FIVE VARIATIONS ON A FEEDBACK THEME

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ABSTRACT

This is a study on a set of feedback amplitude modulation oscillator equations. It is based on a very simple and inexpensive algorithm which is capable of generating a complex spectrum from a sinusoidal input. We examine the original and five variations on it, discussing the details of each synthesis method. These include the addition of extra delay terms, waveshaping of the feedback signal, further heterodyning and increasing the loop delay. In complement, we provide a software implementation of these algorithms as a practical example of their application and as demonstration of their potential for synthesis instrument design.

1. INTRODUCTION

Audio feedback is one of the most important devices in Musical Signal Processing, as it enables the implementation of important components such as infinite impulse response (IIR) filters, waveguides, reverberators, etc. In sound synthesis, it has been used in the useful technique of feedback frequency modulation (FM) synthesis [1][2] and correlate [3] methods. In this study, we will like to focus on a less well-known approach of feedback amplitude modulation (AM). Feedback AM was first discussed as a sound generation method in [5].

The synthesis technique of AM consists in adding an audio rate periodic source to the amplitude of an oscillator [4]. With sinusoidal modulators and carriers, we find components at the sum and difference of the carrier and modulator frequencies, plus the carrier frequency (by heterodyning). A related technique, ring modulation (RM) differs from AM in that the modulator signal is applied directly to the amplitude, which is equivalent to multiplying the two signals together [4]. Feedback RM is not practical (even though it is mentioned in [3], but not directly used), as the output signal will disappear as soon as the input is zero.

In [5], we find a short technical description of a feedback AM oscillator (fig.1), where the output of a sinusoidal oscillator is applied as a modulator to its own amplitude. The algorithm employs a fixed feedback interval D (in samples) that is dependent on the signal block size. This structure can be defined by the following equation (with the radian frequency $\omega = 2\pi freq/f_s$ and amp=1):

$$y(n) = \cos(\omega n)[1 + y(n-D)] \qquad (1)$$

A small adjustment to the above equation, namely, setting $D = 1$, yields the starting point for our exploration of possible musically interesting applications for this particularly simple and inexpensive algorithm. In this study, we will look at the characteristics of this and five other variations of the technique.

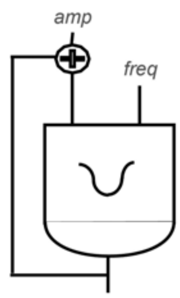

Figure 1. Feedback AM oscillator

2. THEME

The feedback AM equation can be demonstrated to be equivalent to a coefficient-modulated one-pole IIR filter that is being fed a sinusoid. If we recast eq.(1) as

$$y(n) = x(n) + y(n-1)M(n) \qquad (2)$$

equating

$$x(n) = M(n) = \cos(\omega n) \qquad (3)$$

this fact can be clearly demonstrated. The output spectrum of eq.1 (with $D = 1$) is shown on fig.2.

Thus, it can be seen that the effect of modulating the filter coefficient is that of adding a number of extra harmonic components to the sinusoidal input spectrum. We postulate here that this is the result of distorting the linear phase increment, which is caused by the time-varying phase response of the filter. It is actually a similar behaviour to that described in [6], [7] and [8] for the case of allpass filters. However, here, the filter is not allpass, so in addition to the action of the filter's varying phase response, we have some contribution by the magnitude response.

The time-varying phase and magnitude responses for the filter in eq.2 can be derived from the transfer function of its linear time-invariant version (ie. $M(n)$ = C, constant). With time-varying $M(n)$, they are now functions of two variables, time in samples n, and radian frequency ω:

