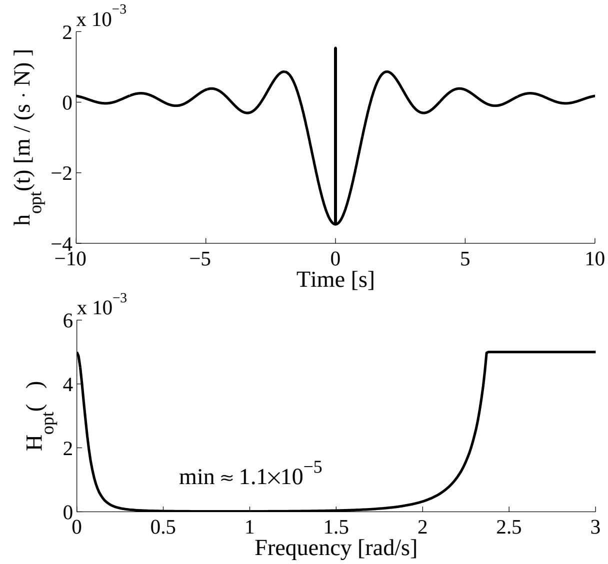

FIGURE 3 – Non-causal transfer function between excitation force and optimal velocity. Example for a heaving cylinder of radius $R = 5m$, draught $h = 6m$ and submerged mass $M = 3.9 \times 10^5\ Kg$.

- The corresponding impulse response $h_{opt}(t)$ is real and even, and therefore non-causal, as $h_{opt}(t) \neq 0$ for $t < 0$. The length of the time-window over which it is significantly different than zero is inversely proportional to the stop-band width of $H_{opt}(\omega)$. In particular, it would converge to an impulse if the stop-band were infinitely wide (transfer function constant).

It may be argued that, in practice, if the WEC (cylinder in this case) is well suited to a certain sea location, most of the incident waves would be contained in the band of resonance of the device, which corresponds to the interval of frequencies where $H_{opt}(\omega)$ has minimum value (radiation maximum) and is quite flat. This indicates that in most situations, the reference-generator in Eq. 11 would work within its stop-band characteristics, and that a constant approximation of $H_{opt}(\omega)$ could offer a good approximation of the ideal reactive control condition, while at the same time removing the complexities involved with the non-causality.

In the following, a methodology to determine such an approximation is presented, followed by a comparison of the performance of the full reactive non-causal reactive control against its causal approximation.

2ND-ORDER MODEL REDUCTION AND CAUSAL REACTIVE CONTROL

In this section a methodology for the causal realisation of reactive control, illustrated in Fig. 2, is presented, where the reference generation logic of Eq. 11, is approximated with a real constant transfer function, on the basis of the previous discussion. The approach is based on the model order reduction of the floating system to 2^{nd} order, which in the single-body case is able to adequately describe its dynamics. It is then shown that the transfer function $H_{opt}(\omega)$, as from Eq. 13, reduces to a real constant which can be used for the required causal approximation of reactive control.

