# Development of the PD/PI extended state observer to detect sensor and actuator faults simultaneously

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Abstract— This paper discusses about an observer based fault detection scheme to detect sensor and actuator faults simultaneously in LTI system. The proposed strategy is to add derivative action on the extended state observer (ESO) in addition to proportional-integral action, so that the structure of the proposed observer is PD/PI or called PD/PI-ESO. The derivative action is performed both in state estimation and fault estimation. This is to achieve fast state estimation as well as fast fault estimation. Furthermore, the effects of disturbance are attenuated by using the H<sub> $\infty$ </sub> performance approach. The observer gains are then determined based on Linear Matrix Inequalities (LMI) technique. Simulation results of a DC motor speed control system are presented to illustrate the effectiveness of the proposed method.

Keywords—fault estimation; sensor fault; actuator fault; extended state observer; LMI.

# I. Introduction

Recently, the control system development has encountered the problems of safety and reliability. One of the research topics contributing in these problems is fault detection and diagnosis (FDD) scheme for any component of a control system, such as sensor and actuator. The study of FDD has been much conducted. Some of the results can be found in several excellent books [1-2] and a survey paper [3]. The basic idea of this technique is to compare the observed system behavior with the desired system behavior. This difference is often known as a residual. But nowadays, it has emerged the new concept to develop FDD scheme without using residual analysis, that is a fault estimation approach. If the fault can be estimated accurately then all fault information includes type, amount, location, and time could be obtained. Thus the fault estimation approach gives more direct way to get any fault information (for detection and diagnosis) than the residual based approach. The estimator can be performed by artificial intelligence schemes as in [4] and [5], or by observer schemes as in this paper. The used observer is commonly called the extended state observer (ESO).

At ESO techniques, the estimation of state and fault are conducted in one design as long as the robustness and boundedness conditions are met. The observer structure initially is proportional/integral (P/I). Here, the used general assumption is that fault or unknown input changes slowly. It become lack of this technique because the fault may have unpredictable behavior and in most of the time is not constant, which means the derivative of it is not zero and may further cause high gains. However many researchers have tried to improve this approach related to more general fault and fast estimation, by using the classical observer such as in [6] for sensor and/or actuator faults; [7] for sensor faults; [8] for actuator faults; [9] and [10] for simultaneous sensor and actuator faults, or using the descriptor observer such as in [11] for actuator faults; [12] and [13] for sensor faults, [14] for simultaneous sensor and actuator faults. The classical observer method used a proportional gain for the state estimation. While the descriptor observer method had used derivative action in estimating states in addition to proportional action, but it was still assumed that the dynamic information of fault had been known previously (*a priori*).

This paper proposes the development of descriptor observer to simultaneously detect any general faults (no need a priori knowledge) for both sensor and actuator faults eventually. Most fault detection researches deal with sensor faults or actuator faults alone. In this case, sensor and actuator faults are subjected as one vector in which the sensor faults appear as unknown inputs as adopted from [15]. This technique was also used by [10] to be able to estimate the sensor and actuator faults. However, the effects of disturbances have not been reviewed in that study. The disturbances could come from the model uncertainty which greatly affects the performance of observer based fault detection system. Thus the objective of this paper is to develop a robust ESO in order to estimate sensor and actuator faults simultaneously and correctly even the disturbances exist.

The proposed observer structure is proportionalderivative/proportional-integral (PD/PI), hereinafter referred to as PD/PI-ESO. By using this structure, it is provided that the accurately estimation process can be made without impulsive behavior and faster, with the result that false alarm and time detection problems of FDD scheme can be overcome. The robustness of the proposed observer is obtained by using the  $H_{\infty}$ performance approach through linear matrix inequalities (LMI) formulation.

This paper is organized as follows. In section II, the LTI system with sensor and actuator faults and the extended state observer problems dealing with fast estimation are presented. In

section III, the development of the extended state observer using derivative action based on descriptor system is proposed. A numerical example of a DC motor speed control system and its simulation results are given in section IV. Finally, concluding remarks are given in section V.