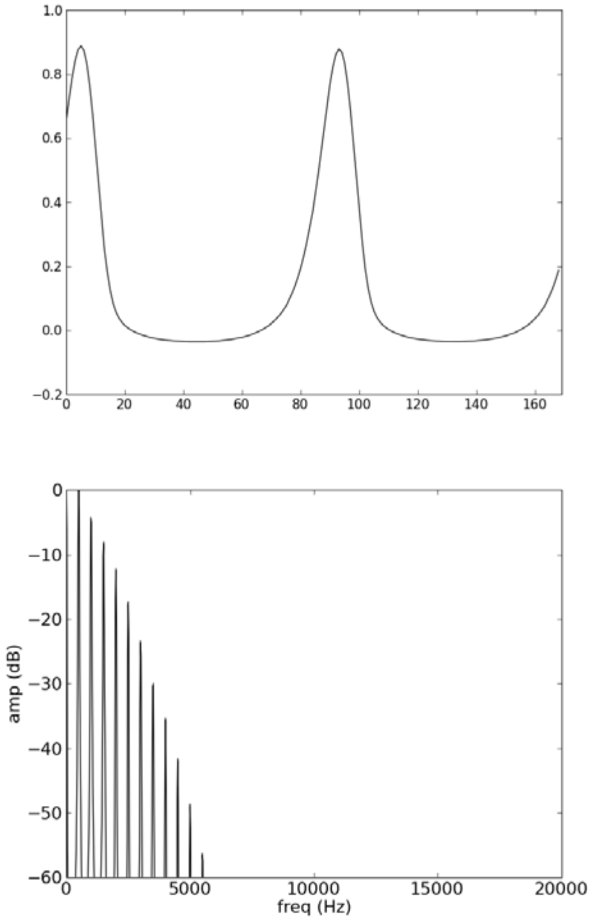

Figure 2. The waveform and spectrum of Feedback AM, with freq = 500 Hz.

$$\phi(\omega,n)=\tan^{-1}\left(\frac{-M(n)\sin(\omega)}{1-M(n)\cos(\omega)}\right) \qquad (4)$$

$$A(\omega,n)=\frac{1}{\sqrt{1-2M(n)\cos(\omega)+M(n)^2}} \qquad (5)$$

It is difficult to isolate the effects of the phase and magnitude in this system, but experiments seem to indicate that the phase distortion is responsible for the generation of new harmonics and the magnitude response helps to give the feedback AM oscillator spectrum a lowpass shape.

The spectrum of the impulse response of the filter in eq.2, with *M(n)* sinusoidal, has a lowpass countour, seen in fig.3 (top). However, when we feed white noise to the same filter, its spectrum displays a gentle dip towards $f_s/4$ (fig.3, bottom). Eq.5, averaged out over time, will confirm this band-reject shape. Fig.3 (bottom) also shows the strong peaks at frequencies generated by the effect of phase distortion. In general, these features of the filter's spectrum are dependent on the modulation frequency.

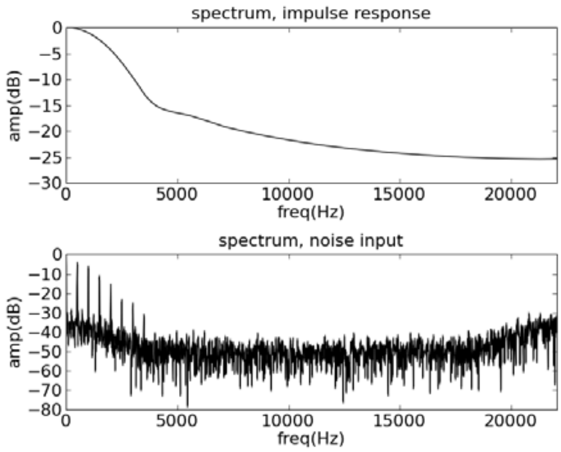

Figure 3. Spectra of coefficient-modulated filter (eq.2): impulse response (top), white noise (bottom), both modulated with a 500 Hz sinusoid, f_s = 44.1 kHz.

By introducing a new parameter β, we can vary the amount of feedback (fig.4):

$$y(n)=\cos(\omega n)[1+\beta y(n-1)] \qquad (6)$$

The net effect of varying this parameter is to produce dynamic spectra. However, we have to be careful not to render the system unstable. The analysis of the equation as a filter structure would suggest that β needs to be below one for the pole to remain inside the unit circle. However, in practice, due to the combination of modulation and sinusoidal input, the system appears to allow values up to around 1.7, but this is dependent on the fundamental frequency.

