International Journal on Advanced Science Engineering Information Technology
Vol.6 (2016) No. 4
ISSN: 2088-5334

A Time-Critical Investigation of Parameter Tuning in Differential Evolution for Non-Linear Global Optimization

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***Abstract*— Parameter searching is one of the most important aspects in getting favorable results in optimization problems. It is even more important if the optimization problems are limited by time constraints. In a limited time constraint problems, it is crucial for any algorithms to get the best results or near-optimum results. In a previous study, Differential Evolution (DE) has been found as one of the best performing algorithms under time constraints. As this has help in answering which algorithm that yields results that are near-optimum under a limited time constraint. Hence to further enhance the performance of DE under time constraint evaluation, a throughout parameter searching for population size, mutation constant and f constant have been carried out. CEC 2015 Global Optimization Competition's 15 scalable test problems are used as test suite for this study. In the previous study the same test suits has been used and the results from DE will be use as the benchmark for this study since it shows the best results among the previous tested algorithms. Eight different populations size are used and they are 10, 30, 50, 100, 150, 200, 300, and 500. Each of these populations size will run with mutation constant of 0.1 until 0.9 and from 0.1 until 0.9. It was found that population size 100, Cr = 0.9, F=0.5 outperform the benchmark results. It is also observed from the results that good higher Cr around 0.8 and 0.9 with low F around 0.3 to 0.4 yields good results for DE under time constraints evaluation**

***Keywords*— evolutionary optimization; time-limited optimization; DE; expensive optimization problems; parameter searching**

I. INTRODUCTION

Optimization problems are mostly evaluated by using number of evaluation budget. Within the given number of evaluations, an algorithm has to solve the optimizations problems without taking the amount of time used into consideration. Top conferences such as GECCO and CEC were among the platform use by researcher to show their works done on solving and finding the best solutions for the given test problems or on a particular optimization problems. CEC test suites only focus on finding best solutions without taking time consideration into account. CEC 2014 introduce a competition of real-parameter single objective expensive optimization that focus on achieving the optimum solution although it was called an expensive optimization competition, their focus was on the solutions provided by the algorithms with more dimensions to be solved. The organizers also allows participant to implement surrogates-model to aid their algorithms. Some of the works exhibited in GECCO 2010 are Zhou and Tan [1] who presented their work on PSO with triggered mutation, Chen [2] presented PSO with self-adjusting neighbors. Hildebrandt [3] presented the usage of GP in solving the complex shop floor scenarios. Similar to CEC conferences, the main focus of the papers presented is to solve optimization problems by providing the best solutions no matter how much time is taken.

Estimation or approximating the fitness is one of the method used by researcher to try and solved the problem face in expensive optimization problems. Instance-based learning method, machine learning method and statistical learning method are three popular method used in fitness approximation. Instance-based method entails transforming the original functions to linear ones, and then using a linear programming technique, such as the Frank-Wolfe method [4] or Powell's quadratic approximation [5]. In machine learning, the techniques available are Clustering, Multilayer Perception Neural Networks and decision tree. Statistical Learning methods for fitness approximation (basically statistical learning models) as applied to EAs have gained much interest among researchers, and have been used in several successful GA packages. In these methods, single or multiple models are built during the optimization process to approximate the original fitness function. These models are also referred to as approximate models, surrogates or meta-models. Among these models, Polynomial Models, Kriging Models, and Support Vector Machines (SVM) are the most commonly used. Although fitness approximation were able to decrease the time of convergence, the question of which algorithms performs the best is a given critical time frame left unanswered. Likewise the focus of fitness approximation is to achieve best solution faster.

Researches that focus on stopping criteria [6], [7], [8] focus on how to stop the optimization process when the solutions reached optimum results. Conventional optimization process use number of evaluations as the termination criteria but it is not practical as the concern of these researches is to save cost and time in real world and expensive optimization problems. Some of the suggestion mentions in these researches are to compare other algorithms with the stopping criteria mention. But still the question of how and what is the performance of PSO, DE and SEA algorithms in a given time frame optimizations problems are not answer.

In expensive optimization problems, researcher address the problems of limited resources and time in running the large number of evaluations in order to obtain the best solutions. Chen [9] used PSO aided MIMO in transceiver design in order to obtain to the best solutions and at the same time lower the computational complexity and time complexity. Vasile and Croisard [10] tackle the space mission design in their work. The main focus of their work is to reduce the time take to compute the space mission design under uncertainty. Researcher work on engineering problems [11], network design [12], word analysis [13], digital circuits[14] all these real-world expensive optimization applications focus on reducing the complexity of the optimizations process. It can be observed that reducing time taken to obtain best solution were the focus of these researchers. This shows how important time is in real world applications. It is crucial to obtain solutions as fast as possible where expensive resources are involved. Yet if the questions of which algorithms that can produce ideal solutions in a given short time frame cannot be answer even though it is observe that time plays an important aspect in real-world optimization problems.