Model Order Reduction Based on Hankel Singular Values

Given that usually a single-body floating system, like the cylinder in

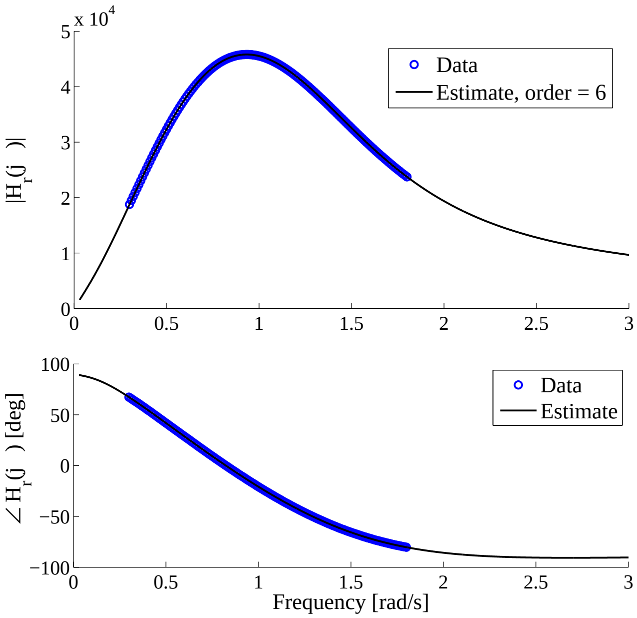

FIGURE 4 – Frequency domain identification of the radiation impedance of the heaving cylinder 2 in Table 1, with a 6th-order transfer function 1, using Matlab toolbox developed by (Perez and Fossen, 2009).

Fig. 1, has a resonant-like behavior similar to a mechanical oscillator, it may be argued that its dynamics is dominated by a resonant 2^{nd} order system (mass-spring-damper mechanical oscillator), to a greater or lesser extent, depending on the specific geometry. In order to verify such a statement it would be convenient to have the system described by classical state space equations, which allow the isolation of the individual components of the system's dynamics.

However, the equation of motion of a heaving floating system, either in the time domain, Eq. 1, or in the frequency domain, Eq. 2, represents, in general, a system of infinite order, so that an infinite number of states would be required. This is due to the fact that, in general, there is no explicit finite order representation for the radiation force, in particular for the radiation kernel $z(t)$ or for its Fourier transform $Z(\omega)$, that are determined numerically. Such difficulty is usually overcome by identifying a finite-order system to model the radiation, a well developed topic in the area of offshore and marine structures. Most common solutions approach the identification in the time-domain, through the Prony's coefficients (Babarit and Clement, 2006) or state space models (Taghipour et al., 2008), or in the frequency domain (Perez and Fossen, 2009, 2008; Taghipour et al., 2008), the latter having been demonstrated to better suit the problem (Perez and Fossen, 2008). As an example, consider Fig. 4, where a 6^{th}-order approximation of the radiation for a heaving cylinder is compared with the numerical data calculated from the hydrodynamic software (WAMIT Inc., 2008). The identification is performed with a MATLAB toolbox developed by Perez and Fossen (2009). The identified transfer function accurately fits the numerical data from the hydrodynamic software and in fact, typically, 2^{nd} to 4^{th} order systems are documented to provide accurate enough estimates of the radiation impedance (Perez and Fossen, 2008).

Once a finite-order approximation of the radiation impedance, namely $\hat{H}_r(s)$, has been identified:

$$F_r(s) = -\hat{H}_r(s)V(s), \tag{14}$$

cylinder	R $[m]$	h $[m]$	m $[Kg]$	σ_2/σ_3
1	5	6	2.6×10^5	230
2	5	8	3.9×10^5	464
3	5	15	1×10^6	1336
4	5	20	1.3×10^6	3204

TABLE 1 – 2^{nd} order model reduction applied to some floating cylinders. R is the radius, h is the draught and m is the submerged mass.

the floating system in Eq. 2 can be approximately modelled with the following SISO finite-order system:

$$\frac{V(s)}{F_{ex}(s)+F_u(s)} = \frac{s}{s^2(m+M_\infty)+s\hat{H}_r(s)+sK_f+K_s} \triangleq \text{H(s)}, \tag{15}$$

where s is the complex frequency in the Laplace domain, $s=\alpha+\jmath\omega$. The input is a superposition of the excitation force from the waves plus the control force from the PTO, while the output is the system's velocity. The finite-order SISO system of Eq. 15 is equivalently described by state space equations of the type:

$$\begin{cases} \dot{x}(t) = Ax(t) + B\left[f_{ex}(t)+f_r(t)\right] \\ v(t) = Cx(t) \end{cases}, \tag{16}$$

where $x(t)$ is a state vector of dimension equal to the order of the transfer function in Eq. 15 (2+ the order of $\hat{H}_r(s)$), if the state space realisation is minimal. Each component of the state vector $x(t)$ can be associated with a positive real quantity, called the Hankel singular value, that quantifies its energy (Chen, 1999), as explained in more detail in the Appendix. State components with a relatively low Hankel singular value can then be removed from the system with no or little effect on the overall dynamics (again refer to the Appendix for the details).