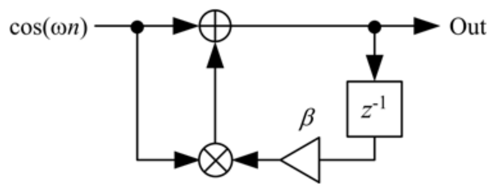

Figure 4. Flowchart of eq.6 ("theme")

Another way of looking at the problem is to expand the feedback loop in eq.6 as a sum of signals:

$$y(n)=\sum_{k=0}^{\infty}\beta^k\prod_{m=0}^{k}\cos(\omega[n-m]) \qquad (7)$$

This description is perhaps less revealing than the previous one, given the complexity involved in evaluating the product terms inside the sum. Nevertheless, it tells us two things: that the output is composed of harmonics of ω, and that smaller values of β will produce less components.

Finally, the system will require some means of normalisation, which can be made dependent on β. An alternative is to apply an RMS balancing process [4] using the input sinusoid as a comparator signal.

3. VARIATIONS

We will now propose a set of variations on the above feedback AM oscillator. Each one will provide a means of obtaining different spectra that might be useful in specific applications.

3.1. Variation I

The first variation is played by adding an extra delay of the input, as in

$$y(n) = \cos(\omega[n-1]) - \cos(\omega n)[1 - \beta y(n-1)] \qquad (8)$$

resulting in the waveform and spectrum of fig.5.

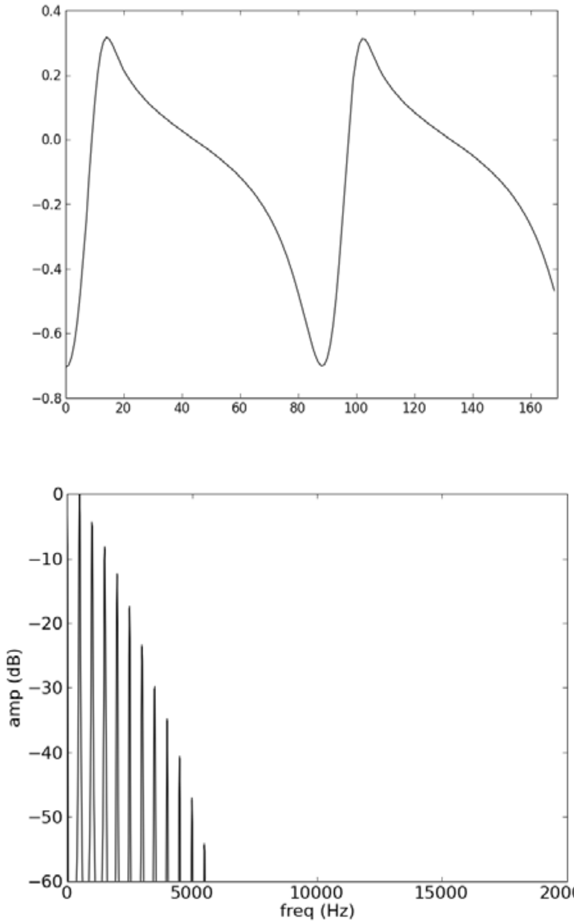

Figure 5. Waveform and spectrum of variation I, with freq = 500 Hz and $\beta = 1$

Interestingly, while the time-domain exhibits some change, there is very little spectral variation. This is due to the fact that the extra term just adds a zero at DC, adding a gentle high-pass contour to it. The results are seen on the higher harmonics, if only very lightly on fig.4.

3.2. Variation II

The second variation is similar to the coefficient modulated allpass filter system described in [6] and [7]. The only difference in eq. (9) is the presence of scaling coefficients α and β.

$$y(n) = \alpha\cos(\omega[n-1]) - \cos(\omega n)[\cos(\omega n) - \beta y(n-1)] \qquad (9)$$

The output of this system is shown in Figure 6. The output spectrum is similar to that of Variation I in that it is also lowpass, however, the magnitude of the fundamental to the other harmonics is relatively higher, and the spectral roll-off is faster.

