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# Simulation of Internal Undular Bores Propagating over a Slowly Varying Region

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***Abstract*— Internal undular bores have been observed in many parts of the world. Studies have shown that many marine structures face danger and risk of destruction caused by internal undular bores due to the amount of energy it carries. This paper looks at the transformation of internal undular bore in two-layer fluid flow under the influence of variable topography. Thus, the surface of the bottom is considered to be slowly varying. The appropriate mathematical model is the variable-coefficient extended Korteweg-de Vries equation. We are particularly interested in looking at the transformation of KdV-type and table-top undular bore over the variable topography region. The governing equation is solved numerically using the method of lines, where the spatial derivatives are first discretised using finite difference approximation so that the partial differential equation becomes a system of ordinary differential equations which is then solved by 4th order Runge-Kutta method. Our numerical results show that the evolution of internal undular bore over different types of varying depths regions leads to a number of adiabatic and non-adiabatic effects. When the depth decreases slowly, a solitary wavetrain is observed at the front of the transformed internal undular bore. On the other hand, when the depth increases slowly, we observe the generation of step-like wave and weakly nonlinear trailing wavetrain, the occurrence of multi-phase behaviour, the generation of transformed undular bore of negative polarity and diminishing transformed undular bore depending on the nature of the topography after the variable topography.**

***Keywords*— Internal undular bores; extended KdV equation; method of lines; two-layer fluid system; solitary wavetrain.**

## I. INTRODUCTION

Internal undular bores have been observed propagating in coastal ocean in many parts of the world [1]. An undular bore refers to a flow of oscillatory fluid connecting two different constant depth of streams. These two streams are connected by a hydraulic jump and propagate in horizontal velocity and exhibiting a solitary wave as the leading wave [2], [3]. The occurrence of internal undular bore can be generated by transcritical flow over topography [4]. In past few decades, the trace of internal undular bore have been found in some stratified fluid areas around the world, e.g. continental shelf off Point Sal, California [5], Japan/East Sea shelf-coastal region [6] and Peter the Great Bay [7]. Similar to internal waves, the occurrence of these strongly nonlinear internal elevation waves packets or internal undular bores have been studied to know the impacts affect to the sea currents, fluid density, and temperature fields of the fluid [6], [8]. Moreover, their propagation in the fluid layers considerably affects the vertical mixing of nutrient contents, transmission of acoustic wave and dominate the direction of the fluid-flow under water [9]–[11].

The appropriate model to describe the nonlinear internal waves in a fluid system with stratification is the extended Korteweg–de Vries (eKdV) equation [12], [13]. This model is extended from the well-known Korteweg–de Vries (KdV) equation, which is widely used to describe the nonlinear shallow water waves. Here, internal undular bores are considered to be propagating in a system that consist of two layers of stratified fluid. Therefore, the KdV equation is not appropriate due to the difference of densities of both layers. When the difference is very small, the coefficient of nonlinearity term in the KdV equation is nearly vanished. Therefore, it is important to introduce an additional higher order nonlinear term in the KdV equation in order to raise up the nonlinearity effects for dynamic balancing with the dispersion effects [14], [15].

It is very important to study the behaviour of the internal waves as they pose danger to the buildings or structures near the coastal regions due to it carried large amount of energy

[16]. To prevent the coastal erosion, there are many coastal structures built to reduce the impact from internal waves for instances, concrete walls [17], stone revetments and tetra pods [18]. Furthermore, internal undular bores are evolving over different kinds of variable depth regions. The primary aim of this article is to look at how the variable topography influences the evolution of internal undular bores. In the next section, the problem formulation and the numerical method adopted in this study are discussed. Numerical results are presented in Section III and the final section would be our conclusion.