II. Method

In CEC 2015, a competition on expensive optimization problems were organized. The benchmark problems used in the competition are used in this study. It comprises from *f*1 to *f*15 benchmark optimization problems as shown in Table I.

In Table II, the results for the previous study [15] are shown. These results are used as benchmark in the following results. The settings for the previous DE are as follow:

- population size 100, Cr = .9, F = .2,

The benchmark results allow us to have a measurement of how each parameter setting is performing.

TABLE I.
SUMMARY OF CEC 2015 EXPENSIVE OPTIMIZATION PROBLEMS

No.	Function	Fi*
1	Rotated Bent Cigar Function	100
2	Rotated Discus Function	200
3	Shifted and Rotated Weierstrass Function	300
4	Shifted and Rotated Schwefel's Function	400
5	Shifted and Rotated Katsuura Function	500
6	Shifted and Rotated HappyCat Function	600
7	Shifted and Rotated HGBat Function	700
8	Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	800
9	Shifted and Rotated Expanded Scaffer's F6 Function	900
10	Hybrid Function 1 (N=3)	1000
11	Hybrid Function 2 (N=4)	1100
12	Hybrid Function 3 (N=5)	1200
13	Composition Function 1 (N=5)	1300
14	Composition Function 2 (N=3)	1400
15	Composition Function 3 (N=5)	1500

TABLE II.
DE RESULTS FOR PREVIOUS STUDY

F	DE
1	8.68E-05
2	0.00E+00
3	1.10E+01
4	9.32E+02
5	1.83E+00
6	9.65E-02
7	8.01E-02
8	1.56E+00
9	3.01E+00
10	1.58E+01
11	5.95E+00
12	4.88E+01
13	3.16E+02
14	1.98E+02
15	4.87E+02

III. Experiment Setup

For this experiment a time threshold is set to 300 milliseconds and once this threshold is reached the algorithm have to stop immediately and the best solution up to that moment are saved. The number of evaluations done in 300 milliseconds was recorded as well, in order to know how many evaluations can be done using different parameter under 300 milliseconds. There will be eight different population size and they are 10, 30, 50, 100, 150, 200, 300 and 500. Each of these population sizes will be run using different mutation and *f* constants. The mutation and *f* constants are as follow:

- Cr = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9
- *F*= 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9

IV. Experiment Results

Due to the large amount of tables results obtained, only the best set of parameter from each population are shown in the Appendix section. In population size of 10, all the results obtained performs much worse that the benchmark results. Form all of the parameter, Cr=0.5 F=0.7 performs the best within population size of 10. Hence in Table III, the overall results for Cr=0.5 are shown and the best Cr=0.5 F=0.7 are shown in Table IV together with the percentage of change from the benchmark results. Since the test suite is a minimization problems, negative percentage highlighted in red shows that the current results are actually performing better than the benchmark results. The overall change shown in Table IV indicates the overall changes of the fitness against the benchmark results. Cr=0.5 F=0.7 overall changes is 561.23%, although it is very high but this figure is the lowest in population size 10. In population size of 30, Cr=0.8 F=0.6 performs the best within population size of 30. Hence in Table V, the overall results for Cr=0.8 are shown and the best Cr=0.8 F=0.6 are shown in Table VI together with the percentage of change from the benchmark results. Cr=0.5 F=0.7 overall changes is -51.47%,

In population size of 50, Cr=0.9 F=0.6 performs the best within population size of 50. Hence in Table VII, the overall results for Cr=0.9 are shown and the best Cr=0.9 F=0.6 are shown in Table VIII together with the percentage of change from the benchmark results. Cr=0.5 F=0.7 overall changes is -58.86%, only *f* number 6, 7 and 10 results is worse than the benchmark results while other *f* number performs better in Cr=0.5 F=0.7. In population size of 100, Cr=0.8 F=0.4 performs the best within population size of 100. Hence in Table IX, the overall results for Cr=0.8 are shown and the best Cr=0.8 F=0.4 are shown in Table X together with the percentage of change from the benchmark results. Cr=0.8 F=0.4 overall changes is -34.73%, only *f* number 1, 2 and 10 results are worse than the benchmark results while other *f* number performs better in Cr=0.8 F=0.4. For population size of 150, Cr=0.8 F=0.4 performs the best within population size of 150. Hence in Table XI, the overall results for Cr=0.8 are shown and the best Cr=0.8 F=0.4 are shown in Table XII together with the percentage of change from the benchmark results. Cr=0.8 F=0.4 overall changes is -35.18% and only *f* number 1, 2 ,10 and 12 results are worse than the benchmark results while other *f* number performs better in Cr=0.8 F=0.4.

