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Semi Non-Standard Trimean Algorithm for Rosenzweig-MacArthur Interaction Model

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***Abstract*— Most real world natural systems are shows seasonal behaviour due to seasonal environmental or climate change. As a results, many species display seasonal changes in their life history parameters. It is crucial to comprehend how the seasonal forcing controls the behaviour of the population dynamics. The Rosenzweig-MacArthur model is a system with at least two ordinary differential equations used in population dynamics to model the interaction of predator and prey bonding. Rosenzweig-MacArthur model overcome the weakness of Lotka-Volterra model to simulate interaction between two species. In Rosenzweig-MacArthur model, logistic growth rate of prey is resource limited. The model utilizes Holling type II as the functional response representation. The purpose of the study is to construct method to improve simulation on the behaviour of interactions between species and predicting equilibrium point accurately and fast. Current methods seem to predict accurately the equilibrium point if only small mesh size used. Using small mesh size will require long simulation time to predict the equilibrium point. Able to increase the mesh size will increase the speed of predicting the equilibrium point. In this paper, we propose three new semi non-standard trimean algorithms to simulate the behaviour of interaction between species represented by Rosenzweig-MacArthur model. The new algorithms apply a hybrid of semi non-standard approach and trimean to approximate the nonlinear terms in the differential equation model. Two cases of experiment conducted to examine the performance of all three semi non-standard schemes. Result shows that all three new semi non-standards schemes accurately predict the equilibrium point (0.25, 0.46875) even using big mesh size ($h = 4.6$ and $h = 2.1$) for both cases. Thus, all three semi non-standard schemes fulfil the purpose of this study.**

***Keywords*— semi non-standard discretization; trimean; Rosenzweig-MacArthur model; interaction between species.**

I. Introduction

Ecological populations in real world typically expose oscillations behaviour due to predator-prey interactions among species. In mathematical models, the situations usually represented in the structure of ordinary differential equation systems are widely being employed to comprehend and forecast the dynamics behaviour of interacting species. Lotka-Volterra dynamic model is the simplest and pioneered dynamic model used to simulate species interaction. Researcher modified the model in many ways since its original formulation in the 1920s.

For many decades, various interactions between populations explored via various versions of Lotka-Volterra equations, which comprise Modifications to the original Lotka-Volterra to gain better understanding for the dynamics of population interactions was included later. The Lotka-Volterra model composes of two impractical assumptions. First, in the absence of predators, the prey population will increase exponentially. Second, the individual predator's stomach never gets full.

Rosenzweig and MacArthur improve Lotka-Volterra model in 1963 [1] by proposing Rosenzweig-MacArthur model. The model corrects the impractical of Lotka-Volterra assumptions. Rosenzweig and MacArthur admit prey growth reliant on density and saturating uptake of prey by the predator. The Rosenzweig-MacArthur model is becoming an alternative fundamental models used in exploring interaction between systems. The differences are the growth and death rates are not totally depending on prey and predator population. The model has taken into consideration the environment around particular area. Predator can also find other source of food beside prey and prey death can cause by disease and inter fight amongst predators. The model serves as a starting point for extensions to simulate interactions that are more complex. Some example of researchers that apply of Rosenzweig-MacArthur model are Weitz & Levin [2], Salomon & Stotle [3], Feng, Rock & Hinson [4], and Huincahue-Archos & Gonzales-Olivares [5].

Weitz & Levin [2] suggest a scaled Rosenzweig-MacArthur model using both Type 1 and Type II functional responses. They offered an analytical framework how to integrate scaled term in Rosenzweig-MacArthur model. They evaluated predator-prey ratio for a broad class of interaction system. Salomon & Stolte[3] used Rosenweig-MacArthur model to forecast dynamic population of Amoebophrya and dinoflagellate. The model is useful to explain how Amoebophrya can control dinoflagellate. Feng, Rock & Hinson [4] apply Rosenzweig-MacArthur to simulate two-patch predator-prey interactions with migration elements in both species. His finding shows that both species population keep on fluctuating at the same level. Interactions of both species are longer than Rosenzweig-MacArthur original model. Huincahue-Archos & Gonzales-Olivares [5] apply the Rosenzweig-MacArthur by incorporating the Allee effect on the prey equation. They modify the Rosenzweig-MacArthur to incorporate Gause type model. The study shows that population with strong Allee effects can be extinct by predation.

Standard finite difference method seldomly used to solve differential equations problems [6]-[9]. Besides standard finite difference, Non-standard (NS) finite difference was proposed in [10] as a practical solution to conserve qualitative characteristic of differential equation. Mickens has construct the basic theory for non-standard approaches [10]-[13]. Standard finite difference approaches seldomly require small meshes to approximate a problem with high accuracy. NS eliminate numerical nonstability in standard approaches. Other contribution on extending non-standard approach are in [14]-[19].