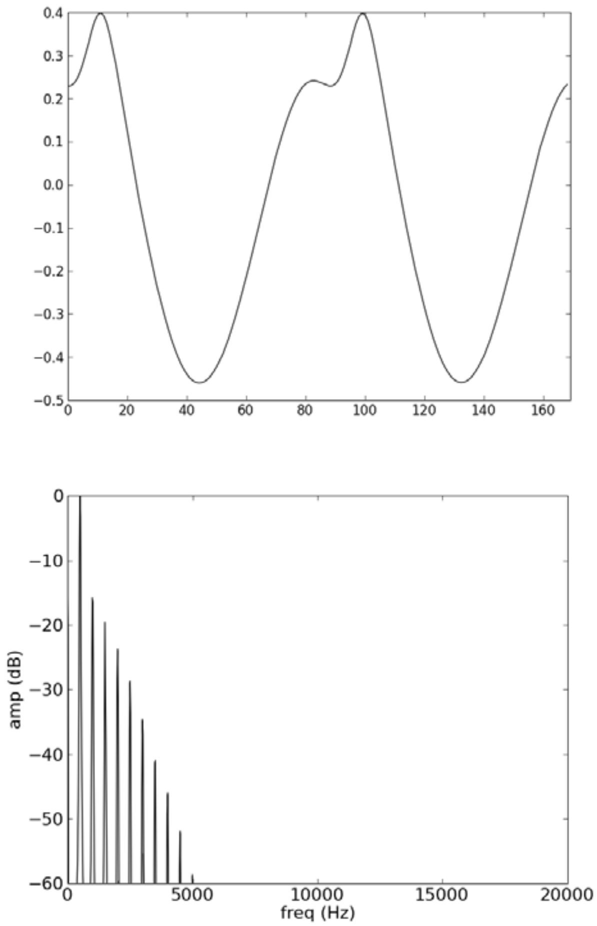

Figure 6. Waveform and spectrum of variation II, with freq = 500 Hz, $\alpha = 1$ and $\beta = 1$

3.3. Variation III

This variation utilises a secondary oscillator to act as a carrier for yet another heterodyning operation. The effect, similar to the techniques discussed in [9], is to create a resonance region around the carrier (fig.7). The width of this region can then be controlled by changing β.

$$y(n) = \cos(\theta n)[\cos(\omega n)(1+\beta y(n-1))],\ \theta = N\omega \qquad (10)$$

Note that this variation requires the evaluation of the cosine function twice per sample, which can be implemented with a second oscillator.

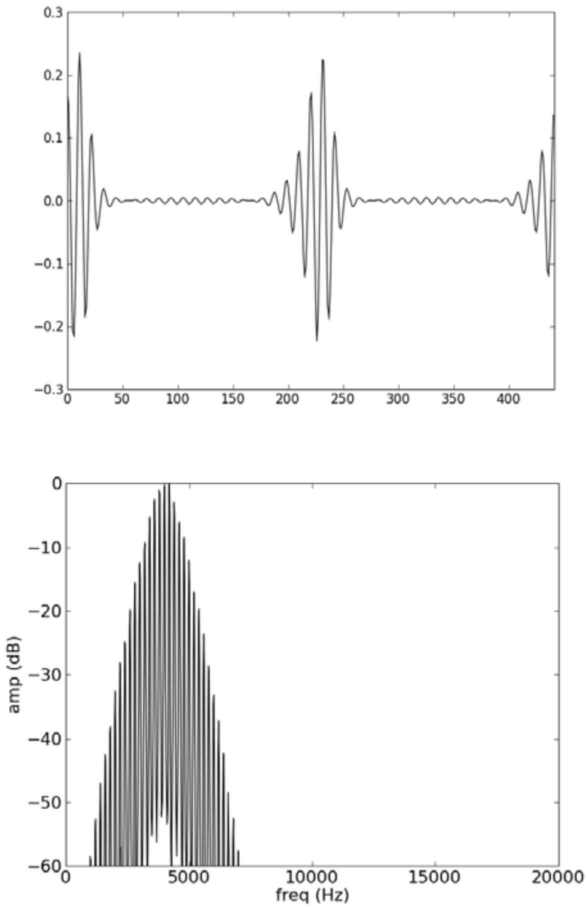

Figure 7. Waveform and spectrum of variation III, with carrier freq = 4000 Hz, feedback oscillator freq = 200 Hz and $\beta = 1$

3.4. Variation IV

This system in eq.11 is similar to the one proposed in [3], where the feedback signal is non-linearly amplified by the cosine mapping. The net result is that of a partial feedback FM spectrum with missing even harmonics, given that the equation implements only one term of a heterodyne-based FM equation (for details see [8] and [10]).

$$y(n) = \cos(\omega n)[1+\cos(\beta y(n-1)] \qquad (11)$$

Figure 8 shows the spectrum of the output with the even harmonics missing. The waveform shape is bipolar and smooth, as the high frequency energy in this example is below 5000 Hz.