Now, suppose that the order of the system (after identification of the radiation), that is the dimension of A or the order of $H(s)$, is n, and that the Hankel singular values are arranged in descending order, $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \geq 0$, each of them associated with a certain state component of the system. If, from the discussion above, the floating system is expected to be dominant 2^{nd} order, then the value of the first two singular values should be much bigger than the value of the other singular values, or equivalently $\sigma_2 >> \sigma_3$. In this case, any state component other than the two associated with the σ_1 and σ_2 may be neglected with little effect on the system's response. As an example, the model reduction procedure is applied to a range of four floating cylinders, whose geometries are specified in Table 1. They all have the same density and radius, but different height. The ratio σ_2/σ_3, which determines how well the system is described with a 2nd order dynamics, is also shown in Table 1. It is clear how this ratio increases with the increasing height of the cylinder, which means decreasing bandwidth, although in general it stays quite large (minimum is 98). In Fig. 5, the frequency response of the 2^{nd} order reduction of cylinder 1, as denoted in Table 1, is shown against the real frequency responses calculated from the hydrodynamic software.

Reactive Control for a 2^{nd} Order Oscillator

Once a 2^{nd} order model has been identified, the floating system of Eq.

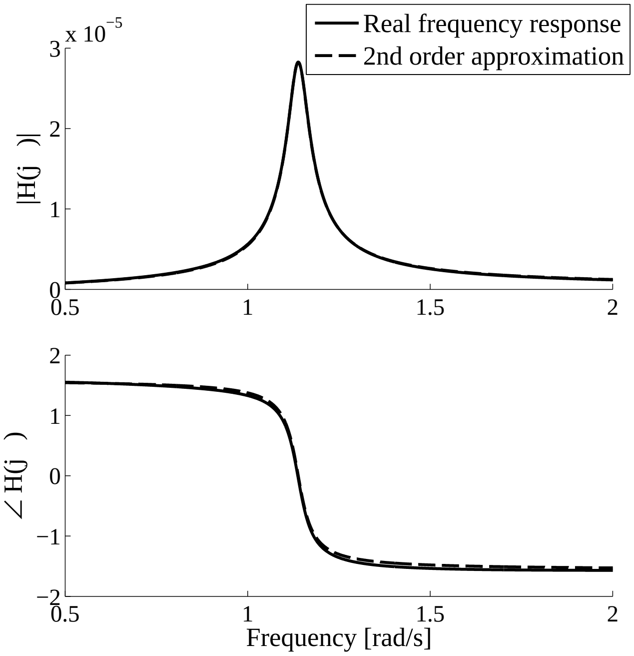

FIGURE 5 – Comparison of real frequency response $H(\jmath\omega)$ against 2^{nd} order approximation frequency response, for cylinder 1 of Table 1.

4 is approximated with the following frequency domain model:

$$V(\omega) = \frac{b_1(\jmath\omega)}{(\jmath\omega)^2 + a_1(\jmath\omega) + a_2}\left[F_{ex}(\omega) + F_u(\omega)\right] \tag{17}$$

which, in the time domain, using simple properties of the Fourier transform, corresponds to:

$$\hat{M}\dot{v}(t) + \hat{K}v(t) + \hat{K}_s\int_0^{\infty} v(\tau)d\tau = f_{ex}(t) + f_u(t), \tag{18}$$

where:

$$\hat{M} = \frac{1}{b_1} \qquad \hat{K} = \frac{a_1}{b_1} \qquad \hat{K}_s = \frac{a_2}{b_1}. \tag{19}$$

The parameters a_1, a_2, b_2 are directly related to the state space matrices after the $2nd$-order model reduction.