Trimean was proposed by Tukey in 1977 as follows

$$TM = (H_1 + 2M + H_2)/4,$$

where M is median and H_1 and H_2 is the upper and lower quartile [22]. Trimean have been used to estimate wavelet coefficient [23] while [21] apply trimean in colour model constancy and [25] combine Trimean and Quartile to estimate the variance.

In this paper, we develop three new schemes for simulating Rosenzweig-MacArthur model by proposing a semi non-standard approximation with trimean approach. Previously, [26],[27] developed non-standard approach (SNST) for simulating Lotka-Volterra. The algorithm proposed used Scilab to code. We modified and apply method used in [11],[18],[26] to simulate Rosenzweig-MacArthur model.

Rosenzweig-MacArthur model is given by

$$\left.\begin{aligned} \frac{dx}{dt} &= bx(1-x) - ag(x)xy, x(t_0) = x_0 \geq 0, \\ \frac{dy}{dt} &= g(x)xy - dy, y(t_0) = y_0 \end{aligned}\right\} \quad (1)$$

x and y are prey and predator population, b prey's growth rate $(b > 0)$, $a =$ predation rate $(a > 0)$, $d =$ predator's death rate rate $(d > 0)$.

In Eq. (1), it is assumed that $g(x) \geq 0, g'(x) \leq 0, [xg(x)]' \geq 0$ and $xg(x)$ is the boundary if $x \to \infty$. Following [28], we assumed that if food volume increased, predator population would increase. This will increase prey consumption since number of predator increase. Thus, prey population will decrease, and then predator population will decreased because of lack of food. The equilibrium point for (1) is defined by

$$\begin{aligned} bx(1-x) - ag(x)xy &= 0, \\ g(x)xy - dy &= 0. \end{aligned} \quad (2)$$

Depending on parameter values and reaction function, $xg(x)$, Eq. (1) will have equilibrium points given below :

1) $E_0 = (0,0)$;
2) $E_1 = (1,0)$ and
3) $E^p = (x^p, y^p)$ where x^p is solution for $xg(x) = d$ and $y^p = \frac{bx^p(1-x^p)}{ad}$. Equilibrium point E^p exists if and only if $g(1) > d$.

By following stability theorem for nonlinearity, the given statement for equilibrium point is true.

1) Equilibrium point E_0 is always linearly non-stable.
2) Equilibrium point E_1 is linearly stable if $g(1) > d$ and non-linear otherwise;
3) Equilibrium point E^p is linear stabil if $b + ay^p g'(x^p) > 0$ and non-stabil linear if $b + ay^p g'(x^p) < 0$.

II. Material and Method

To construct the Semi Non-standard Trimean (SNST) scheme for Eq. (1), we approximate $\frac{dx}{dt}$ with $\frac{x_{i+1} - x_i}{h}$, and $\frac{dy}{dt}$ with $\frac{y_{i+1} - y_i}{h}$. For the right hand side we proposed three new non-standard schemes of Eq. (1),

A. Scheme 1

For Eq. (1), the x, x^2 and xy is approximate by non-local representation as follows.

$$x = 2x_i - x_{i+1}$$

$$x^2 = x_i x_{i+1}$$

$$xy = x_{i+1} y_i$$

which equal to

$$\frac{x_{i+1}-x_i}{h} = Bx - Bx^2 - AGxy$$

where $G(x_i)$ is

$$G(x_i) = \frac{1}{C+x_i}$$

Therefore,

$$\frac{x_{i+1}-x_i}{h} = B(2x_i - x_{i+1}) - B(x_i x_{i+1}) - AG(x_{i+1}y_i)$$
$$x_{i+1}-x_i = hB(2x_i - x_{i+1}) - hB(x_i x_{i+1}) - AhG(x_{i+1}y_i)$$
$$x_{i+1}-x_i = 2hBx_i - hBx_{i+1} - hBx_i x_{i+1} - AhGx_{i+1}y_i$$
$$x_{i+1} + hBx_{i+1} + hBx_i x_{i+1} + AhGx_{i+1}y_i = 2hBx_i + x_i$$
$$x_{i+1}(1 + hB + hBx_i + AhGy_i) = 2hBx_i + x_i$$
$$x_{i+1} = \frac{2hBx_i + x_i}{(1+hB+hBx_i+AhGy_i)} \quad (3)$$