### **II.** Problem Statement

### A. System description

Consider the following linear time invariant system (LTI) with disturbance, noise, sensor and actuator faults

$$\dot{x}(t) = Ax(t) + Bu(t) + D_d d(t) + F_a f_a(t)$$
(1)

$$y(t) = Cx(t) + D_{\omega}\omega(t) + F_s f_s(t)$$
(2)

where  $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, y(t) \in \mathbb{R}^p, d(t) \in \mathbb{R}^n, \omega(t) \in \mathbb{R}^q, f_a(t) \in \mathbb{R}^r, f_s(t) \in \mathbb{R}^q$  are the states, the input, the measurement output, the disturbance, the measurement noise, the actuator faults, and the sensor faults respectively.  $A, B, C, D_d, D_\omega, F_a, F_s$  are constant real matrices of appropriate dimension. It is supposed that the pairs (A, C) are observable.

The effect of actuator and sensor faults can be represented as an additional unknown input vector acting on the dynamics of the system or on the measurements [16]. An actuator fault corresponds to the variation of the global control input applied to the system, whereas a sensor fault corresponds to the variation of the global measurement output of the system. It is assumed that the derivative of fault with respect to time is norm bounded. In the rest of this paper, the dependence on time of the used variables will be suppressed when no confusion might arise.

### B. Classical PI Observer

Introducing the new state  $x_z \in \mathbb{R}^p$  as follows [15]:

$$\dot{x}_z = A_z(y-z) = -A_z x_z + A_z C x + A_z D_\omega \omega + A_z F_s f_s \quad (3)$$

where  $A_z$  is a stable matrix with appropriate dimension.

Defining the augmented state X as  $X = [x \ x_z]^T$ , then it is obtained the augmented state space of (1) and (3):

$$\dot{X} = A_{a_z}X + B_a u + D_{a\omega}\omega + F_z f + D_d d \tag{4}$$

$$Y = C_a X \tag{5}$$

with

$$A_{az} = \begin{bmatrix} A & 0 \\ A_z C & -A_z \end{bmatrix}; B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}; D_{a\omega} = \begin{bmatrix} 0 \\ A_z D_{\omega} \end{bmatrix};$$
$$F_z = \begin{bmatrix} F_a & 0 \\ 0 & -A_z F_s \end{bmatrix}; C_a = \begin{bmatrix} 0 & I_p \end{bmatrix}; f = \begin{bmatrix} f_a \\ f_s \end{bmatrix}$$

Furthermore, the PI observer can be obtained from (4)-(5) in the following form:

$$\begin{cases} \hat{X} = A_a \hat{X} + B_a u + F \hat{f} + L_X (Y - \hat{Y}) \\ \hat{f} = L_f (Y - \hat{Y}) \\ \hat{Y} = C_a \hat{X} \end{cases}$$
(6)

where  $\hat{X}$ ,  $\hat{f}$ ,  $\hat{Y}$  are the estimated augmented state, the estimated fault, and the estimated output respectively. The second equation in (6) describes the integral loop added to the proportional one, in the first equation. This observer type is therefore termed PI observer.  $L_X$  is the proportional observer gain and  $L_f$  is the integral observer gain which are computed using the LMI formulation as in [10]. These observer gains are used to ensure the stability of the estimated error dynamics.

From (6), it is implied that the fault estimation is only pure integral term so this observer may fail to deal with fast time varying fault [6]. In addition, the fast state estimation can also accelerate the fault estimation and derivative action could realize no impulsive behavior [17][18]. Furthermore  $A_z$  in (3) is chosen on an empirical manner and the disturbances have not a been considered. Therefore this paper proposes derivative action in order to improve performances and to provide more systematic way of the observer based fault estimation.