## II. Materials and Method

The mathematical model for nonlinear internal waves in stratified fluid over variable topography is the variable coefficient eKdV (veKdV) equation [19].

$$A_t + cA_x - \frac{cQ_x}{Q}A + \mu A A_x + \mu_1 A^2 A_x + \delta A_{xxx} = 0, \qquad (1)$$

where $A$ denotes the wave amplitude. $x$, and $t$ are the temporal and spatial variables respectively. Here, the speed of the linear long wave is represented by $c(x)$. $Q(x)$ represents the linear modification factor. It is defined such that $Q^{-2}A^2$ is the linear long wave action flux. There are three important coefficients i.e. $\mu(x)$, $\mu_1(x)$ denote the nonlinearity terms and $\delta(x)$ denotes the dispersive term. The coefficients of the veKdV equation, i.e. $\mu$, $\mu_1$, and $\delta$ are slowly varying functions of $x$ and are defined by

$$\mu = \frac{3c\left(\rho_2 H_1^2 - \rho_1 H_2^2\right)}{2H_1 H_2\left(\rho_2 H_1 + \rho_1 H_2\right)},$$

$$\mu_1 = \frac{-3c}{8\left(H_1^2 H_2^2\right)\left(\rho_2 H_1 + \rho_1 H_2\right)^2}\begin{bmatrix}\left(\rho_1 H_2^2 - \rho_2 H_1^2\right)^2 \\ +8\rho_1\rho_2 H_1 H_2\left(H_1 + H_2\right)^2\end{bmatrix},$$

$$\delta = \frac{cH_1 H_2\left(\rho_1 H_1 + \rho_2 H_2\right)}{6\left(\rho_2 H_1 + \rho_1 H_2\right)},$$

where

$$c = \sqrt{\frac{g\left(\rho_2 - \rho_1\right)H_1 H_2}{2\rho_1 H_2}}, \quad Q = \sqrt{\frac{1}{2g\left(\rho_2 - \rho_1\right)c}}.$$

The densities of the fluid for both layers are constants denoted by $\rho_1$ and $\rho_2$ respectively. $H_1$ and $H_2$ denote the depths for upper and lower layers respectively. We suppose that $H_1$ remains constant all the time and $H_2(x)$ changes monotonically varies from $h_0$ to $h_1$ in the region $x_0 < x \le x_1$.

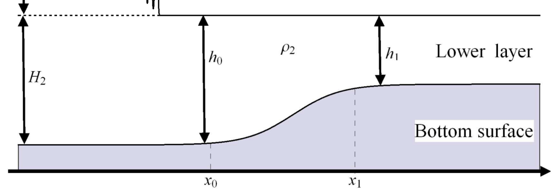



Fig. 1 Schematic illustration of an internal undular bore propagates over a varying slope

Here, we shall suppose $x_0 \ge 0$ and $x_1 - x_0 >> 1$. We will consider two kindss of variable region in the interval $x_0 < x \le x_1$:

- slowly increasing slope
- slowly decreasing slope

The schematic of our problem is illustrated in Fig. 1.

The first two terms in equation (1) are the dominant terms. Therefore, equation (1) can be transfromed using the following new variables [19],

$$A = QU, \qquad T = \int^x \frac{dx}{c}, \qquad X = c(T - t).$$

By substituting the new variables into the veKdV equation yields the following equation, to the same leading order of approximation where equation holds

$$U_T + \alpha U U_X + \beta U^2 U_X + \lambda U_{XXX} = 0, \qquad (2)$$

where

$$\alpha = Q\mu, \qquad \beta = Q^2\mu_1, \qquad \lambda = \delta.$$

In terms of the new variables after the tranformation, $\alpha$, $\beta$, and $\lambda$ are functions of $T$. For the depth profile, we consider $H_1 = 1.5$ is constant for all $T$ and $H_2$ varies according to

$$H_2(T) = \begin{cases} h_0 & : 0 \le T < T_0, \\ f(T) & : T_0 \le T < T_1, \\ h_1 & : T \ge T_1, \end{cases}$$

where $f(T)$ is a function in terms of $T$.

