Abstract

Differential Equations constrained by algebraic equations and inequalities on the states occur naturally in the modeling of many engineering systems such as simulation and control of multibody dynamical systems and electrical circuits, in robotic path planning and sliding mode control systems, in guidance design for aerospace vehicles and in chemical reactor systems. These systems are often known as path constrained Differential Algebraic Equation (DAE) systems. In this work we investigate the numerical methods for (1) simulation of path constrained DAEs with high differential index, (2) computational solution of optimal control problems with high index constraints and (3) dynamic optimization problem with disjunctive algebraic constraints.

At first, this thesis develops the generalized-α method and analyzes it in detail. The generalized-α method is shown to have regularization property which becomes a key advantage in solving high index path constrained dynamical systems.

We show that the α-method is a good simulator for high index DAEs due to the regularization property involved in the discretization. This solves effectively the index 3 Euler-Lagrange equations, Kane's equations formulation and other higher index DAEs occurring naturally.

We study a direct method known as transcription method, a numerical approach, equipped with the generalized-α discretization to solve an Optimal Control Problem (OCP). State-constrained path constraints in an OCP pose numerical challenges because (1) along the (active) constraints a high index DAE is often formed and (2) the number and interval of active inequality constraints (even if not state constrained) is not known in priori. It is shown that the regularization property of the generalized-α method is useful

in these situations. The proposed method does not perturb the inequality constraints and hence the physics of the problem is not diluted. The convergence analysis of the discretized dynamic optimization problem to the continuous OCP is proved for a wide range of control problems by assuming low continuity conditions on the controls.

Also this thesis solves OCPs having either-or (disjunctive) constraints by considering an equivalent simple convex hull of the disjunctive constraints. This idea is implemented to a control problem in vehicle dynamics to stabilize the rollover of a road vehicle undergoing severe maneuver. From a practical point of view we obtain the control forces as sensory output.

This thesis also focuses on the computation of the radial hedgehog solution, solution of a Singular Boundary Value Problem (SBVP) arising from the energy minimization of nematic liquid crystal, by solving an equivalent boundary value DAE. The equivalent DAE, which is index 3 at the point of singularity and index 1 in the rest of the simulation interval, is solved by the α-method. It is shown that the point of singularity can be removed by the DAE approach. Advantages of the DAE approach over classical solvers for SBVP and transcription method for the radial hedgehog problem are also shown. Some computational findings of the radial hedgehog solution completes the thesis.

Overall, the thesis shows that the singularity and ill-conditioning inherent in path constrained DAEs can be dealt with numerically in a consistent and correct way by a discretization with regularization property.