# III. PD/PI Extended State Observer

# A. Observer Algorithm

The disturbance could represent the model uncertainty. Because the observer is developed based on a model, the modeling error has an excessive affect on the observer performance. Therefore the proposed observer must be robust to the disturbance. On the other hand, the existence of measurement noise is not able to be avoided and need to be handled without amplifying it as the conventional observers do [19]. For this purpose, the noise is considered as the disturbance also. Then the augmented state space is redefined as follow:

$$\dot{X} = A_a X + B_a u + Ff + D_a d_a \tag{7}$$

with

$$A_{a} = \begin{bmatrix} A & 0 \\ C & -I_{p} \end{bmatrix}; D_{a} = \begin{bmatrix} D_{d} & 0 \\ 0 & D_{\omega} \end{bmatrix}; F = \begin{bmatrix} F_{a} & 0 \\ 0 & -F_{s} \end{bmatrix}; d_{a} = \begin{bmatrix} d \\ \omega \end{bmatrix}$$

The matrix *I* is the identity matrix with appropriate dimension shown in its subscribe. Note that there is no more  $A_z$  in (7).

The structure of the proposed observer is as follows:

$$\begin{cases} \hat{X} = A_a \hat{X} + B_a u + F \hat{f} + L_X (Y - \hat{Y}) + L_{Xd} (\dot{Y} - \dot{Y}) \\ \hat{f} = L_f (Y - \hat{Y}) + L_{fd} (\dot{Y} - \dot{Y}) \\ \hat{Y} = C_a \hat{X} \end{cases}$$
(8)

 $L_X, L_{Xd}, L_f, L_{fd}$  are the observer gains.

Letting  $e_X = X - \hat{X}$ ,  $e_f = f - \hat{f}$ , the error dynamic of the proposed observer (8) is:

$$\begin{bmatrix} I_{n+p} + L_{Xd}C_a & 0\\ L_{fd}C_a & I_{r+q} \end{bmatrix} \begin{bmatrix} \dot{e}_X\\ \dot{e}_f \end{bmatrix} = \begin{bmatrix} A_a - L_XC_a & F\\ -L_fC_a & 0\\ 0 & I_{r+q} \end{bmatrix} \begin{bmatrix} e_X\\ e_f \end{bmatrix} + \begin{bmatrix} D_a & 0\\ 0 & I_{r+q} \end{bmatrix} \begin{bmatrix} d_a\\ \dot{f} \end{bmatrix}$$
(9)

The compact form of (9) lead to the state space system of the error, i.e.

$$\begin{cases} E_o \dot{e} = A_o e + B_o d_D \\ v = C_o e \end{cases}$$
(10)

with

$$e = \begin{bmatrix} e_X \\ e_f \end{bmatrix}, d_D = \begin{bmatrix} d_a \\ \dot{f} \end{bmatrix}, B_o = \begin{bmatrix} D_a & 0 \\ 0 & I_{r+q} \end{bmatrix}, C_o = I_{n+p+r+q}$$
$$A_o = \tilde{A} - L\tilde{C}$$
(11)

where

$$\tilde{A} = \begin{bmatrix} A_a & F\\ 0 & 0 \end{bmatrix} \tag{12}$$

$$\tilde{C} = \begin{bmatrix} C_a & 0 \end{bmatrix} \tag{13}$$

$$L = \begin{bmatrix} L_X \\ L_f \end{bmatrix} \tag{14}$$

$$E_o = I_{n+p+r+q} + L_d \tilde{C} \tag{15}$$

where

$$L_d = \begin{bmatrix} L_{Xd} \\ L_{fd} \end{bmatrix}$$
(16)

From (10), it is concluded that the error dynamic of the proposed observer can be formulated as a descriptor system. So the analysis and synthesis of descriptor systems are used to determine the observer gains.