In population size of 200, Cr=0.9 F=0.4 performs the best within population size of 200. Hence in Table XIII, the overall results for Cr=0.9 are shown and the best Cr=0.9 F=0.4 are shown in Table XIV together with the percentage of change from the benchmark results. Cr=0.9 F=0.4 overall changes is -30.28%.

While population size of 300, Cr=0.9 F=0.3 performs the best within population size of 300. Hence in Table XV, the overall results for Cr=0.9 are shown and the best Cr=0.9 F=0.3 are shown in Table XVI together with the percentage of change from the benchmark results. Cr=0.9 F=0.3 overall changes is -29.69%. In population size of 500, Cr=0.9 F=0.3 performs the best within population size of 500. Hence in Table XVII, the overall results for Cr=0.9 are shown and the best Cr=0.9 F=0.3 are shown in Table XVIII together with the percentage of change from the benchmark results. Cr=0.9 F=0.3 overall changes is -32.08%.

From all the results obtained, population size 50 Cr=0.5 F=0.7 has the best overall changes which is -58.86% but it did not perform better for function number 6,7 and 10. While in Table XIX population size 100, Cr = 0.9, F=0.5, manage to outperform benchmark results in all of the functions although with only overall changes of -32.42%. Hence it can concluded that population size 100, Cr = 0.9, F=0.5 is the ideal parameter settings for DE under time evaluation constraints.

V. Conclusions and Future Works

It is observed that with high Cr and lower F, it can yield better results for DE under time constraints evaluations. The sweet spots for Cr are 0.8 and 0.9 and F is 0.3 to 0.5. From the results obtain it is observed that population size 50, 100 and 150 yields much better results that the other population size. For future works, different variant of DE should be study under time constraint evaluations. In hopes that with different variant of DE, the performance and results obtained under time constraint evaluations can improve to near optimum solutions.

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VI. Appendix

TABLE III.
Overall Results for population size 10, Cr = 0.5

F/ *f*	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2.52E+09	1.02E+09	2.89E+08	1.18E+08	2.22E+07	1.99E+05	9.22E+03	9.01E+03	1.98E+04
2	2.36E+04	1.65E+04	1.09E+04	7.64E+03	2.68E+03	2.69E+01	4.33E-01	1.90E+01	1.39E+02
3	7.31E+00	5.90E+00	6.14E+00	7.41E+00	9.46E+00	1.08E+01	1.15E+01	1.12E+01	1.15E+01
4	3.37E+02	2.04E+02	9.43E+01	8.38E+01	4.89E+01	6.33E+01	5.67E+01	4.28E+01	4.45E+01
5	1.95E+00	1.90E+00	1.90E+00	1.99E+00	2.00E+00	1.98E+00	1.93E+00	1.95E+00	1.87E+00
6	2.39E+00	1.05E+00	6.08E-01	2.69E-01	1.36E-01	9.66E-02	1.09E-01	1.28E-01	1.50E-01
7	2.21E+01	9.00E+00	5.30E+00	8.97E-01	1.35E-01	9.48E-02	6.50E-02	6.76E-02	7.61E-02
8	2.16E+03	4.85E+02	5.57E+01	1.58E+00	1.31E+00	1.41E+00	1.65E+00	1.78E+00	1.85E+00
9	3.02E+00	2.70E+00	2.10E+00	2.15E+00	2.26E+00	2.83E+00	3.20E+00	3.49E+00	3.72E+00
10	1.54E+05	7.80E+04	1.02E+04	2.68E+03	1.37E+03	1.54E+03	2.88E+03	5.46E+03	1.37E+04
11	7.58E+00	7.02E+00	4.59E+00	4.59E+00	4.75E+00	6.54E+00	7.77E+00	9.71E+00	1.29E+01
12	1.21E+02	9.34E+01	7.28E+01	6.71E+01	7.02E+01	7.73E+01	1.10E+02	1.73E+02	2.06E+02
13	3.78E+02	3.38E+02	3.29E+02	3.19E+02	3.16E+02	3.15E+02	3.19E+02	3.26E+02	3.26E+02
14	2.05E+02	2.00E+02	1.98E+02	1.98E+02	1.98E+02	1.99E+02	2.01E+02	2.01E+02	2.01E+02
15	4.60E+02	4.23E+02	3.99E+02	4.44E+02	4.78E+02	5.66E+02	5.66E+02	5.85E+02	5.91E+02