The oscillatory structure of the undular bore can be evolved from a simple unit step using Heaviside function. Here, we shall consider the initial condition for the veKdV equation (2) to be in the form of a sharp step,

$$U = U_0 P(-X).$$

Here, we let $U_0 > 0$ and $P$ is a Heaviside function to generate a hydraulic jump connects two different constant depth.

$$P(X) = \begin{cases} 1, & \text{if } X > 0, \\ 0, & \text{if } X < 0. \end{cases}$$

This study adopts the method of lines (MOL) in order to solve the veKdV equation (2) numerically. First, we make approximation to the spatial derivatives so that the governing equation (2) will be reduced into a set of ordinary differential equations (ODEs). Then, this system of ODEs can be solved by any time integrator. The MOL is widely used in solving many partial differential equations, e.g. the eKdV equation [20–21], KdV equation with forcing term [22], and forced KdVB equation [23]. To begin, we rewrite veKdV equation as follows

$$U_T = -\alpha U U_X - \beta U^2 U_X - \lambda U_{XXX}.$$

The spatial derivatives are discretized using central finite difference formulae as follows,

$$U_X \approx \frac{U_{j+1} + U_{j-1}}{2\Delta X},$$

$$U_{XXX} \approx \frac{U_{j+2} + U_{j+1} + U_{j-1} + U_{j-2}}{2(\Delta X)^2},$$

$j$ indicates the position on the $X$ axis. $\Delta X$ is the step-size for the spatial axis. Hence, the MOL approximation for the veKdV equation (2) is given by

$$\frac{\partial U_j}{\partial T} = -(\alpha U_j + \beta U_j^2)\frac{U_{j+1}+U_{j-1}}{2\Delta X} - \lambda \frac{U_{j+2}+U_{j+1}+U_{j-1}+U_{j-2}}{2(\Delta X)^2} = f(U_j).$$

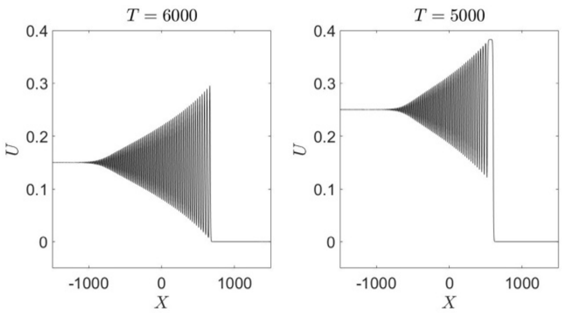
We apply the classical 4th order Runge-Kutta method to solve the time integration.

## III. Results and Discussion

In this section, we present the numerical results of internal undular bores evolving over two different kinds of slowly varying depth region, i.e. slowing increasing slope and slowing decreasing slope. In order to generate a fully developed undular bore in our problem, the initial condition of veKdV equation is taken as

$$U(X,T=0) = \frac{b}{2}\left(1-\tanh\left(\frac{X}{20}\right)\right), \tag{3}$$

where $b$ is the height of the sharp step. Here, we consider two values for $b$, i,e, $b$ = 0.15 and $b$ = 0.25 so that we have KdV-type solitary wave (see Fig. 2(a)) and a table-top solitary wave (see Fig. 2(b)) as the leading wave of internal undular bore. The depth of the top layer, $H_1$ = 1.5 while the bottom layer has depth $H_2$ = 1.0.




(a) (b)

Fig. 2 The structure of internal undular bore: (a) KdV-type internal undular bore where $b$ = 0.15; (b) Table-top internal undular bore where $b$ = 0.25

The KdV-type internal undular bore has amplitude of

$$U_{\lim 0} \approx 2b = 0.3,$$

at the leading edge. Due to the limiting amplitude, i.e.

$$U_{\lim 0} \approx \frac{-\alpha}{\beta} = 0.3827,$$

the lead wave of the internal undular bore is a table-top solitary. Our main concern on this paper is to see how the varying depth will affect the behaviour of the undular bore as it propagates over the slope.