Note that the heaving cylinder in water approximately behaves as a mass-spring-damper mechanical oscillator with of mass $\hat{M}$, damper $\hat{K}$ and spring coefficient $\hat{K}_s$. The fluid memory effect, modelled by the frequency dependance of the radiation, has disappeared and the radiation impedance has been reduced to a constant. If reactive control is solved for such a 2^{nd} order system, it is straightforward to show how the optimal conditions in Eq. 11 and Eq. 12 reduce to:

$$V_{opt}(\omega) = \frac{1}{2\hat{K}_f}F_{ex}(\omega) \tag{20}$$

$$F_{u,opt}(\omega) = -\left[\frac{1}{2} + \jmath\omega\frac{\hat{M}}{2\hat{K}} - \frac{K_s}{\jmath\omega 2\hat{K}}\right]F_{ex}(\omega). \tag{21}$$

For a 2^{nd} order oscillator, maximum wave energy extraction is realised if the velocity, Eq. 20, is simply proportional to the excitation force and if the optimal force, Eq. 21, is related to the excitation force through a

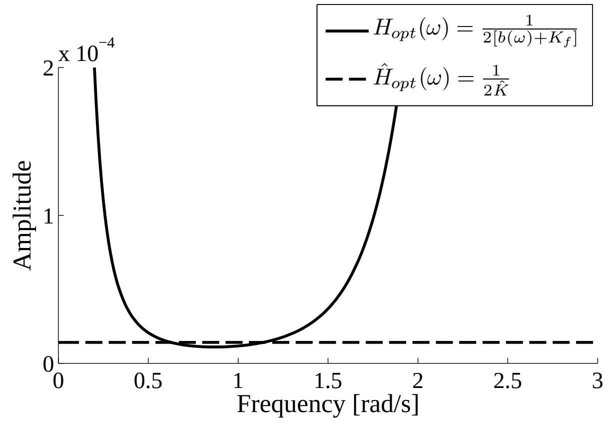

FIGURE 6 – Velocity-generation law, $V(\omega)/F_{ex}(\omega)$, for reactive control calculated for full system and 2^{nd} order approximation model (constant over frequency). The system is cylinder 2 of Table 1.

classical proportional-derivative-integral (PID) relationship. As mentioned before, the focus in this preliminary study is put on the condition in Eq. 20, but the PID relationship between the excitation force and the optimal force would deserve more attention in future studies and could represent a valid alternative.

For the current study, the implementation of reactive control as a velocity-following loop (Fig. 2) is perfectly realisable, because the reference-generation logic needs simply to implement the following constant transfer function:

$$\hat{H}_{opt}(\omega) = \frac{1}{2\hat{K}}, \tag{22}$$

as opposed to the non-causal $H_{opt}(\omega)$ of Eq. 13. Such a causal realisation of reactive control is illustrated in the block diagram of Fig. 7.

More interestingly, the constant $1/2\hat{K}$, obtained from the model reduction and application of reactive control optimal conditions, is an approximation of the flat part of $H_{opt}(\omega)$ around the resonance frequency of the device, as shown in Fig. 6. The approximation is close to the real transfer function in the frequency range $[0.6, 1.2]$ rad/s while is quite far from ideal outside such a range. Ocean waves, however are most likely to occur within the specified range, for two main reasons:

- The device is most likely designed such that its resonance curve matches the most common wave spectra at the deployment location.
- Waves at higher frequency than the flat part of $H_{opt}(\omega)$ are filtered out by the excitation transfer function of the system (Falnes, 2002), so that the device does not experience significant forces from waves at high frequencies.

RESULTS

In order to test the effectiveness of the presented causal approximation of reactive control, which adopts the the constant $\hat{H}_{opt}(\omega)$, as shown in Fig. 7, this is compared against the full implementation of reactive control, using the non-causal $H_{opt}(\omega)$, Fig 2, in terms of wave energy capture efficiency. Such a comparison is carried out for a few different wave energy converters and over a variety of wave conditions.