TABLE IV.
Results for population size 10, Cr = 0.5, 0.7

f	Fitness	Percentage of Change
1	9.22E+03	>1000.00%
2	4.33E-01	43.33%
3	1.15E+01	3.84%
4	5.67E+01	-93.91%
5	1.93E+00	5.34%
6	1.09E-01	13.34%
7	6.50E-02	-18.80%
8	1.65E+00	5.38%
9	3.20E+00	6.52%
10	2.88E+03	>1000.00%%
11	7.77E+00	30.73%
12	1.10E+02	124.94%
13	3.19E+02	0.94%
14	2.01E+02	1.15%
15	5.66E+02	16.10%
	Overall Change	561.23%

TABLE V.
OVERALL RESULTS FOR POPULATION SIZE 30 CR=0.8

FR/ *f*	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2.74E+08	3.34E+07	1.00E+06	9.56E+03	8.53E+03	2.81E+01	1.47E-05	1.56E-01	2.35E+02
2	1.62E+04	9.06E+03	6.34E+03	2.65E+03	4.61E+00	6.48E-09	6.02E-09	5.56E-09	5.60E-09
3	4.87E+00	4.68E+00	5.47E+00	6.49E+00	7.25E+00	7.67E+00	7.71E+00	7.69E+00	7.81E+00
4	5.04E+01	1.87E+01	9.05E+00	1.34E+01	2.10E+01	5.23E+01	1.02E+02	5.67E+01	1.39E+02
5	1.30E+00	1.32E+00	1.31E+00	1.28E+00	1.24E+00	1.27E+00	1.22E+00	1.26E+00	1.27E+00
6	7.79E-01	1.58E-01	6.48E-02	5.56E-02	6.64E-02	7.49E-02	8.95E-02	9.87E-02	1.20E-01
7	3.48E+00	4.66E-01	1.97E-01	1.05E-01	7.29E-02	8.14E-02	7.29E-02	7.03E-02	6.91E-02
8	1.25E+01	1.03E+00	6.80E-01	6.02E-01	8.25E-01	1.09E+00	1.14E+00	1.23E+00	1.27E+00
9	1.86E+00	1.52E+00	1.15E+00	1.25E+00	1.61E+00	2.17E+00	2.32E+00	2.22E+00	2.31E+00
10	6.81E+04	1.52E+04	1.22E+03	1.47E+03	1.64E+02	1.33E+02	1.25E+02	1.54E+02	2.18E+02
11	3.38E+00	2.61E+00	2.23E+00	2.15E+00	2.81E+00	3.81E+00	5.06E+00	6.20E+00	8.30E+00
12	5.52E+01	2.99E+01	2.49E+01	1.87E+01	1.84E+01	3.41E+01	4.27E+01	7.88E+01	1.01E+02
13	2.22E+02	2.14E+02	2.11E+02	2.10E+02	2.10E+02	2.15E+02	2.14E+02	2.10E+02	2.11E+02
14	1.35E+02	1.31E+02	1.31E+02	1.33E+02	1.33E+02	1.33E+02	1.33E+02	1.33E+02	1.33E+02
15	2.17E+02	2.45E+02	2.35E+02	2.99E+02	3.55E+02	3.69E+02	3.79E+02	3.90E+02	3.96E+02

TABLE VI.
RESULTS FOR POPULATION SIZE 30, CR = 0.8, F=0.6

f	Fitness	Percentage of Change
1	2.81E+01	>1000.00%%
2	6.48E-09	0.00%
3	7.67E+00	-30.54%
4	5.23E+01	-94.39%
5	1.27E+00	-30.55%
6	7.49E-02	-22.40%
7	8.14E-02	1.68%
8	1.09E+00	-30.24%
9	2.17E+00	-27.67%
10	1.33E+02	742.89%
11	3.81E+00	-35.93%
12	3.41E+01	-30.08%
13	2.15E+02	-31.77%
14	1.33E+02	-32.76%
15	3.69E+02	-24.30%
	Overall Changes	-51.47%