Since trimean approaches need two previous nodes to calculate the next node, thus we use non-trimean equation, which is Eq. (3) for the first two nodes.
For $i=0$, the equation (3) become

$$x_1 = \frac{2hBx_0 + x_0}{(1+hB+hBx_0+AhGy_0)}$$

For $i=1$, Eq. (3) become

$$x_2 = \frac{2hBx_1 + x_1}{(1+hB+hBx_1+AhGy_1)}$$

Applying Trimean formula, $\left(\frac{x_{i-1}+2x_i+x_{i+1}}{4}\right)$ to Eq. (3) for x, without changing value in xy and for $i>1$ resulting

$$\frac{x_{i+1}-x_i}{h} = B\left(\frac{x_{i-1}+2x_i+x_{i+1}}{4}\right) - Bx_i x_{i+1} - AGx_{i+1}y_i$$
$$x_{i+1} - x = hB\left(\frac{x_{i-1}+2x_i+x_{i+1}}{4}\right) - hBx_i x_{i+1} - hAGx_{i+1}y_i$$
$$x_{i+1}-x_i = 0.25hBx_{i-1} + 0.5hBx_i + 0.25hBx_{i+1}$$
$$- hBx_i x_{i+1} - hAGx_{i+1}$$
$$x_{i+1} - 0.25hBx_{i+1} + hBx_i x_{i+1} + hAGx_{i+1}$$
$$= 0.25hBx_{i-1} + 0.5hBx_i + x_i$$
$$x_{i+1}(1 - 0.25hB + hBx_i + hAG)$$
$$= 0.25hBx_{i-1} + 0.5hBx_i + x_i$$
$$x_{i+1} = \frac{0.25hBx_{i-1} + 0.5hBx_i + x_i}{(1-0.25hB+hBx_i+hAG)} \quad (4)$$

While for y and xy in the right-hand-side of (1) also is replaced by non-local representation as follows.

$$y = -y_i + 2y_{i+1}$$
$$xy = 2x_{i+1}y_i - x_i y_{i+1}$$

Thus,

$$\frac{y_{i+1}-y_i}{h} = Gxy - Dy,$$

where $G(x_i)$ is

$$G(x_i) = \frac{1}{C+x_i}$$

$$\frac{y_{i+1}-y_i}{h} = G(2x_{i+1}y_i - x_i y_{i+1}) - D(-y_i + 2y_{i+1})$$
$$y_{i+1}-y_i = hG(2x_{i+1}y_i - x_i y_{i+1}) - hD(-y_i + 2y_{i+1})$$
$$y_{i+1}-y_i = 2hGx_{i+1}y_i - hGx_i y_{i+1} + hDy_i - 2hDy_{i+1}$$
$$y_{i+1} + hGx_i y_{i+1} + 2hDy_{i+1} = 2hGx_{i+1}y_i + hDy_i + y_i$$
$$y_{i+1}(1 + hGx_i + 2hD) = 2hGx_{i+1}y_i + hDy_i + y_i$$
$$y_{i+1} = \frac{2hGx_{i+1}y_i + hDy_i + y_i}{(1+hGx_i+2hD)}. \quad (5)$$

For $i=0$, Eq. (5) become

$$y_1 = \frac{2hGx_1y_0 + hDy_0 + y_0}{(1+hGx_0+2hD)}.$$

For $i=1$, Eq. (5) become

$$y_2 = \frac{2hGx_2y_1 + hDy_1 + y_1}{(1+hGx_1+2hD)}.$$

Trimean formulation $\left(\frac{y_{i-1}+2y_i+y_{i+1}}{4}\right)$ to Eq. (5) for y, without any changes in xy and for $i>1$,

$$y_{i+1} = \frac{2hGx_{i+1}y_i + hD(0.25y_{i-1}+0.5y_i+0.25y_{i+1}) + (0.25y_{i-1}+0.5y_i+0.25y_{i+1})}{(1+hGx_i+2hD)}.$$
$$y_{i+1} = \frac{2hGx_{i+1}y_i + (hD+1)(0.25y_{i-1}+0.5y_i+0.25y_{i+1})}{(1+hGx_i+2hD)}. \quad (6)$$

B. *Scheme 2*

The value of x and xy at the right-hand-side of Eq. (1) is similar to scheme 1 while the approximate value of y and xy was replaced by non-local representation as follows.

$$y = y_{i+1}$$
$$xy = 2x_{i+1}y_i - x_i y_{i+1}$$

Which is equal to

$$\frac{y_{i+1}-y_i}{h} = Gxy - Dy,$$

where $G(x_i)$ is

$$G(x_i) = \frac{1}{C+x_i}$$

$$\frac{y_{i+1}-y_i}{h} = G(2x_{i+1}y_i - x_i y_{i+1}) - D(y_{i+1})$$
$$y_{i+1}-y_i = hG(2x_{i+1}y_i - x_i y_{i+1}) - hD(y_{i+1})$$
$$y_{i+1}-y_i = 2hGx_{i+1}y_i - hGx_i y_{i+1} - hDy_{i+1}$$
$$y_{i+1} + hGx_i y_{i+1} hDy_{i+1} = 2hGx_{i+1}y_i + y_i$$
$$y_{i+1}(1 + hGx_i + hD) = 2hGx_{i+1}y_i + y_i$$
$$y_{i+1} = \frac{2hGx_{i+1}y_i + y_i}{(1+hGx_i+hD)}. \quad (7)$$