This variation is significant in that it brings the concept of waveshaping into play. In fact, we could generalise it by substituting *cos(.)* by an arbitrary function *f(.)*. Experiments have indicated that using *abs(.)* as the waveshaper produces triangle-like waveforms. In addition, this variation is reminiscent of some recent work on non-linear oscillators utilising sine [11] and circle [12] maps.

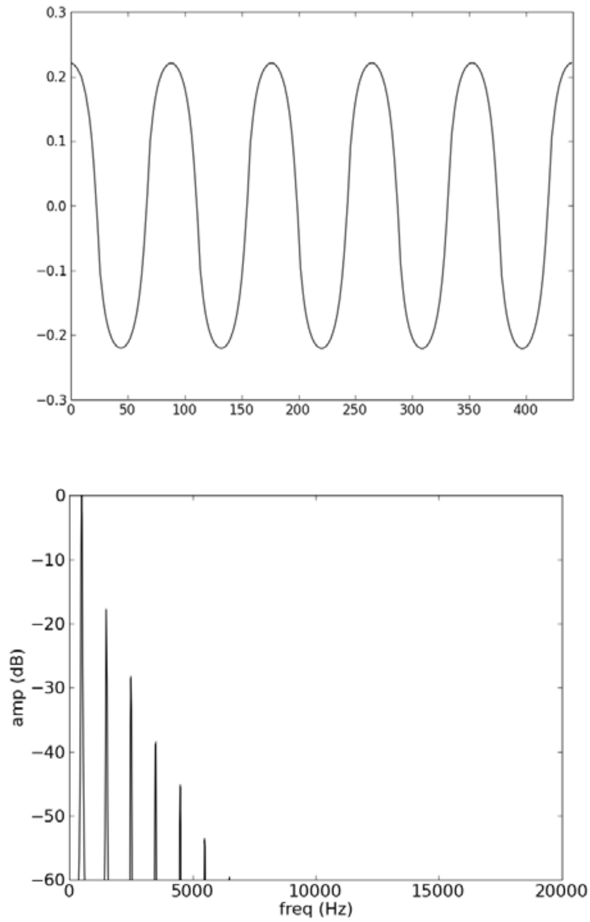

Figure 8. Waveform and spectrum of variation IV, with freq = 500 Hz and $\beta = 1$

3.5. Variation V

This variation goes back to the original formulation in [5] and introduces a means of changing the delay *D* in samples.

$$y(n) = \cos(\omega n)[1+\beta y(n-D)] \qquad (12)$$

If we look at this expression as a filter, then we might consider this as a comb filter tuned to f_s/D samples. The particular case where $D = f_s/freq$ is worth noting. Here we will have a very sharp pulse wave at the output (fig.9). This case is where the phases of the cosine products on eq.7 should be aligned, so theoretically, for $\beta = 1$, we will have

$$y(n)=\sum_{k=1}^{\infty}\cos^{k}(\omega n)=$$
$$\sum_{k=1}^{\infty}\left[\frac{1}{2^{2k}}\binom{2k}{k}+\frac{2}{2^{2k}}\sum_{m=0}^{k-1}\binom{2k}{m}\cos([2k-2m]\omega n)\right] \quad (13)$$
$$+\sum_{k=1}^{\infty}\left[\frac{2}{2^{2k-1}}\sum_{m=0}^{k-1}\binom{2k-1}{m}\cos([2k-2m-1]\omega n)\right]$$