### A. Slowly increasing slope

In this case, we assume the profile for the bottom layer varies as follows

$$H_2(T) = \begin{cases} 1.0 & : 0 \le T < 100, \\ -0.00006T + 1.006 & : 100 \le T < 5100, \\ 0.7 & : T \ge 5100. \end{cases}$$

When $b$ = 0.15, our numerical result shows that the solitary wave at the leading edge deforms adiabatically and behaves like an isolated solitary wave as it evolves through the variable topography region and thus a solitary wavetrain is generated. On the new region with constant depth, the undular bore retains its structure, i.e. a slowly modulated nonlinear periodic wavetrain. The depth variation does not affect jump across the undular bore after the slope (see Fig. 3). The generation of solitary wavetrain has been observed as well when a surface undular bore evolves over a slowly increasing depth region. Also, we observed that there is an occurrence of multi-phase behaviour during the evolution process of the internal undular bore. The multi-phase interaction continues for quite some time and it diminishes after the transformed bore has settled down on the new constant region. The amplitude of the lead wave of the transformed bore remains the same as in the initial undular bore.

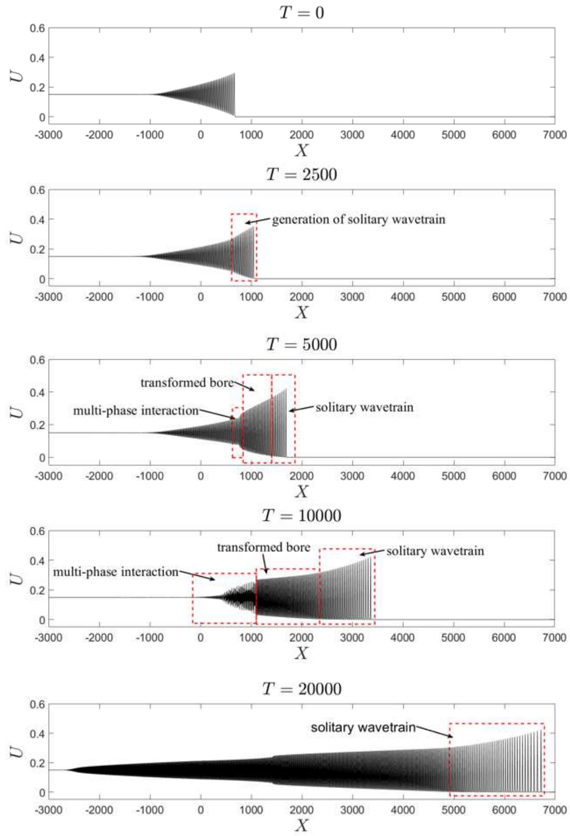



Fig. 3 2D plot of an undular bore with a leading KdV-type solitary wave at the leading edge evolving over a slowly increasing slope

For table-top internal undular bore, i.e. when $b$ = 0.25, the leading table-top solitary wave also behaves like an isolated solitary wave such that it deforms adiabatically and reaches a new amplitude limit, i.e. $U_{\lim 1}$ = 0.5381 after the slope. There is no solitary wavetrain generation in this case because the amplitude of the lead solitary wave has hit the limiting value throughout the entire evolution. Similarly, we observe multi-phase behaviour during the entire evolution of the internal undular bore. Fig. 4 shows the evolution of the table-top undular bore over the slowly increasing slope region.

### B. Slowly decreasing slope

For the case where the slope is decreasing slowly, there are three cases of the bottom layer to be considered here, i.e.

- $h_1 < H_1$,
- $h_1 = H_1$,
- $h_1 > H_1$.

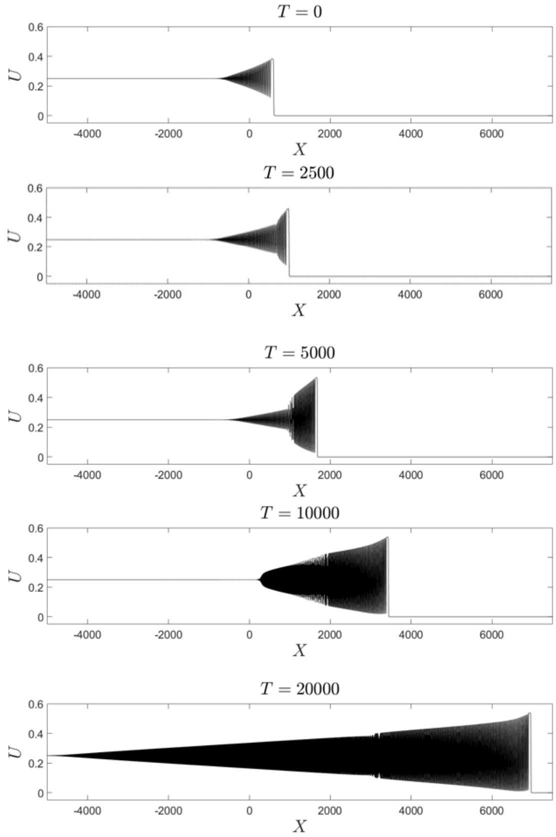


Fig. 4 2D plot of an undular bore with a leading table-top solitary wave evolving over a slowly increasing slope