For $i=0$, Eq. (7) become

$$y_1 = \frac{2hGx_1y_0 + y_0}{(1+hGx_0+hD)}.$$

For $i=1$, Eq. (7) become

$$y_2 = \frac{2hGx_2 y_1 + y_1}{(1+hGx_1+hD)}.$$

Trimean $\left(\frac{y_{i-1}+2y_i+y_{i+1}}{4}\right)$ replace y in Eq. (7), without changes in xy for $i>1$,

$$y_{i+1} = \frac{2hGx_{i+1}y_i + (0.25y_{i=1} + 0.5y_i + 0.25y_{i+1})}{(1+hGx_i+hD)}.$$

$$y_{i+1} = \frac{0.25y_{i=1} + (2hGx_{i+1} + 0.5)y_i + 0.25y_{i+1})}{(1+hGx_i+hD)}. \quad (8)$$

C. Scheme 3

The value of x and xy at the right-hand-side of Eq. (1) is similar to scheme 1 while the approximate value of y and xy was replaced by non-local representation as follows.

$$y = 2y_i - y_{i+1}$$
$$xy = 2x_{i+1}y_i - x_i y_{i+1}$$

Which equal to

$$\frac{y_{i+1}-y_i}{h} = Gxy - Dy,$$

where $G(x_i)$ equal to

$$G(x_i) = \frac{1}{C+x_i}$$

$$\frac{y_{i+1}-y_i}{h} = G(2x_{i+1}y_i - x_i y_{i+1}) - D(2y_i - y_{i+1})$$

$$y_{i+1} - y_i = hG(2x_{i+1}y_i - x_i y_{i+1}) - hD(2y_i - y_{i+1})$$

$$y_{i+1} - y_i = 2hGx_{i+1}y_i - hGx_i y_{i+1} - 2hDy_i + hDy_{i+1}$$

$$y_{i+1} + hGx_i y_{i+1} - hDy_{i+1} = 2hGx_{i+1}y_i - 2hDy_i + y_i$$

$$y_{i+1}(1+hGx_i - hD) = 2hGx_{i+1}y_i - 2hDy_i + y_i$$

$$y_{i+1} = \frac{2hGx_{i+1}y_i - 2hDy_i + y_i}{(1+hGx_i-hD)}. \quad (9)$$

For $i=0$,

$$y_1 = \frac{2hGx_1 y_0 - 2hDy_0 + y_0}{(1+hGx_0-hD)}.$$

For $i=1$,

$$y_2 = \frac{2hGx_2 y_1 - 2hDy_1 + y_1}{(1+hGx_1-hD)}.$$

Trimean $\left(\frac{y_{i-1}+2y_i+y_{i+1}}{4}\right)$ replace y in Eq. (9) without changes in xy for $i>1$,

$$y_{i+1} = \frac{2hGx_{i+1}y_i - 2hD(0.25y_{i-1}+0.5y_i+0.25y_{i+1}) + (0.25y_{i-1}+0.5y_i+0.25y_{i+1})}{(1+hGx_i-hD)}.$$

$$y_{i+1} = \frac{2hGx_{i+1}y_i + (1-2hD)(0.25y_{i-1}+0.5y_i+0.25y_{i+1})}{(1+hGx_i-hD)}. \quad (10)$$

Algorithms for all semi non-standard trimean (SNST) schemes proposed in this study are given in Algorithm 1-3.

III. Results and Discussion

In this paper, three SNST schemes simulate Rosenzweig-MacArthur model. We analyse the behaviour of these SNST schemes for two set of initial values with two mesh size values each. We conduct the experiment with two sets of parameters value [29]:

$$A=2.0, B=1.0, C=1.0, D=0.2.$$

a. Case 1: $x_0 = 0.4; y_0 = 0.4;$ with $h=1.3$ and $h=4.6$.

b. Case 2: $x_0 = 0.1; y_0 = 0.2;$ with $h=2.1$ and $h=0.001$.