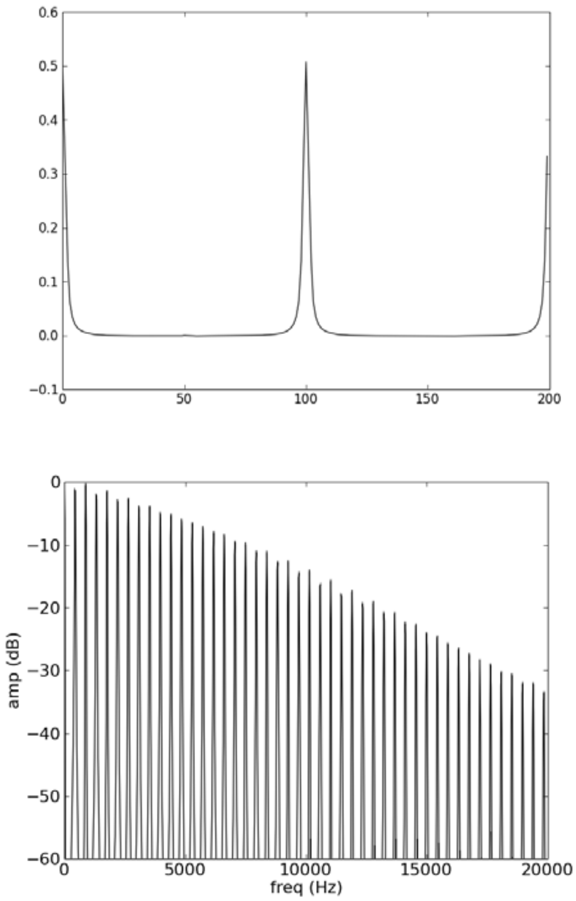

Figure 9 Waveform and spectrum of variation V, with freq = 441 Hz, f_s = 44100 Hz, D = 100 and β = 1

Despite the complicated nature of eq.13, this hints at a slowly-decaying spectrum containing all harmonics of the fundamental frequency. Partially evaluating eq.13, with $k \leq 4$, we have:

$$\frac{11}{8}+\frac{7}{4}\cos(\omega n)+\cos(2\omega n)+\frac{1}{4}\cos(3\omega n)+\frac{1}{8}\cos(4\omega n) \quad (14)$$

Since this expression generates a non-bandlimited spectrum, some aliasing will inevitably occur. In the case of integral delays, this is not noticeable, as the aliased components will fall over the harmonics in the baseband.

In the case of non-integral delays, if interpolation is used, it will have some filtering effect in the predicted spectrum of eq.13. Varying the delays will have the effect of changing the timbre of the sound, as harmonic weights change. The flowchart for this variation is shown in fig.10.

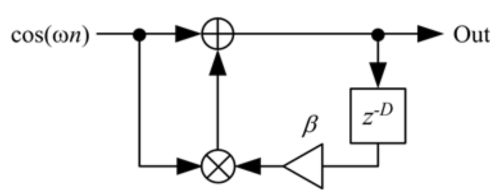

Figure 10. Variation V flowchart

3.6. Coda

In addition to the above, it is also possible to realise the equations above successfully as audio effects with arbitrary complex inputs. In this case, the best results were obtained by inserting a simple LP filter (1st order IIR) in the feedback loop. This will attenuate high frequencies and allow for an increase in β above the usual limit (ca. 1.7) increasing distortion but avoiding instabilities in the system. With this simple measure it is possible realise the effects above. The implementation of variation III, however, will require pitch tracking as there is an auxiliary oscillator involved.

4. THE PIECE

This section describes an implementation of the feedback AM oscillator and its variations I-V, and demonstrates three example applications utilizing the synthesis method.

4.1. Verse

There are four types of signals in the basic feedback AM equation (eq. 6) and its variations (eqs. 8-12). The source signal $\cos(\omega n)$ and the frequency shifter signal $\cos(\theta n)$ can be realized as sinusoidal oscillators, the feedback signal $y(n-D)$ as a delay line, and the delayed source signal $\cos(\omega[n-1])$ as a state variable. The parameters of these building blocks are shown in Table 1.

Table 1: *Synthesis parameters.*

parameter	description
f_x	source frequency
a_x	source amplitude
β	feedback amount
D	feedback delay amount
N	frequency shift amount
v	variation number

The parametrized building blocks were implemented in software as C language objects, while equations 6 and 8-12 were implemented as block-based C functions operating on these objects. The DSP processing logic invokes one of these functions at runtime, based on the value of the variation number parameter v. The source code was compiled then into an external library (fbam~), implementing the feedback AM oscillator in the Pure Data (PD) environment [13]. Finally, two envelope generators (EGs) were inserted in front of the oscillator in order to control the amplitude a_x and the feedback amount β of the oscillator (see Fig. 11). For convenience, the resulting patch was encapsulated into a PD abstraction so that it could be handled as a single unit generator. This composite unit is the equivalent of an FM operator used in the commercial FM implementations, and therefore, we call the unit the *feedback AM operator*.