In order to correctly estimate the system state in the presence of the fault and the disturbance, it is proposed to use the  $H_{\infty}$ approach. For standard systems, the well-known approach is to use the Bounded Real Lemma [20]. Consider Lemma 1 provided in the pioneering work of [21] gives the Bounded Real Lemma for descriptor system. **Lemma 1:** The pair (*E*, *A*) is admissible and its transfer function from exogenous disturbance (with a distribution matrix of *B*) to performance variable (with a distribution matrix of *C*), i.e  $G = C(sE - A)^{-1}B$ , satisfy  $||G|| < \gamma$  if and only if there exists *P* with appropriate dimension such that:

$$EP = P^T E^T \ge 0 \tag{17}$$

$$\begin{bmatrix} AP + P^T A^T & B & P^T C^T \\ * & -\gamma I & 0 \\ * & * & -\gamma I \end{bmatrix} < 0$$
(18)

The conditions of convergence of the proposed observer are then formulated by the following result.

**Theorem 1**: The estimation error dynamics (10) are stable and the  $H_{\infty}$  performance index is guaranteed with attenuation level  $\gamma$ , if there exists a symmetric positive definite matrix  $P_1 \in R^{(n+p+q+r)\times(n+p+q+r)}$ ,  $P_2 \in R^{(n+p+q+r)\times(n+p+q+r)}$ ,  $P_3 \in R^{(n+p+q+r)\times(n+p+q+r)}$ and non singular matrix  $Y_1 \in R^{h\times(n+p+q+r)}$ ,  $Y_2 \in R^{h\times(n+p+q+r)}$  such that:

# min y subject to

$$\begin{bmatrix} P_2 + P_2^T & P_1\tilde{A} - P_2^T - Y_1^T\tilde{C} + P_3 & 0 & P_1B_0 \\ * & -P_3 - P_3^T - Y_2^T\tilde{C} - \tilde{C}^TY_2 & I & 0 \\ * & * & -\gamma I & 0 \\ * & * & * & -\gamma I \end{bmatrix} < 0$$
(19)

where  $\tilde{A}$  and  $\tilde{C}$  are given in (12) and (13) respectively, \* denotes the symmetric elements in a symmetric matrix.

# Proof:

Consider the descriptor system whose the error state equation is described in (10), i.e:

$$(I + L_d \tilde{C})\dot{e} = (\tilde{A} - L\tilde{C})e + B_o d_D$$
(20)

$$y_e = e \tag{21}$$

The dual of system (20) and (21) is given as:

$$\left(I + \tilde{C}^T L_d^T\right) \dot{z} = (\tilde{A}^T - \tilde{C}^T L^T) z + d_D$$
(22)

$$y_z = B_o^T z \tag{23}$$

Introducing  $z_a = \dot{z}$ , then it is obtained the augmented equation of (22)-(23):

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{z} \\ \dot{z}_a \end{bmatrix} = \begin{bmatrix} 0 & I \\ \tilde{A}^T - \tilde{C}^T L^T & -I - \tilde{C}^T L_d^T \end{bmatrix} \begin{bmatrix} z \\ z_a \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} d_D \quad (24)$$

$$y_z = \begin{bmatrix} B_o^T & 0 \end{bmatrix} \begin{bmatrix} z \\ z_a \end{bmatrix}$$
(25)

The compact form of (24)-(25) can be written as:

$$E_{ea}\dot{x}_{ea} = (A_{ea} - B_{ea}L_{ea})x_{ea} + D_{ea}d_D$$
(26)

$$y_z = C_{ea} x_{ea} \tag{27}$$

where

$$\begin{aligned} x_{ea} &= \begin{bmatrix} z \\ z_a \end{bmatrix}; E_{ea} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}; A_{ea} = \begin{bmatrix} 0 & I \\ \tilde{A}^T & -I \end{bmatrix}; B_{ea} = \begin{bmatrix} 0 \\ \tilde{C}^T \end{bmatrix} \\ L_{ea} &= \begin{bmatrix} L^T & L_d^T \end{bmatrix}; D_{ea} = \begin{bmatrix} 0 \\ I \end{bmatrix}; C_{ea} = \begin{bmatrix} B_o^T & 0 \end{bmatrix}; \end{aligned}$$