The WECs consist of four bottom-referenced heaving cylinders, as in Fig. 1, with the dimensions specified in Table 1. The sea states consist

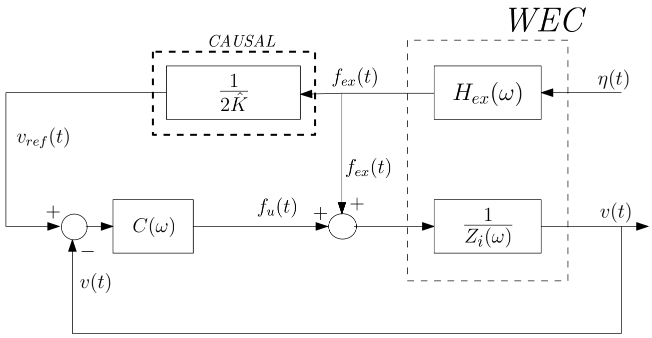

FIGURE 7 – Real-time implementation of reactive control as a velocity-following. If system model is 2^{nd}-order, reference generation is a purely proportional block, which is *causal* and realisable.

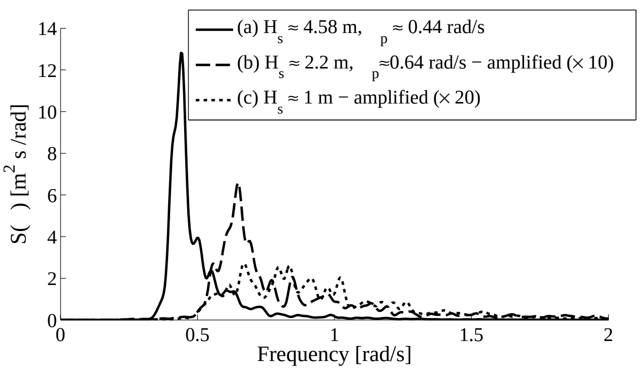

FIGURE 8 – Variance spectra of some real data collected off the coast of Pico Island, in the Azores Archipelago. Significant wave height, H_s, and peak period, T_p, is also indicated. Some spectra have been amplified for clarity.

of three time series (30 minutes sets sampled at $1.28\ Hz$) of real measurements collected from a data buoy off the north-west coast of Pico island, in the Azores (Esteves et al., 2009; Azevedo and Rodrigues, 2008; Barrera et al., 2008): *(a)* a high-energy and low-frequency swell; *(b)* a quite low-energy sea state widely spread at high frequencies and *(c)* a situation in between. The spectra of these sea states are shown in Fig. 8. For both the control schemes, the lower-level loop is assumed to be ideal so that the velocity generated by the reference is perfectly followed by the WEC. This allows a fair comparison of the two control strategies, that differ only in the way they generate the reference, one requiring prediction and another implementing a simple gain. Additionally, in order to maintain the focus of the paper, the excitation force is supposed to be measurable (in reality an observer based on system's motion measurements shall be deployed (Bacelli et al., 2009)) and full knowledge of its future values (for the non-causal control) is assumed.

The performance of the control strategies is measured in terms of the well known Relative Capture Width (RCW) (Cruz, 2008):

$$RCW = \frac{\overline{P_u}}{2R \cdot P_{wave}}, \tag{23}$$

where $\overline{P_u}$ is the average power absorbed by the PTO, as from Eq. 7, R is the radius of the cylinder and P_{wave} is the wave power per meter of wavefront. For the ideal non-causal reactive control, the reference velocity (and therefore the performance) is calculated considering a range