A single feedback AM operator instance can be used as a simple and inexpensive virtual analog (VA) synthesizer implementation. The basic feedback AM equation produces a sawtooth-like spectrum, while variation IV can be used in pseudo square and triangle wave generation. Thus, the variation number v can be used as a waveform selector. The amount of feedback β can be used to simulate the cutoff frequency parameter of a subtractive synthesis lowpass filter. This simple VA synthesizer model can be extended by patching additional control rate signal generators, such as low frequency oscillators (LFOs), into the f_x, a_x and β inputs of the operator.

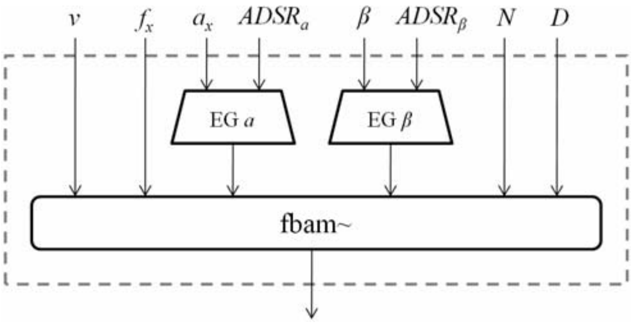

Figure 11. *Feedback AM operator.*

4.2. Bridge

Although LFOs and EGs provide simple means for sound animation in time domain, modulation of the delay time parameter D results in more intense dynamic spectra. This is due to the fact that the equivalent filter structure changes with the amount of delay in the feedback path. Another way of looking at this is to think that the delayed signals will have different phases and this will contribute to the appearance and disappearance of harmonics. We call this effect *phase difference modulation*.

4.3. Chorus

The generated timbral space can be further expanded by interconnecting the feedback AM operators into parallel or serial configurations. Continuing with the FM parlance, these interconnection topologies are the equivalents of FM algorithms. However, instead of modulating the frequency (or phase) of the attached operator, cascaded feedback AM operators modulate the amplitude input a_x of the attached unit.

Fig. 12 shows a spectrogram of an electric piano timbre that was synthesized using four feedback AM operators. Two operators were cascaded in series as to produce the higher frequency attack portion of the sound (variation III, f_x = 440 Hz, N = 8 and β = 0.24 in the modulating operator, N = 21 and β = 0.5 in the modulated operator). The output of the cascade was mixed with a slightly detuned pair of operators arranged in parallel configuration, producing the lower frequency sustaining portion of the sound (variation II, β = 0.42, f_{x1} = 440 Hz, f_{x2} = 441 Hz). Amplitude inputs were modulated with linearly decaying envelopes, but the feedback amount β was kept constant.

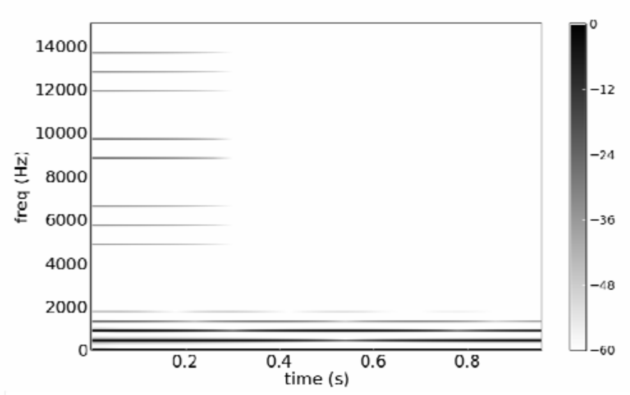

Figure 12. Spectrogram of an electric piano patch.

5. CONCLUSION

We have presented here a brief study of a series of simple and low-cost synthesis methods based on feedback structures, which prove to be useful for a variety of applications. In this work, we have examined five variations of a feedback AM equation, each one providing a different signal output. A reference software implementation in PD was presented. The encapsulation of these methods in an ‘operator’ abstraction has been shown to be a useful way of providing them for general-purpose synthesis applications.

6. ACKNOWLEDGEMENTS

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