*1)* $h_1 < H_1$:

Here, depth profile of the bottom layer is assumed to vary according to

$$H_2(T) = \begin{cases} 1.0 & : 0 \le T < 100, \\ 0.00006T + 0.994 & : 100 \le T < 5100, \\ 1.3 & : T \ge 5100. \end{cases}$$

When $b$ = 0.15, one can observe the amplitude of the leading wave is decreasing as it enter the increasing depth region (see Fig. 5 $T$ = 2500 and $T$ = 5000). Therefore, there is no series of solitary wave is generated ahead of the transformed bore. However, the interaction between the leading wave and the nonlinear wavetrain behind it will prevent the amplitude continues to decrease. Instead it will cause the leading wave to grow and thus the leading wave amplitude will increase. This is clearly shown in Fig. 5 at $T$ = 10000. On the new area with constant depth at large time, the transformed bore consists of two distinct wave structures, i.e. a new undular bore at the front and weakly nonlinear wave structure which is a part of the initial internal undular bore at the rear part of the transformed bore.

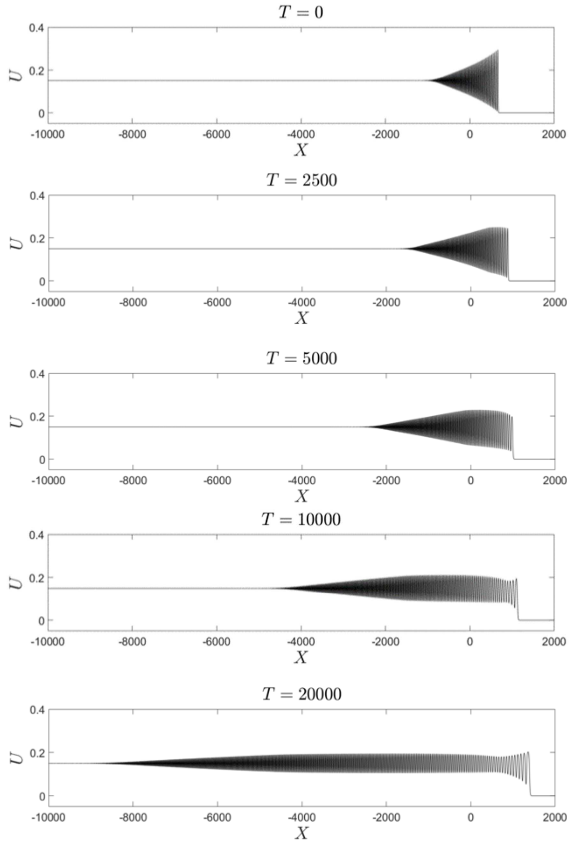


Fig. 5 2D plot of an undular bore with a leading KdV-type solitary wave at the leading edge propagating over a slowly decreasing slope where $h_1 < H_1$