Simulated results presented in Table 1-6. We analyzed the pattern of behavior and equilibrium value compared to analytic solution. From the theory of equilibrium described in section I, we can calculate the third linear stable point of equilibrium. The theory mention that $E^p = (x^p, y^p)$, where x^p is solution for $xG(x) = D$ and $y^p = \frac{Bx^p(1-x^p)}{AD}$. Equilibrium point E^p exists if and only if $G(1) > D$. Thus, in this experiment, $G(x) = \frac{1}{C+x}, C=1.0, D=0.2,$ $G(1) = 0.5 > 0.2$. Therefore, the E^p equilibrium point is linearly stable.

Algorithm 1: SNST scheme 1

---

initialize:
mesh size, h, solution domain i, initial value of predator and prey, relevant parameters in prey and predators differential equations

approximate predator and prey for each i from 1 to $n-1$:

$$x_1 = \frac{2hBx_0 + x_0}{(1+hB+hBx_0+AhGy_0)}$$

$$x_2 = \frac{2hBx_1 + x_1}{(1+hB+hBx_1+AhGy_1)}$$

$$y_1 = \frac{2hGx_1 y_0 + hDy_0 + y_0}{(1+hGx_0+2hD)}.$$

$$y_2 = \frac{2hGx_2 y_1 + hDy_1 + y_1}{(1+hGx_1+2hD)}.$$

else

$$x_{i+1} = \frac{0.25hBx_{i-1} + 0.5hBx_i + x_i}{(1-0.25hB+hBx_i+hAG)}$$

$$y_{i+1} = \frac{2hGx_{i+1}y_i + (hD+1)(0.25y_{i-1}+0.5y_i+0.25y_{i+1})}{(1+hGx_i+2hD)}.$$

.

Display Output: $x_{\min}, x_{\max}, y_{\min}$ and $y_{\max}$

Plot

1) Prey and predator
2) Interaction between prey and predator

Algorithm 2: SNST scheme 2

initialize:
mesh size, h, solution domain i, initial value of predator and prey, relevant parameters in prey and predators differential equations

approximate predator and prey for each i from 1 to $n-1$:

$$x_1 = \frac{2hBx_0 + x_0}{(1 + hB + hBx_0 + AhGy_0)}$$
$$x_2 = \frac{2hBx_1 + x_1}{(1 + hB + hBx_1 + AhGy_1)}$$
$$y_1 = \frac{2hGx_1 y_0 + y_0}{(1 + hGx_0 + hD)}.$$
$$y_2 = \frac{2hGx_2 y_1 + y_1}{(1 + hGx_1 + hD)}.$$

else

$$x_{i+1} = \frac{0.25hBx_{i-1} + 0.5hBx_i + x_i}{(1 - 0.25hB + hBx_i + hAG)}$$
$$y_{i+1} = \frac{0.25y_{i=1} + (2hGx_{i+1} + 0.5)y_i + 0.25y_{i+1})}{(1 + hGx_i + hD)}.$$

Display Output: $x_{\min}, x_{\max}, y_{\min}$ and $y_{\max}$

Plot

1) Prey and predator
2) Interaction between prey and predator

Algorithm 3: SNST scheme 3

initialize:
mesh size, h, solution domain i, initial value of predator and prey, relevant parameters in prey and predators differential equations

approximate predator and prey for each i from 1 to $n-1$:

$$x_1 = \frac{2hBx_0 + x_0}{(1 + hB + hBx_0 + AhGy_0)}$$
$$x_2 = \frac{2hBx_1 + x_1}{(1 + hB + hBx_1 + AhGy_1)}$$
$$y_1 = \frac{2hGx_1 y_0 - 2hDy_0 + y_0}{(1 + hGx_0 - hD)}.$$
$$y_2 = \frac{2hGx_2 y_1 - 2hDy_1 + y_1}{(1 + hGx_1 - hD)}.$$

else

$$x_{i+1} = \frac{0.25hBx_{i-1} + 0.5hBx_i + x_i}{(1 - 0.25hB + hBx_i + hAG)}$$
$$y_{i+1} = \frac{2hGx_{i+1}y_i + (1 - 2hD)(0.25y_{i-1} + 0.5y_i + 0.25y_{i+1})}{(1 + hGx_i - hD)}.$$