TABLE VII.
OVERALL RESULTS FOR POPULATION SIZE 50 CR=0.9

FR/ *f*	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1	2.63E+08	1.58E+07	9.12E+05	7.96E+03	9.25E+02	5.30E-09	5.51E-09	7.77E-03	4.00E+02
2	1.27E+04	7.67E+03	3.89E+03	7.31E+02	1.51E+00	5.14E-09	4.96E-09	5.40E-09	5.58E-09
3	5.32E+00	5.39E+00	5.98E+00	6.79E+00	7.22E+00	7.61E+00	7.64E+00	7.62E+00	7.83E+00
4	3.81E+01	1.78E+01	1.16E+01	1.88E+01	4.80E+01	5.87E+01	1.41E+02	1.30E+02	1.38E+02
5	1.18E+00	1.26E+00	1.25E+00	1.19E+00	1.21E+00	1.28E+00	1.23E+00	1.24E+00	1.28E+00
6	4.23E-01	8.66E-02	4.28E-02	6.47E-02	7.65E-02	1.02E-01	8.07E-02	8.50E-02	1.15E-01
7	1.71E+00	2.98E-01	1.65E-01	1.18E-01	9.88E-02	8.89E-02	8.15E-02	9.03E-02	9.37E-02
8	1.51E+01	7.90E-01	5.78E-01	5.81E-01	9.22E-01	1.04E+00	1.22E+00	1.28E+00	1.06E+00
9	1.81E+00	1.48E+00	1.20E+00	1.44E+00	1.87E+00	1.91E+00	2.06E+00	2.31E+00	2.00E+00
10	5.97E+04	5.61E+03	3.93E+02	1.07E+02	4.63E+01	1.76E+01	3.45E+01	7.32E+01	1.04E+02
11	3.89E+00	2.35E+00	1.89E+00	2.03E+00	2.68E+00	3.32E+00	4.22E+00	5.06E+00	7.04E+00
12	5.70E+01	2.32E+01	1.87E+01	1.51E+01	1.51E+01	2.17E+01	3.21E+01	4.85E+01	7.28E+01
13	2.22E+02	2.13E+02	2.10E+02	2.10E+02	2.10E+02	2.10E+02	2.10E+02	2.10E+02	2.12E+02
14	1.33E+02	1.30E+02	1.33E+02	1.34E+02	1.33E+02	1.33E+02	1.32E+02	1.33E+02	1.33E+02
15	2.64E+02	2.50E+02	2.59E+02	3.09E+02	3.43E+02	3.75E+02	3.73E+02	3.78E+02	3.85E+02

TABLE VIII.
RESULTS FOR POPULATION SIZE 50, CR = 0.9, F=0.6

f	**Fitness**	**Percentage of Change**
1	5.30E-09	-99.99%
2	5.14E-09	0.00%
3	7.61E+00	-31.10%
4	5.87E+01	-93.70%
5	1.28E+00	-29.84%
6	1.02E-01	5.40%
7	8.89E-02	11.05%
8	1.04E+00	-33.20%
9	1.91E+00	-36.51%
10	1.76E+01	11.77%
11	3.32E+00	-44.19%
12	2.17E+01	-55.60%
13	2.10E+02	-33.35%
14	1.33E+02	-32.99%
15	3.75E+02	-23.01%
	Overall Changes	-58.86%

TABLE IX.
OVERALL RESULTS FOR POPULATION SIZE 100 CR=0.8

FR/ *f*	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	8.02E+05	7.39E+03	4.89E+03	1.50E+02	2.53E+00	3.34E+03	2.78E+05	8.55E+06	6.65E+07
2	5.44E+03	1.91E+03	1.58E+02	1.61E-05	5.45E-09	5.39E-09	2.82E-04	1.33E+00	2.27E+02
3	6.77E+00	6.93E+00	6.90E+00	7.10E+00	7.30E+00	7.70E+00	7.63E+00	7.58E+00	7.68E+00
4	1.72E+00	9.24E-01	5.74E+01	3.80E+02	5.15E+02	6.76E+02	7.40E+02	8.52E+02	9.25E+02
5	1.25E+00	1.21E+00	1.28E+00	1.30E+00	1.30E+00	1.31E+00	1.28E+00	1.27E+00	1.34E+00
6	3.55E-02	3.79E-02	4.72E-02	5.66E-02	7.44E-02	9.00E-02	1.10E-01	1.34E-01	1.58E-01
7	1.65E-01	7.64E-02	5.17E-02	5.10E-02	5.30E-02	6.74E-02	7.87E-02	7.99E-02	9.35E-02
8	5.83E-01	4.82E-01	9.19E-01	1.09E+00	1.20E+00	1.33E+00	1.52E+00	1.59E+00	1.91E+00
9	1.22E+00	1.06E+00	1.32E+00	1.84E+00	2.22E+00	2.42E+00	2.55E+00	2.64E+00	2.71E+00
10	1.27E+03	4.43E+02	7.80E+01	6.13E+01	1.42E+02	2.61E+02	4.69E+02	2.05E+03	1.85E+04
11	2.96E+00	3.13E+00	3.89E+00	4.79E+00	5.43E+00	6.44E+00	8.02E+00	1.08E+01	1.16E+01
12	2.06E+01	2.24E+01	2.71E+01	4.28E+01	6.19E+01	8.77E+01	9.95E+01	1.35E+02	1.64E+02
13	2.11E+02	2.10E+02	2.10E+02	2.10E+02	2.10E+02	2.11E+02	2.12E+02	2.14E+02	2.23E+02
14	1.29E+02	1.28E+02	1.33E+02	1.33E+02	1.33E+02	1.33E+02	1.34E+02	1.35E+02	1.36E+02
15	2.98E+02	2.96E+02	3.12E+02	3.27E+02	3.44E+02	3.40E+02	3.50E+02	3.55E+02	3.59E+02