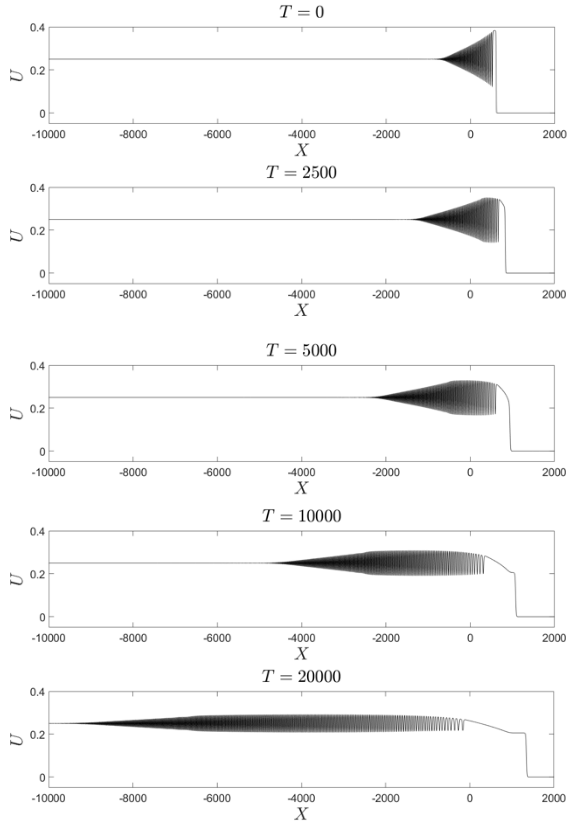


Fig. 6 2D plot of an undular bore with a leading table-top solitary wave at the leading edge propagating over a slowly decreasing slope where $h_1 < H_1$

When $b$ = 0.25, the new limiting amplitude after the slope is

$$U_{\lim 1} \approx 0.2057\,,$$

which is smaller than the jump across the undular bore, i.e. $b$ = 0.25. Therefore, we do not observe nonlocal interaction at the leading edge. Instead, the leading wave amplitude decreases and reaches the new limiting amplitude value as it enters the decreasing slope region.

Thus, instead of growing leading solitary wave, we observe the formation of a step-like wave propagating over time. The whole structure of the initial undular bore is slowly destroyed as time increases. Fig. 6 shows the evolution of the internal undular bore over a slowly decreasing slope region.

*2)* $h_1 = H_1$:

Next, we suppose that the depth of bottom layer after the slope is equivalent to the depth of top layer.

$$H_2(T) = \begin{cases} 1.0 & : 0 \le T < 100, \\ 0.0001T + 0.99 & : 100 \le T < 5100, \\ 1.5 & : T \ge 5100. \end{cases}$$

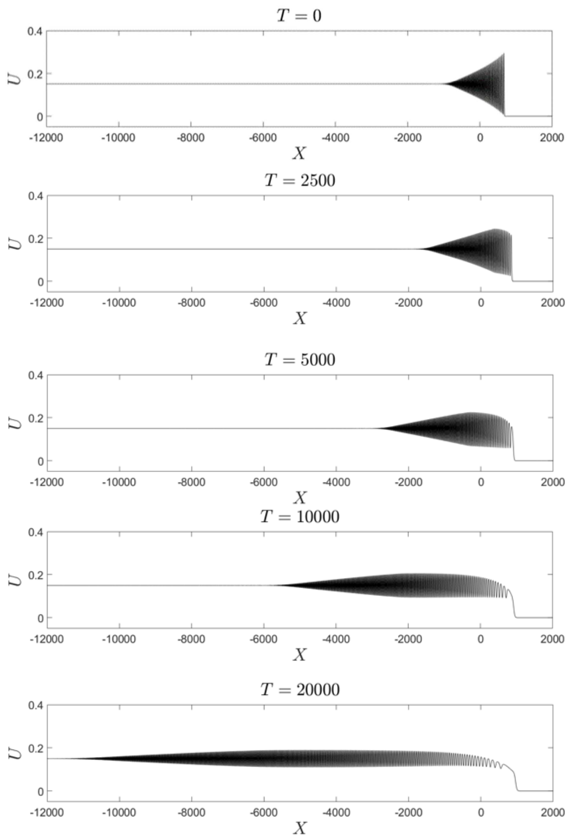



Fig. 7 2D plot of an undular bore with a leading KdV-type solitary wave at the leading edge propagating over a slowly decreasing slope where $h_1 = H_1$

In this scenario, we observe that for both table-top undular bore and also KdV-type undular bore, the leading wave deforms adiabatically, and its amplitude decreases as it enters the variable topography region. However, the leading wave does not interact with the nonlinear wavetrain at the rear part of the undular bore due to the new limiting amplitude after the slope, i.e. $U_{\lim 1} \approx 0.0855$. The new limiting amplitude is smaller than the jump across the bore for both types of undular bore, i.e. $b$ = 0.15 and $b$ = 0.25. At large time, the structure of these two internal undular bores is diminishing. Fig. 7 and Fig. 8 show the evolution of KdV-type undular bore and table-top undular bore over the slowly decreasing slope region.