Hereafter, Lemma 1 is applied for the system (26)-(27) so it is obtained this matrix inequality:

$$E_{ea}P = P^T E_{ea}^T \ge 0 \tag{28}$$

$$\begin{bmatrix} (A_{ea} - B_{ea}L_{ea})P + P^{T}(A_{ea} - B_{ea}L_{ea})^{T} & D_{ea} & P^{T}C_{ea}^{T} \\ * & -\gamma I & 0 \\ * & * & -\gamma I \end{bmatrix} < 0$$
(29)

The presence of the terms  $L_{ea}P$  let the inequality (29) nonlinear. To linearize it, introduce a new variable  $Y = L_{ea}P$ . The inequality (29) can then be written as:

$$\begin{bmatrix} A_{ea}P + P^{T}A_{ea}^{T} - B_{ea}Y - B_{ea}^{T}Y^{T} & D_{ea} & P^{T}C_{ea}^{T} \\ * & -\gamma I & 0 \\ * & * & -\gamma I \end{bmatrix} < 0(30)$$

Thus the observer gains can be calculated from the solution for P and Y in (30) using

$$L_{ea} = \begin{bmatrix} L^T & L_d^T \end{bmatrix} = YP^{-1} \tag{31}$$

Considering the specific structure of  $E_{ea}$ , (28) can be satisfied by setting:

$$P = \begin{bmatrix} P_1 & 0\\ P_2 & P_3 \end{bmatrix}$$
(32)

then the new variable:

$$Y = \begin{bmatrix} Y_1 & Y_2 \end{bmatrix} \tag{33}$$

Therefore, by (30) and the definition of the related variables, (19) is obtained.

B. Observer Structure

The proposed observer (8) can be rewritten as:

$$\begin{cases} \begin{bmatrix} I + L_{Xd}C_a & 0 \\ L_{fd}C_a & I \end{bmatrix} \begin{bmatrix} \dot{\hat{X}} \\ \dot{\hat{f}} \end{bmatrix} = \begin{bmatrix} A_a & F \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\hat{X}} \\ \hat{f} \end{bmatrix} + \begin{bmatrix} B_a \\ 0 \end{bmatrix} u \\ + \begin{bmatrix} L_X \\ L_f \end{bmatrix} (Y - \hat{Y}) + \begin{bmatrix} L_{Xd} \\ L_{fd} \end{bmatrix} \dot{Y}$$
(34)  
$$\hat{Y} = \begin{bmatrix} C_a & 0 \end{bmatrix} \begin{bmatrix} \hat{\hat{X}} \\ \hat{f} \end{bmatrix}$$

The simplified form of (34) is:

$$\begin{cases} E_o \dot{\tilde{x}} = \tilde{A} \tilde{x} + \tilde{B} u + L (Y - \hat{Y}) + L_d \dot{Y} \\ \dot{\tilde{Y}} = \tilde{C} \tilde{x} \end{cases}$$
(35)

where  $\tilde{x} = \begin{bmatrix} \hat{X} \\ \hat{f} \end{bmatrix}$ ;  $\tilde{B} = \begin{bmatrix} B_a \\ 0 \end{bmatrix}$ ;  $\tilde{A}, \tilde{C}, L, E_o, L_d$  are given in (12), (13), (14), (15) and (16) respectively.

In order to remove the derivatives of the output in the proposed observer (35), it is introduced a new state

$$\xi = \tilde{x} - L_d(Y - \hat{Y}) \tag{36}$$

then the PID-ESO can be formulated as:

$$\begin{cases} \dot{\xi} = \tilde{A}\tilde{x} + \tilde{B}u + L(Y - \hat{Y}) \\ \tilde{x} = (I + L_d \tilde{C})^{-1} \{\xi + L_d Y\} \\ \hat{Y} = \tilde{C}\tilde{x} \end{cases}$$
(37)

The derivative term of the output Y does not appear in the proposed observer (37) thus it is more applicable in practical systems than the original form (35). The observer (37) can give the accurate asymptotic estimates of the system state and the fault. The fault signal may be in any form and even unbounded as long as its first derivative is bounded. From this point of view, the present observer (37) gains an advantageous aspect over the results given by [22][23].