TABLE X.
RESULTS FOR POPULATION SIZE 100, CR = 0.8, F=0.4

f	Fitness	Percentage Change
1	1.50E+02	>1000.00%
2	1.61E-05	0.00%
3	7.10E+00	-35.74%
4	3.80E+02	-59.28%
5	1.30E+00	-28.90%
6	5.66E-02	-41.35%
7	5.10E-02	-36.30%
8	1.09E+00	-30.03%
9	1.84E+00	-38.82%
10	6.13E+01	288.70%
11	4.79E+00	-19.41%
12	4.28E+01	-12.38%
13	2.10E+02	-33.41%
14	1.33E+02	-32.78%
15	3.27E+02	-32.97%
	Overall Changes	-34.73%

TABLE XI.
OVERALL RESULTS FOR POPULATION SIZE 150CR=0.8

FR/ *f*	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	9.18E+03	5.83E+03	5.50E+03	3.53E+00	1.45E+03	1.47E+05	3.48E+06	3.07E+07	1.45E+08
2	3.36E+03	9.47E+02	1.22E+00	5.27E-09	5.13E-09	1.39E-04	3.74E-01	8.96E+01	2.84E+03
3	6.96E+00	7.08E+00	7.22E+00	7.23E+00	7.39E+00	7.54E+00	7.74E+00	7.57E+00	7.59E+00
4	7.49E-01	1.35E+01	1.59E+02	4.65E+02	5.74E+02	6.98E+02	7.78E+02	8.79E+02	9.44E+02
5	1.25E+00	1.28E+00	1.26E+00	1.33E+00	1.15E+00	1.29E+00	1.26E+00	1.28E+00	1.30E+00
6	3.82E-02	3.96E-02	4.69E-02	5.81E-02	8.09E-02	9.67E-02	1.24E-01	1.39E-01	1.68E-01
7	1.24E-01	7.69E-02	5.23E-02	5.69E-02	6.65E-02	7.58E-02	8.38E-02	8.95E-02	9.47E-02
8	5.44E-01	5.76E-01	9.95E-01	1.17E+00	1.22E+00	1.38E+00	1.59E+00	1.71E+00	1.88E+00
9	1.02E+00	1.24E+00	1.56E+00	2.03E+00	2.29E+00	2.47E+00	2.56E+00	2.64E+00	2.69E+00
10	8.30E+02	1.82E+02	6.14E+01	1.15E+02	1.95E+02	3.63E+02	1.20E+03	8.92E+03	4.78E+04
11	3.55E+00	4.08E+00	4.56E+00	5.22E+00	6.41E+00	7.84E+00	9.29E+00	9.70E+00	1.46E+01
12	2.20E+01	2.61E+01	3.35E+01	4.92E+01	7.32E+01	8.79E+01	1.30E+02	1.39E+02	1.63E+02
13	2.11E+02	2.09E+02	2.09E+02	2.10E+02	2.11E+02	2.12E+02	2.15E+02	2.22E+02	2.36E+02
14	1.28E+02	1.29E+02	1.33E+02	1.33E+02	1.33E+02	1.34E+02	1.34E+02	1.35E+02	1.37E+02
15	2.95E+02	2.80E+02	3.03E+02	3.18E+02	3.30E+02	3.28E+02	3.53E+02	3.42E+02	3.66E+02

TABLE XII.
RESULTS FOR POPULATION SIZE 150, CR = 0.8, F=0.4

f	Fitness	Percentage Change
1	3.53E+00	>1000.00%
2	5.27E-09	0.00%
3	7.23E+00	-34.58%
4	4.65E+02	-50.18%
5	1.33E+00	-27.30%
6	5.81E-02	-39.77%
7	5.69E-02	-28.93%
8	1.17E+00	-25.11%
9	2.03E+00	-32.37%
10	1.15E+02	629.65%
11	5.22E+00	-12.26%
12	4.92E+01	0.80%
13	2.10E+02	-33.35%
14	1.33E+02	-32.78%
15	3.18E+02	-34.85%
	Overall Changes	-35.18%