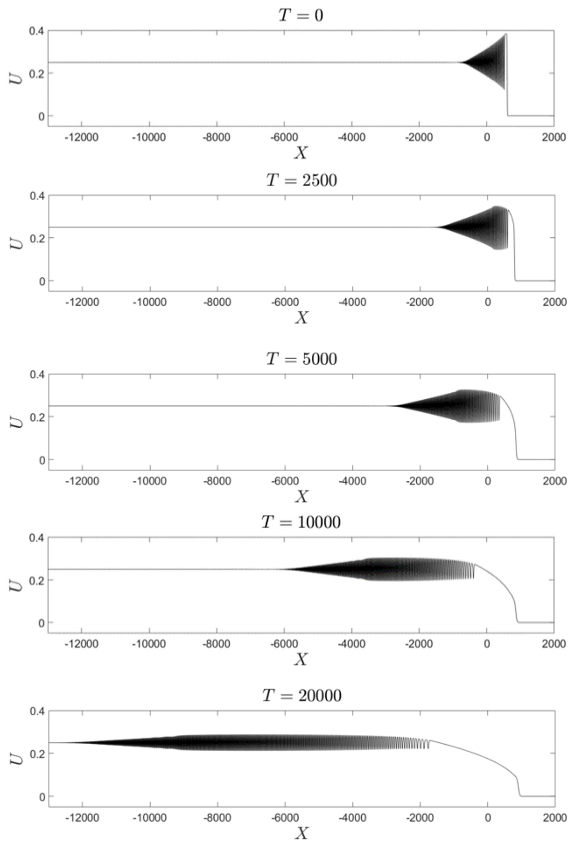



Fig. 8 2D plot of an undular bore with a leading table-top solitary wave at the leading edge propagating over a slowly decreasing slope where $h_1 = H_1$

*3)* $h_1 > H_1$:

Lastly, we suppose that the depth of bottom layer after slope is greater than the depth of top layer. Hence, the profile for $H_2(T)$ is given by

$$H_2(T) = \begin{cases} 1.0 & : 0 \le T < 100, \\ 0.00014T + 0.986 & : 100 \le T < 5100, \\ 1.7 & : T \ge 5100. \end{cases}$$

In this case, the transformation of internal undular bore involves polarity change for both cases, i.e. $b$ = 0.15 and $b$ = 0.25 when they propagate into the region where the deepness of bottom layer after the slope is greater than the deepness of top layer. The polarity of the internal undular bore is determined by sign of the coefficient $\alpha$ of the nonlinearity term in veKdV equation. In this case, the polarity of the internal undular bore changes from positive to negative. As the initial internal undular bore evolves over the slowly decreasing slope region, the amplitude of the leading wave decreases. Once the polarity has changed, an internal undular bore of depression is generated. We can observe the transformed internal undular bore is riding a positive pedestal. As time increases, the transformed bore of negative polarity is slowly diminishing due to the pedestal. One could observe that there is no change to the jump across the transformed bore. These can be observed clearly through the 2D plots of the numerical simulation of internal undular bore in the propagation over slowly increasing slope region where $h_0 > H_1$ in Fig. 9 for $b = 0.15$ and Fig. 10 for $b = 0.25$.