Note, in accordance with observability requirement, the number of faults to be detected must be less or equal with the number of the output, or  $r + q \le p$ . Because of that, it is necessary to determine first the appropriate state space model of the considered system.

### **IV. Simulation Results**

Consider a linear model of the DC-motor speed control system described in [24]. This example is concerned only with the rotational speed of the shaft as the output and the armature voltage as the input. Its state space representation is given by

$$\dot{x} = \begin{bmatrix} -\frac{b}{J} & \frac{K_t}{J} \\ -\frac{K_e}{L} & -\frac{R}{L} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

where the rotational speed and electric current are chosen as the state variables. The nominal values of the physical parameters are given in table 1.

Parameters	Symbol	Value
moment of inertia of the rotor	J	0.01 kg.m <sup>2</sup>
motor viscous friction constant	b	0.1 N.m.s
electromotive force constant	K <sub>e</sub>	0.01 V/rad/sec
motor torque constant	K <sub>t</sub>	0.01 N.m/Amp
electric resistance	R	1 Ohm
electric inductance	L	0.5 H

TABLE VII. THE NOMINAL VALUES OF THE PARAMETERS IN THE LINEAR MODEL OF THE DC-MOTOR

The system is subjected to a reference signal r in the form of unit pulse with a period of 4 sec. The control signal is given by

$$u = -K_c x + K_i \int (y - r) dt$$

where  $K_c$  is the state feedback gain, and  $K_i$  is the error integral gain. Both those controller gains are computed by using the pole placement technique in order to obtain the settling time of less than 2 sec and the maximum overshoot of less than 5% in the step response.

It is supposed that this control system is subject to a sensor fault  $f_a$  and an actuator fault  $f_s$ , so  $F_s = [1 \ 0]^T$ , E = B. Because there are two faults, the output matrix C must have two rows, that is  $I_2$ . It is easy to verify that A is a stable matrix, and the pair (A,C) is observable, so the proposed method is applicable. Furthermore, to show that the proposed method is superior to the PI observer as in [10], this paper compares them with the following simulation.

The faults are simulated as follows:

$$f_{s} = \begin{cases} 0 & , t < 1.5 \\ 0.1 & , 1.5 \le t < 4 \\ 10^{-2}tsin(0.5\pi t) & , 4 \le t < 10 \\ 0.2 & , t \ge 10 \\ .t < 1.5 \\ 5 \times 10^{-2}tsin(0.25\pi t) & , 1.5 \le t < 15 \\ 0.2 & , t \ge 15 \end{cases}$$

In order to show that the proposed observer is robust against disturbances and noise, it is introduced

$$D_d = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$
 and  $D_\omega = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ 

The disturbances are uniformly distributed random signals whose the minimum and maximum values are [-0.2 - 0.1] and [-0.2 0.1]respectively. The noise is a normal distributed random signal with the noise power of  $10^{-4}$ . The system is subjected to only the disturbances in the first simulation. Next the system is subjected to both the disturbances and the noise in the second simulation.

By implementing the algorithm in Theorem 1, it is obtained  $P_1$ ,  $P_2$ ,  $P_3$ ,  $Y_1$ , and  $Y_2$ . Next they are arranged to form P and Y as in (32) and (33). Then the gains observer are computed using (31), i.e:

$$\begin{bmatrix} L\\ L_d \end{bmatrix} = L_{ea}^T$$

Therefore the PD/PI ESO in the form (37) has been designed.

The estimates of the actuator and sensor faults using the classical PI observer and the PD/PI ESO for no noise case are shown in Fig. 1-2. It can be seen that excellent estimates of the faults have been obtained using the two methods for constant fault. But for time-varying fault, the PD/PI ESO can improve the rapidity of fault estimation. Furthermore, the effects of disturbances have been attenuated by the two methods in the same level. These effects appear in the actuator fault estimation as well as in the sensor fault estimation.