Display Output: $x_{\min}, x_{\max}, y_{\min}$ and $y_{\max}$

Plot

1) Prey and predator
2) Interaction between prey and predator

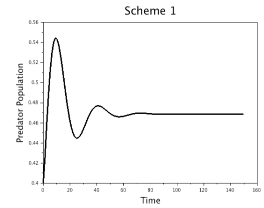
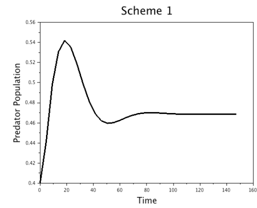
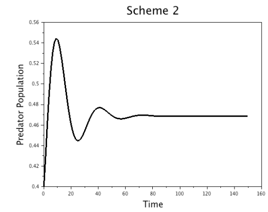
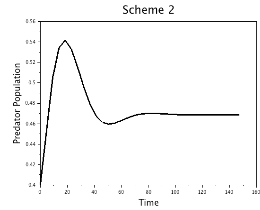
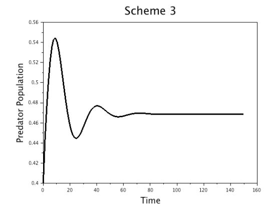
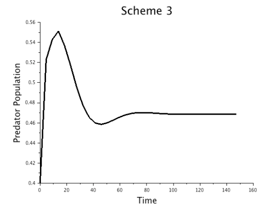
We calculate the equilibrium point as follows.

$$x^p G(x) = D \rightarrow x^p = DG(x)^{-1} \rightarrow D\left(\frac{1}{C + x^p}\right)$$
$$D = 0.2, C = 1.0 \rightarrow x^p = 0.2 + 0.2x^p \rightarrow x^p = 0.25.$$
$$y^p = \frac{Bx^p(1 - x^p)}{AD}, B = 1.0, A = 2.0, D = 0.2,$$
$$y^p = \frac{x^p(1 - x^p)}{2(0.2)} = \frac{0.25(0.75)}{0.4} = 0.46875.$$

Therefore, the equilibrium point, E^p is $(0.25, 0.46875)$.

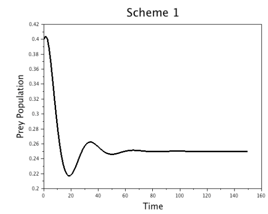
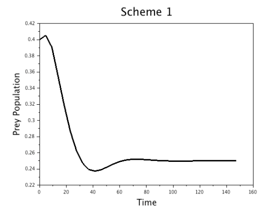
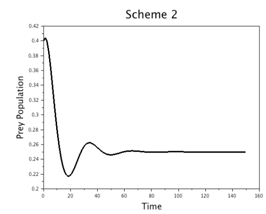
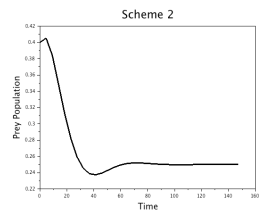
From Table 1, all SNST schemes for both mesh size values saturated at value $y = 0.4687$ at around the same time. The only difference is that less behaviour fluctuation occurs for bigger mesh size.

TABLE I
SIMULATED PREDATORS FOR ROSENZWEIG-MACARTHUR MODEL FOR CASE 1

$h = 1.3$	$h = 4.6$
	
	
	

From Table 2, all SNST schemes for both mesh size values saturated at value $x = 0.25$ at around the same time. The only difference is that less behaviour fluctuation occurs for bigger mesh size.

TABLE II
SIMULATED PREY FOR ROSENZWEIG-MACARTHUR MODEL FOR CASE 1

$h = 1.3$	$h = 4.6$
	
	

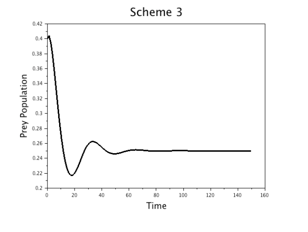
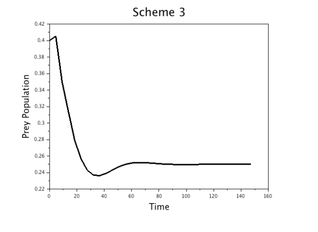
	