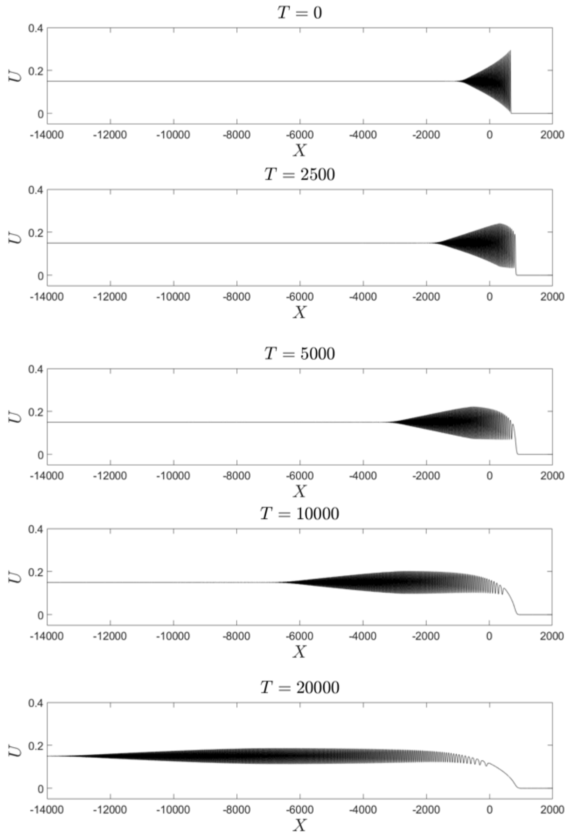



Fig. 9 2D plot of an undular bore with a leading KdV-type solitary wave at the leading edge evolving over a slowly decreasing slope where $h_1 > H_1$

## IV. Conclusion

We have discussed the transformation of internal undular bores over a slowly varying regions in a fluid system that consists of two layers of fluids in the framework of veKdV equation. When the depth of the bottom layer decreases slowly, the leading wave of the initial bore changes its form adiabatically and a non-adiabatically respond is generated in the form of a solitary wavetrain in front of the transformed bore. The long-time behaviour shows that the transformed bore consists a series of solitary waves at the front followed by the transformed undular bore. However, no formation of solitary wavetrain is observed for table-top undular bore. For the slowly decreasing slope case where the depth of bottom layer fluid after the slope is smaller than the depth of top layer fluid, we observe the generation of either weakly nonlinear trailing wavetrain behind the transformed bore for the KdV-type undular bore and the generation of a step-like wave for table-top undular bore. When the depth of the bottom layer after the slope is equivalent to the depth of the top layer, we observe the diminishing initial undular bore due to the new limiting amplitude value after the slope for both KdV-type and table-top undular bores. For the case where the depth of the bottom layer is greater than the depth of the top layer, a transformed bore of negative polarity riding on a positive pedestal is observed. The transformed bore is also diminishing as time increases.

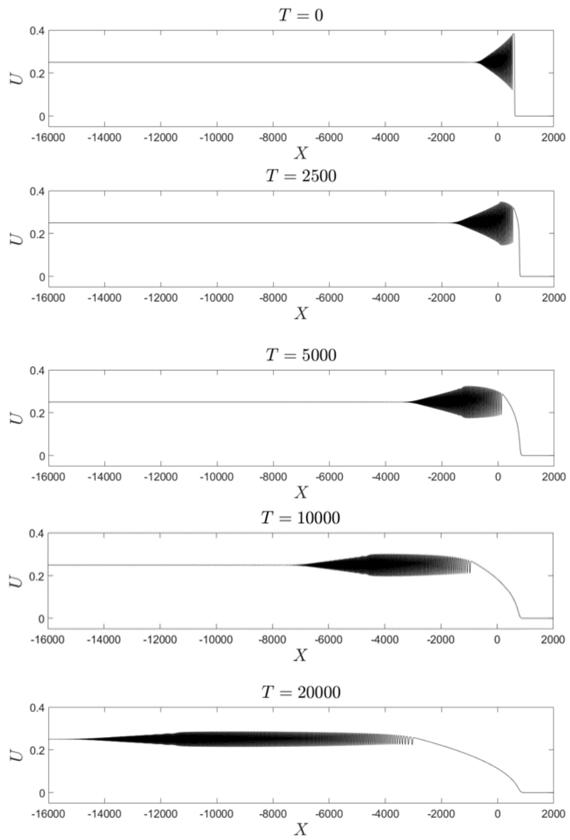



Fig. 10 2D plot of an undular bore with a leading table-top solitary wave at the leading edge propagating over a slowly decreasing slope where $h_1 > H_1$

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