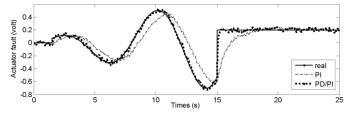


Fig. 2. Actuator fault and its estimate for no noise case

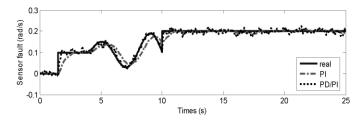
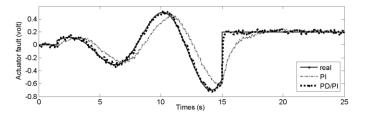
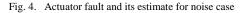


Fig. 3. Sensor fault and its estimate for no noise case





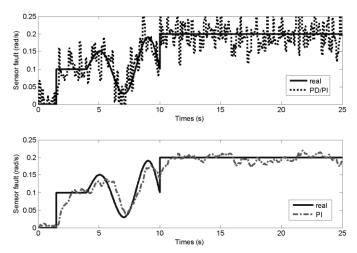


Fig. 5. Sensor fault and its estimate for noise case

**Remark:** The classical PI observer does not have capability to track any fast dynamic faults because its observer gains are obtained through feasibility problem of the related LMI, not optimization problem with subject to the related LMI as in the PD/PI ESO described in (19). If the observer gains of the PI observer are found by using optimization problem with subject to the appropriate LMI, the estimation results of the PI observer are the same with ones of the PD/PI ESO. However the proofs are not shown in this paper.

The simulation results for noise case are shown in Fig. 3-4. There is no noise effect in the actuator fault estimate of both methods as seen in Fig. 3. Meanwhile the noise effect appears in the sensor fault estimate of both methods as seen in Fig. 4. But the noise attenuation level of the PI observer is better than one of the PD/PI ESO. The noise is not attenuated nor amplified in the PD/PI ESO, and appears together with the sensor faults estimate. However, for control purpose, the information of noise need to be known in order to obtain the true value of measurement. So the PD/PI ESO is still acceptable to be used. In addition, a filter can be applied to remove the noise.

## V. Conclusions

In this paper, by adding the derivative action in both state and fault estimation, a descriptor system approach has been introduced to reconstruct the sensor and actuator faults simultaneously for LTI systems. There is no constrain imposed on the faults. The simulation has shown that the proposed observer allow us to reconstruct the fault in any forms even when the sensor and actuator faults occur simultaneously and the disturbance and noise are present. This approach will be extended to T-S fuzzy model viewed as nonlinear system representation.