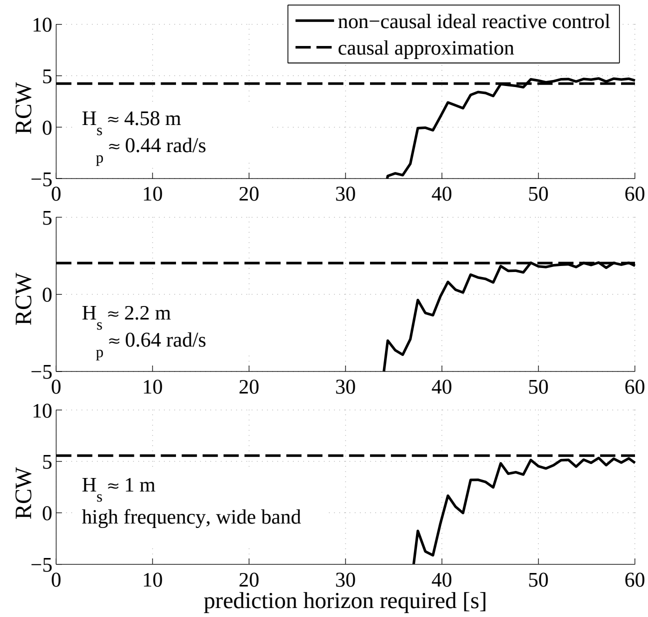

FIGURE 9 – Performance of the causal approximation to reactive control against non-causal realisation.

of finite future horizon (ideally infinite) for the excitation force, from 0 to 60 seconds. This provides useful information about the forecasting horizon required to get a certain performance level (Fusco et al., 2010). As an example, consider Fig. 9, showing the performance of a sample system, cylinder 2 of Table 1, with the two different control strategies and in three different wave conditions. It is clear how the performance of the ideal reactive control improves if more (into the future) information about the wave excitation force is taken into account. In order to get a positive power output ($RCW > 0$), for all sea states, the excitation force needs to be predicted more than 35 seconds into the future and the maximum is approached only if at least 50 seconds of predictions are available. Note that the required forecasting horizon is strictly connected to the bandwidth of the system's radiation and does not depend on the specific wave system (Fusco et al., 2010). On the other hand, the performance of the causal approximation, obviously, does not improve with the future horizon considered, as such information is not required in the calculation of the optimal velocity calculation. The achievable RCW, however, is very close to the maximum obtained with the non-causal control (more than 90%).

Table 2 summarises the results also for the other cylinders. For each sea state, it is shown the RCW obtained through the causal approximation along with the forecasting horizon LT required by the ideal reactive control in order to offer the same performance. The effectiveness of the simple control strategy presented here is confirmed, where the ideal implementation of reactive control is shown to be able to do better only if a prediction of the excitation force is available for more than 40, and in some cases 50 seconds. In some situations, the RCW achieved by the non-causal control is slightly bigger than the maximum achieved with the ideal implementation of reactive control, as for example in case three in Fig. 9. This situation, depicted with an ∞ value for LT in Table 2, can be due to two reasons. One reason is that the maximum future horizon of 60 seconds, considered in the simulations, is not enough. The second reason is that the calculations are affected by numerical errors, as reactive control involves the numerical ill-posed inversion of a function

cylinder	$H_s \approx 4.53\,m$ $RCW\,/\,LT$	$H_s \approx 2.2\,m$ $RCW\,/\,LT$	$H_s \approx 1\,m$ $RCW\,/\,LT$
1	$4.75\,/\,44s$	$3.56\,/\,\infty$	$9.08\,/\,\infty$
2	$4.23\,/\,48s$	$2.02\,/\,49s$	$5.56\,/\,\infty$
3	$4.63\,/\,50s$	$1.95\,/\,49s$	$5.60\,/\,54s$
1	$4.67\,/\,57s$	$1.90\,/\,41s$	$5.73\,/\,58s$

TABLE 2 – Performance of causal approximation of reactive control: RCW is the relative capture width and LT is the forecasting horizon required by the ideal reactive control to obtain the same RCW.

with significant variations, such as the radiation resistance.

Ultimately, it has been shown how a simple proportional approximation of the optimal condition in Eq. 11 can provide a very effective method of calculating the reference velocity to impose on a wave energy conversion system. With a very little loss in terms of performance, such an approach completely eliminates the needs to predict the excitation force. This also means that the control approach is very likely to be more robust, as it does not have to cope with the inevitable forecasting errors which will impact the reference velocity. Finally, note, that the prediction horizons required by the ideal reactive control can be quite large, more than 40 seconds. Accurate predictions with simple stochastic models have shown to feasible only for up to 2 wave periods (Fusco and Ringwood, 2010), which corresponds to 20-30 seconds for the sea states considered here. This means that the complexity of the control design, as well as of the instrumentation required, and therefore the costs required for an implementation of the non-causal reactive control may not justify the marginal power gain.