TABLE XIII.
OVERALL RESULTS FOR POPULATION SIZE 200,CR=0.9

FR/ *f*	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1	5.83E+05	6.80E+03	1.68E+03	4.01E-05	8.49E-01	1.37E+03	3.09E+05	1.05E+07	1.02E+08
2	4.64E+03	8.90E+02	1.62E+00	5.24E-09	5.62E-09	5.41E-09	8.09E-07	4.13E-02	2.40E+01
3	7.21E+00	7.23E+00	7.22E+00	7.38E+00	7.44E+00	7.65E+00	7.70E+00	7.59E+00	7.88E+00
4	1.34E+00	2.92E+01	3.70E+02	6.47E+02	7.11E+02	8.56E+02	9.03E+02	9.98E+02	1.07E+03
5	1.32E+00	1.28E+00	1.28E+00	1.35E+00	1.26E+00	1.23E+00	1.29E+00	1.33E+00	1.25E+00
6	4.68E-02	3.41E-02	4.33E-02	5.96E-02	7.54E-02	1.01E-01	1.22E-01	1.48E-01	1.77E-01
7	1.30E-01	6.57E-02	5.17E-02	5.94E-02	6.52E-02	7.53E-02	8.55E-02	9.20E-02	1.02E-01
8	4.90E-01	4.20E-01	9.21E-01	1.11E+00	1.22E+00	1.43E+00	1.60E+00	1.83E+00	2.11E+00
9	1.28E+00	1.16E+00	1.56E+00	1.99E+00	2.26E+00	2.41E+00	2.55E+00	2.65E+00	2.70E+00
10	1.88E+03	1.24E+02	1.07E+01	5.30E+01	1.18E+02	1.91E+02	3.13E+02	8.63E+02	8.74E+03
11	3.58E+00	3.73E+00	4.25E+00	4.79E+00	5.82E+00	6.71E+00	8.05E+00	1.08E+01	1.14E+01
12	2.08E+01	2.19E+01	2.52E+01	3.91E+01	5.54E+01	7.67E+01	9.25E+01	1.03E+02	1.39E+02
13	2.12E+02	2.10E+02	2.10E+02	2.10E+02	2.10E+02	2.12E+02	2.24E+02	2.38E+02	2.67E+02
14	1.29E+02	1.29E+02	1.33E+02	1.33E+02	1.33E+02	1.33E+02	1.34E+02	1.34E+02	1.36E+02
15	2.94E+02	2.97E+02	3.08E+02	3.11E+02	3.30E+02	3.32E+02	3.42E+02	3.41E+02	3.55E+02

TABLE XIV.
RESULTS FOR POPULATION SIZE 200, CR = 0.9, F=0.4

f	**Fitness**	**Percentage Change**
1	4.01E-05	-53.81%
2	5.24E-09	0.00%
3	7.38E+00	-33.18%
4	6.47E+02	-30.63%
5	1.35E+00	-26.25%
6	5.96E-02	-38.18%
7	5.94E-02	-25.82%
8	1.11E+00	-29.04%
9	1.99E+00	-33.88%
10	5.30E+01	236.25%
11	4.79E+00	-19.49%
12	3.91E+01	-19.97%
13	2.10E+02	-33.45%
14	1.33E+02	-32.82%
15	3.11E+02	-36.25%
	Overall Changes	-30.28%