TABLE III
SIMULATED PREDATOR AND PREY INTERACTION FOR ROSENZWEIG-MACARTHUR MODEL FOR CASE 1

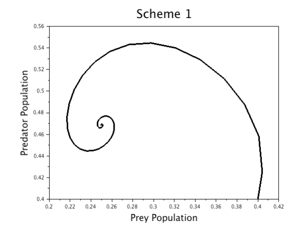
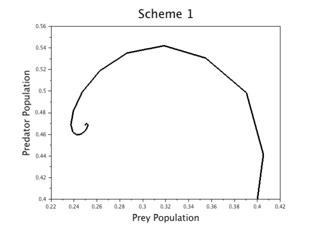
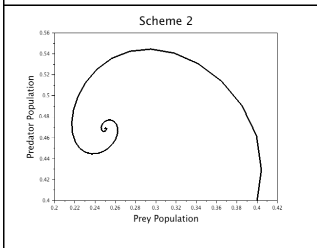
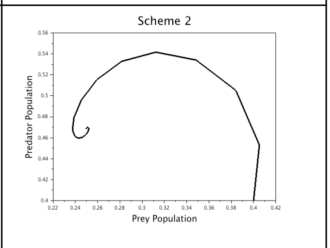
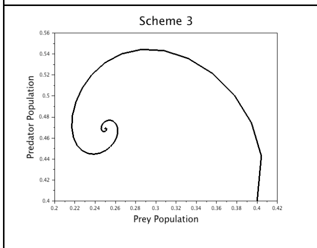
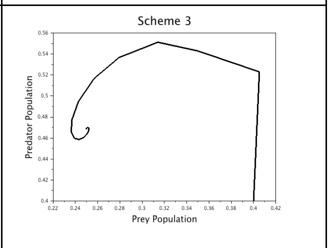
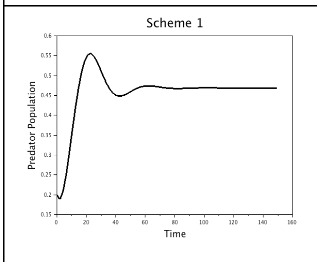
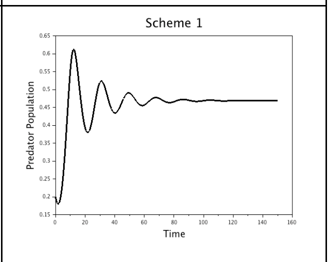
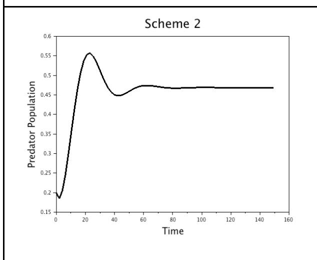
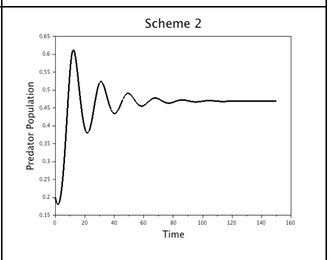
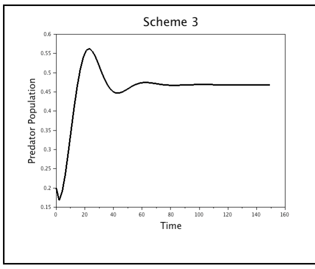
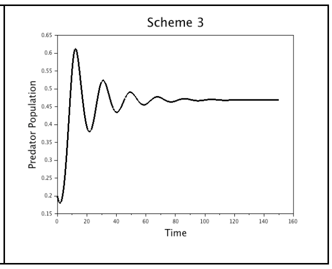
$h = 1.3$	$h = 4.6$
	
	
	

TABLE IV
SIMULATED PREDATORS FOR ROSENZWEIG-MACARTHUR MODEL FOR CASE 2

$h = 2.1$	$h = 0.001$
	
	
	

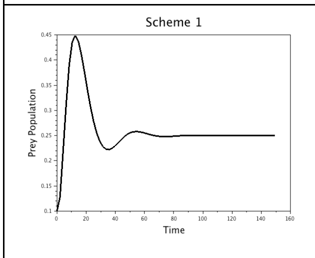
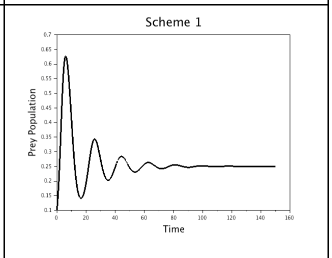
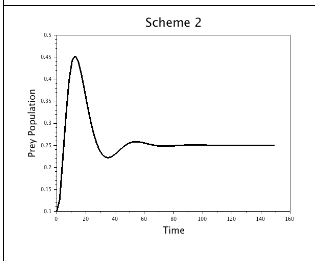
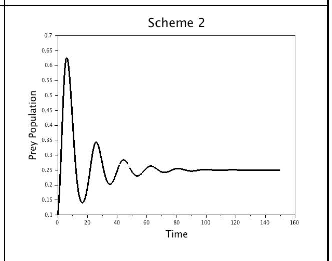
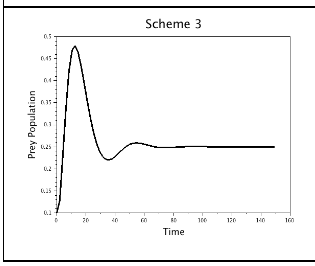
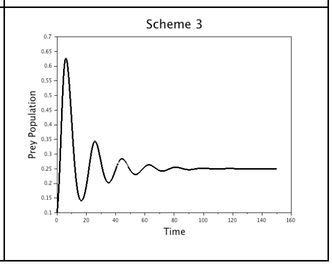
From Table 3, all SNST schemes for both mesh size values saturated at value $x = 0.25, y = 0.4687$ at around the same time. The only difference is that non-smoth behaviour and less circulary behaviour occurs for biger mesh size.