CONCLUSIONS

This paper introduced a detailed discussion about reactive control, focused on the non-causal transfer function relating the wave excitation force to the optimal velocity that a WEC, in one degree of freedom, shall have in order to achieve maximum energy extraction. Such a transfer function may be utilised, in a practical implementation of reactive control, to generate an optimal reference for the velocity which would then be imposed on the system with a lower level control loop and the power take-off system. Non-causality, however, implies that future values of the excitation force need to be predicted, which adds complexity to the problem and can introduce robustness issues related to inevitable errors in the predictions.

A closer analysis of the properties and the behavior of such non-causal transfer function, utilised for the calculation of the optimal velocity, revealed that a simple constant approximation, while removing the non-causality, could perform very close to optimality in most sea conditions for which the device itself is designed. In order to determine this constant approximation a methodology based on the 2^{nd} order reduction of the WEC's model was also introduced. While the approximation could be performed in several other ways (e.g. simple numerical fitting of the optimal curve in the region of interest), the methodology gave this process a significant physical meaning. In the constant approximation of the optimal transfer function, the frequency-varying radiation resistance of the device is replaced by the constant damping coefficient of its dominant 2^{nd} order dynamics.

The results showed how the simple proportional law utilised to generate the reference velocity can perform very much close to the ideal reactive control. In particular, if one considers the simplicity (no need of a predictor) and robustness of the method, the control technique can represent a valid candidate for the control of wave energy conversion. Further study will need to assess the possibility to extend such simple techniques to multi-body systems oscillating in more than one degree of freedom. Also it would be important to determine the performance in the case of non-linearities, which may appear for large body motions. In such situations, probably, more than one constant suitable to the different states of the system may need to be identified and adapted in real time. Further work will also need to address the design of the lower-level control loop, which was supposed ideal to maintain the focus of this paper.

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APPENDIXES

State Space Balanced Realisation and Hankel Singular Values

Given a state space representation for a linear system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}, \tag{24}$$

and a real a non-singular transformation matrix P, the system defined by the matrices $\left(\overline{A} = PAP^{-1},\, \overline{B} = PB,\, \overline{C} = CP^{-1}\right)$ is equivalent to the system $(A,\, B,\, C)$, that is they have the same set of eigenvalues and the same input-output transfer function.

For any state space representation as in Eq. 24, the controllability Gramian matrix, W_c, and the observability Gramian matrix, W_o, are defined such that Chen (1999):

$$AW_c + W_cA^T = -BB^T \tag{25}$$

$$A^TW_o + W_oA = -C^TC. \tag{26}$$

Such matrices, W_c and W_o, are important because their product W_cW_o is similar (same determinant and eigenvalues, among other properties) to the product $\overline{W}_c\overline{W}_o$ corresponding to an equivalent state space representation $\left(\overline{A},\, \overline{B},\, \overline{C}\right)$. Furthermore, the product W_cW_o is similar to the Hankel matrix:

$$\Sigma = diag\left\{\sigma_1,\, \sigma_2,\, \ldots \sigma_n\right\}, \tag{27}$$

where n is the dimensionality of the state of the system and the elements $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \geq 0$ are the Hankel singular values of the system, which represent a quantification of the energy of each state component Chen (1999).

A state space realisation of a system, $(A,\, B,\, C)$, is balanced when $W_c = W_o = \Sigma$ Chen (1999).

Model Reduction Based on Hankel Singular Values

The properties of a balanced state space realisation and the Hankel singular values can be exploited in order to isolate the dominant dynamics of a system. If the state space model in Eq. 24 is decomposed as follows:

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{cases}, \tag{28}$$

and the Hankel matrix is accordingly decomposed as:

$$\Sigma = diag\left\{\Sigma_1,\, \Sigma_2\right\}, \tag{29}$$

then the subsystem $(A_{11},\, B_1,\, C_1)$ is a good approximation of the original system $(A,\, B,\, C)$ if the singular values in Σ_1 are much bigger than the singular values in Σ_2 .