TABLE XV.
OVERALL RESULTS FOR POPULATION SIZE 300,CR=0.9

FR/ *f*	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1	5.83E+05	6.80E+03	1.68E+03	4.01E-05	8.49E-01	1.37E+03	3.09E+05	1.05E+07	1.02E+08
2	4.64E+03	8.90E+02	1.62E+00	5.24E-09	5.62E-09	5.41E-09	8.09E-07	4.13E-02	2.40E+01
3	7.21E+00	7.23E+00	7.22E+00	7.38E+00	7.44E+00	7.65E+00	7.70E+00	7.59E+00	7.88E+00
4	1.34E+00	2.92E+01	3.70E+02	6.47E+02	7.11E+02	8.56E+02	9.03E+02	9.98E+02	1.07E+03
5	1.32E+00	1.28E+00	1.28E+00	1.35E+00	1.26E+00	1.23E+00	1.29E+00	1.33E+00	1.25E+00
6	4.68E-02	3.41E-02	4.33E-02	5.96E-02	7.54E-02	1.01E-01	1.22E-01	1.48E-01	1.77E-01
7	1.30E-01	6.57E-02	5.17E-02	5.94E-02	6.52E-02	7.53E-02	8.55E-02	9.20E-02	1.02E-01
8	4.90E-01	4.20E-01	9.21E-01	1.11E+00	1.22E+00	1.43E+00	1.60E+00	1.83E+00	2.11E+00
9	1.28E+00	1.16E+00	1.56E+00	1.99E+00	2.26E+00	2.41E+00	2.55E+00	2.65E+00	2.70E+00
10	1.88E+03	1.24E+02	1.07E+01	5.30E+01	1.18E+02	1.91E+02	3.13E+02	8.63E+02	8.74E+03
11	3.58E+00	3.73E+00	4.25E+00	4.79E+00	5.82E+00	6.71E+00	8.05E+00	1.08E+01	1.14E+01
12	2.08E+01	2.19E+01	2.52E+01	3.91E+01	5.54E+01	7.67E+01	9.25E+01	1.03E+02	1.39E+02
13	2.12E+02	2.10E+02	2.10E+02	2.10E+02	2.10E+02	2.12E+02	2.24E+02	2.38E+02	2.67E+02
14	1.29E+02	1.29E+02	1.33E+02	1.33E+02	1.33E+02	1.33E+02	1.34E+02	1.34E+02	1.36E+02
15	2.94E+02	2.97E+02	3.08E+02	3.11E+02	3.30E+02	3.32E+02	3.42E+02	3.41E+02	3.55E+02

TABLE XVI.
RESULTS FOR POPULATION SIZE 300, CR = 0.9, F=0.3

f	**Fitness**	**Percentage Change**
1	2.35E+02	>1000.00%
2	3.25E-03	0.32%
3	7.54E+00	-31.79%
4	4.71E+02	-49.44%
5	1.31E+00	-28.66%
6	4.81E-02	-50.14%
7	6.22E-02	-22.36%
8	1.02E+00	-34.93%
9	1.78E+00	-40.89%
10	1.95E+01	23.77%
11	4.92E+00	-17.25%
12	3.04E+01	-37.69%
13	2.09E+02	-33.73%
14	1.33E+02	-32.77%
15	3.06E+02	-37.19%
	Overall Changes	-29.69%

TABLE XVII.
OVERALL RESULTS FOR POPULATION SIZE 500, CR=0.9

FR/ *f*	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1	7.01E+03	5.26E+03	7.16E+01	4.91E+03	2.39E+05	4.56E+06	3.86E+07	1.85E+08	5.05E+08
2	1.84E+03	7.70E+01	5.40E-09	5.34E-09	3.31E-06	1.39E-02	5.67E+00	4.16E+02	6.14E+03
3	7.49E+00	7.59E+00	7.43E+00	7.71E+00	7.53E+00	7.47E+00	7.43E+00	7.40E+00	7.77E+00
4	2.43E+01	1.42E+02	5.47E+02	6.82E+02	7.36E+02	8.84E+02	9.30E+02	9.97E+02	1.05E+03
5	1.35E+00	1.32E+00	1.27E+00	1.21E+00	1.30E+00	1.26E+00	1.22E+00	1.28E+00	1.26E+00
6	3.42E-02	3.71E-02	5.22E-02	7.25E-02	9.02E-02	1.14E-01	1.43E-01	1.82E-01	2.21E-01
7	9.76E-02	6.65E-02	7.63E-02	7.97E-02	8.57E-02	8.90E-02	9.25E-02	9.97E-02	1.12E-01
8	3.59E-01	8.83E-01	1.14E+00	1.19E+00	1.41E+00	1.51E+00	1.75E+00	2.01E+00	2.56E+00
9	1.18E+00	1.59E+00	1.94E+00	2.18E+00	2.34E+00	2.47E+00	2.53E+00	2.58E+00	2.67E+00
10	2.71E+02	4.18E+01	6.48E+01	1.49E+02	2.37E+02	5.12E+02	2.30E+03	1.51E+04	7.62E+04
11	5.57E+00	6.04E+00	6.30E+00	6.96E+00	8.20E+00	8.92E+00	1.05E+01	1.18E+01	1.21E+01
12	2.38E+01	2.98E+01	4.07E+01	5.13E+01	6.53E+01	8.25E+01	1.07E+02	1.30E+02	1.47E+02
13	2.10E+02	2.09E+02	2.10E+02	2.11E+02	2.13E+02	2.21E+02	2.47E+02	2.66E+02	2.74E+02
14	1.27E+02	1.32E+02	1.33E+02	1.33E+02	1.34E+02	1.34E+02	1.34E+02	1.36E+02	1.37E+02
15	3.03E+02	2.95E+02	2.88E+02	3.02E+02	3.06E+02	3.14E+02	3.17E+02	3.24E+02	3.24E+02