### References

- [1] J. Chen and J. J. Patton, *Robust Model-based Fault Diagnosis for Dynamic Systems*, Kluwer Academic Publishers, Boston, MA, 1999.
- [2] M. Blanke, M. Kinnaert, J. Lunze, M. Staroswiecki, *Diagnosis and Fault-Tolerant Control*, 2nd edition, Springer Verlag, Berlin Heidelberg, 2006.
- [3] D. M. Frank, S. X. Ding, B. Koppen-Seliger, "Current developments in the theory of FDI," *Proc. of IFAC Safeprocess*, pp. 16-27, 2000.
- [4] B. Badre, "A Decoupled Parameters Estimators for in Nonlinear Systems Fault diagnosis by ANFIS", *International Journal of Electrical and Computer Engineering* (IJECE), vol 2, no 2, pp. 166-174, 2012.
- [5] E. Wang, T. Pang, M. Chai, Z. Zhang, , "Fault Detection and Isolation for GPS RAIM Based on Genetic Resampling Particle Filter Approach", *TELKOMNIKA Indonesian Journal of Electrical Engineering*, vol 12, no 5, pp. 3911-3919, 2014.
- [6] K. Zhang, B. Jiang, V. Cocquempot, "Adaptive observer-based fast fault estimation", *International Journal of Control, Automation, and Systems*, 6(3), 320-326, 2008.
- [7] A. Khedher, K. Benothman, D. Maquin, M. Benrejeb, "State and sensor faults estimation via a proportional integral observer", In 6th International Multi-conference on Systems Signals & Devices SSD'09, Djerba, Tunisia, March 23-26, pp. 1-6, 2009
- [8] R.J. Patton and S. Klinkhieo, "Actuator fault estimation and compensation based on an augmented state observer approach", in Proceedings of 48th CDC and 28th Chinese Control Conference, Shanghai, 2009
- [9] D. Ichalal, B. Marx, J. Ragot, D. Maquin, "Simultaneous state and unknown inputs estimation with PI and PMI observers for Takagi Sugeno model with unmeasurable premise variables", In *Control and Automation*, *MED'09*, pp. 353-358, 2009.
- [10] A. Khedher, K. Benothman, M. Benrejeb, D. Maquin, "Adaptive observer for fault estimation in nonlinear systems described by a Takagi-Sugeno model", In 18th Mediterranean Conference on Control & Automation (MED), pp. 261-266, 2010
- [11] Z. Gao, and S.X. Ding, "Actuator fault robust estimation and fault-tolerant control for a class of nonlinear descriptor systems", *Automatica*, 43(5), 912–920, 2007
- [12] Z. Gao, and S.X. Ding, "Sensor fault reconstruction and sensor compensation for a class of nonlinear state-space systems via a descriptor system approach", *IET Control Theory & Applications*, 1(3), 578-585, 2007
- [13] M. Bouattour, M. Chadli, A. El.Hajjaji, and M. Chaabane, "H<sub>∞</sub> sensor faults estimation for T-S models using descriptor techniques : Application to fault diagnosis," in *IEEE International Conferences on Fuzzy Systems*, Jeju Island, Korea, August 20-24 2009, pp. 251–255.
- [14] M. Bouattour, M. Chadli, A. El.Hajjaji, and M. Chaabane, "Estimation of state, actuator and sensor faults for TS models". In *49th IEEE Conference* on Decision and Control (CDC), pp. 1613-1618, 2010
- [15] C. Edwards C, "A comparison of sliding mode and unknown input observers for fault reconstruction", 43rd IEEE Conference on Decision and Control, CDC'04, Atlantis, Paradise Island, Bahamas, December 14-17, 2004.
- [16] H. Noura, D. Theilliol, J.C. Ponsart, A. Chamseddine, *Fault-tolerant control systems: Design and practical applications*. Springer Science & Business Media, 2009.
- [17] A. Wu & G. Duan, "Design of PD observers in descriptor linear systems", International Journal of Control Automation and Systems, 5(1), 93, 2007.
- [18] A. Wu, G. Duan, Y.M. Fu, "Generalized PID observer design for descriptor linear systems", *IEEE Transactions on Systems, Man, and Cybernetics*, 37(5), 1390-1395, 2007.
- [19] Z. Gao & H. Wang, "Descriptor observer approaches for multivariable systems with measurement noises and application in fault detection and diagnosis", *Systems & Control Letters*, 55, 304 – 313, 2006.

- [20] S. Boyd, L. El Ghaoui, E. Feron, V. Balakrishnan, *Linear matrix inequalities in systems and control*, SIAM, Philadelphia, 1994
- [21] I. Masubuchi, Y. Kamitane, A. Ohara, N. Suda, "H∞ control for descriptor systems: a matrix inequalities approach", *Automatica*, 33(4), 669-673, 1997
- [22] M. Bouattour, M. Chadli, M Chaabane, A.E. Hajjaji, "Design of robust fault detection observer for Takagi-Sugeno models using the descriptor approach", *International Journal of Control, Automation, and Systems* 9(5), 973-979, 2011
- [23] L. Tao, G. Lei, W. Lingyao, W. Xinjiang, "Robust fault detection filter design for uncertain LTI systems based on new bounded real lemma," *International Journal of Control, Automation, and Systems*, 7(4), 644-650, 2009.
- [24] R.C. Dorf, and R.H. Bishop, *Moderm Control Systems*, Addison-Wesley Publishing Company, 1995.