This finding is accurately simulating the analytic solution, which is $(0.25, 0.46875)$ with percentage relative error of 0.0107%.

From Table 4, all SNST schemes for both mesh size values saturated at value $y = 0.4688$ at around the same time. The only difference is that less behaviour fluctuation occurs for bigger mesh size.

From Table 5, all SNST schemes for both mesh size values saturated at value $x = 0.25$ at around the same time. The only difference is that less behaviour fluctuation occurs for bigger mesh size.

TABLE V
SIMULATED PREY FOR ROSENZWEIG-MACARTHUR MODEL FOR CASE 2

$h = 2.1$	$h = 0.001$
	
	
	

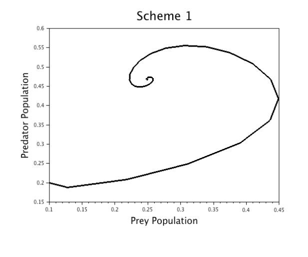
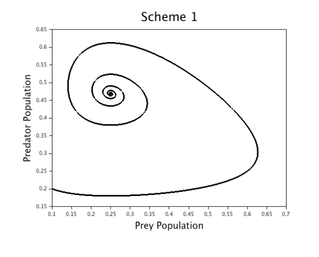
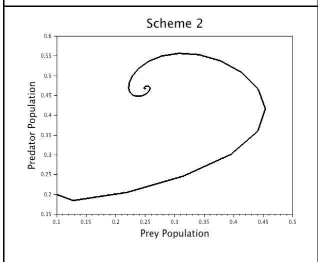
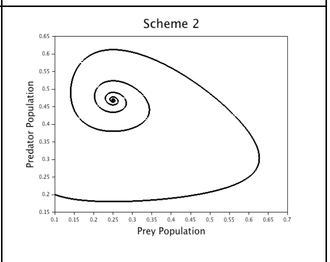
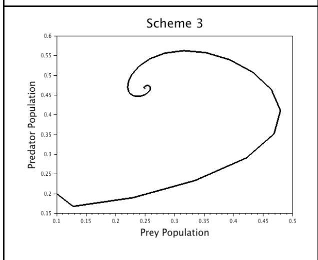
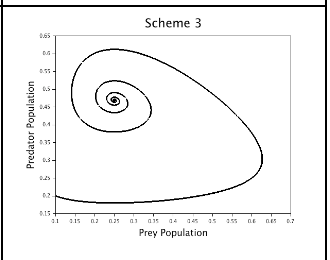
From Table 6, all SNST schemes for both mesh size values saturated at value $x = 0.25, y = 0.4688$ at around the same time. The only difference is that non-smoth behaviour and less circulary behaviour occurs for biger mesh size.

This finding is accurately simulating the analytic solution, which is $(0.25, 0.46875)$ with percentage relative error of 0.0107%.

Table 3 and 6 shows the interaction between predator and prey. Both tables show that prey population will increase if predator population decrease. This phenomena is as such because the decreasing of predator population will increase the possibilities of prey to stay alive. High ratio of prey to predator will increase the possibility of predator to consume prey, and again will increase the predator population. This explain why predator population will increase if the prey population increase. A continuous decrease in prey population will decrease the population of predator. Since, predator will be lack of food. This is the prime dying factor of predator. This phenomena follows exactly the food chain relationship between prey and predators.

All three SNST schemes precisely simulate the equilibrium point both cases. Mesh size used greatly impact the behavior of simulated interaction between both parties. Using high resolution of mesh size will increase the accuracy of the simulation.

TABLE VI
SIMULATED PREDATOR AND PREY INTERACTION FOR ROSENZWEIG-MACARTHUR MODEL FOR CASE 2

$h = 2.1$	$h = 0.001$
	
	
	

Thus, Table 4-6 clearly shows that using smaller mesh size simulate the interaction clearer than using bigger mesh size. This exhibited by more fluctuations predator and prey behaviour and more circular activity of interactions between prey and predator.

IV. CONCLUSION

Ordinary differential equations have been widely utilized in engineering, biology, medicine, economics, and wide range of areas. We proposed three new algorithms called Semi Non-Standard Trimean algorithms in this paper. We have shown that the ordinary differential solutions via three new numerical algorithms accurately simulate the equilibrium point of the interaction. It is always a good idea to implement simulation to verify the performance of these new algorithms. We also suggest varying the initial values when examining the accuracy of these algorithms. The algorithm shows almost similar behaviour between each other. Mesh size selection play an important role in simulating precise simulation behaviour. However, the equilibrium point is precisely approximate even though by using bigger mesh size. This is the main advantage of the proposed schemes.

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