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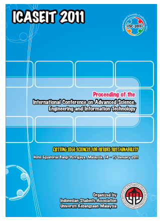

Reliability Evaluation of Slopes Using Particle Swarm Optimization

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***Abstract*— The objective of this research is to develop a numerical procedure to reliability evaluation of earth slope and locating the critical probabilistic slip surface. The performance function is formulated using simplified Bishop's limit equilibrium method to calculate the reliability index. The reliability index defined by Hasofer and Lind is used as an index of safety measure. Searching the critical probabilistic surface that is associated with the lowest reliability index will be formulated as an optimization problem. In this paper, particle swarm optimization is applied to calculate the minimum Hasofer and Lind reliability index and critical probabilistic failure surface. To demonstrate the applicability and to investigate the effectiveness of the algorithm, two numerical examples from literature are illustrated. Results show that the proposed method is capable to achieve better solutions for reliability analysis of slope if compared with those reported in the literature.**

***Keywords*— Slope stability, reliability evaluation, particle swarm optimization.**

I. Introduction

Slope stability problems play a crucial role in both mining and geotechnical engineering fields, which primarily deal with earthen structures. The stability assessment of slopes, where their instability will cause major damage to the surrounding environments, should be carried out using suitable technique. Most soil slope stability analyses are based on the deterministic methods in which soil layers are assumed to be uniform and average soil properties are used. In deterministic methods a factor of safety is commonly used to express the safety of a slope. The factor of safety is a ratio of some expression of resistance to some corresponding expression for factors causing instability of the slope. In general, the factor of safety is not a consistent measure of risk. Slopes with the same values of the factor of safety may exist at different risk levels depending on the variability in soil properties. It is impossible to quantify how much safer a slope becomes as the factor of safety is increased. This indicates a need for more objectively structured and quantitative approach toward handling uncertainties involved in the problems. The probabilistic approach is a natural choice for this type of analysis, because it allows for the direct incorporation of uncertainties into the analytical model. In recent years, several attempts have been done to develop a probabilistic slope stability analysis [1-5].

The results of probabilistic analysis may be expressed as a probability of failure or reliability index. Hasofer and Lind [6] proposed an invariant definition of the reliability index. They defined the reliability index β as the minimum distance from the origin in the standard normal space to the limit state surface. To apply the probabilistic analysis using Hasofer-Lind reliability index (β_{HL}) it is necessary to solve a constraint optimization problem to find the minimum reliability index or maximum probability of failure utilize the appropriate optimization technique.

As a newly developed subset of evolutionary algorithm optimization, the particle swarm optimization has demonstrated its many advantages and robust nature in recent decades. It is derived from social psychology and the simulation of the social behavior of bird flocks in particular. Inspired by the swarm intelligence theory, Kennedy created a model which Eberhart then extended to formulate the practical optimization method known as particle swarm optimization (PSO)[7]. The PSO algorithm has some advantages compared with other optimization algorithms. It is a simple algorithm with only a few parameters to be adjusted during the optimization process, rendering it compatible with any modern computer language. It is also a very powerful algorithm because its application is virtually unlimited. In this paper, we propose a particle swarm optimization (PSO) for minimizing the Hasofer-Lind reliability index (β_{HL}) and determine the critical probabilistic slip surface of earth slope.

II. Probabilistic slope stability analysis

The problem of the probabilistic analysis is formulated by a vector, $\boldsymbol{X}=[X_1,X_2,X_3,\ldots,X_n]$, representing a set of random variables. From the uncertain variables, a performance function $g(\boldsymbol{X})$ is formulated to describe the limit state in the space of $\boldsymbol{X}$. The performance function divides the vector space $\boldsymbol{X}$ in to two distinct regions. The safety region for $g(\boldsymbol{X})>0$ and the failure region $g(\boldsymbol{X})<0$, while the limit state surface is $g(\boldsymbol{X})=0$. The performance function for the slope stability is a function of the factor of safety (FS) usually defined as:

$$g(\boldsymbol{X}) = FS - 1 \tag{1}$$

In the above equation, FS is a factor of safety and can be evaluated using any limit equilibrium method. In this paper a simplified Bishop's method is used to calculate the safety factor.

Bishop [8] considered only circular slip surfaces for analysis. In Bishop's method, the safety factor is determined by trial and errors procedure, because the factor of safety appears in both sides of Eq. (1). In his method, the inter slice shear forces are ignored, and only the normal forces are used to define the inter slice forces. The details of forces acting on a typical slice are shown in Fig. 1. The equation for the factor of safety is derived from the moment equilibrium as follows:

$$FS = \frac{\sum_{i=1}^{n}[c'b\sec\alpha + (m_\alpha[W - \frac{c'b\tan\alpha}{F} - u.b]\tan\varphi')]}{\sum_{i=1}^{n} W\sin\alpha + \sum_{i=1}^{n} k_h W(\cos\alpha - \frac{h_a}{R})} \tag{2}$$

All the parameters of Eq. (2) are defined in Fig. 1 and m_α is defined as:

$$m_\alpha = 1/[\cos\alpha + \frac{\sin\alpha\tan\phi'}{F}] \tag{3}$$

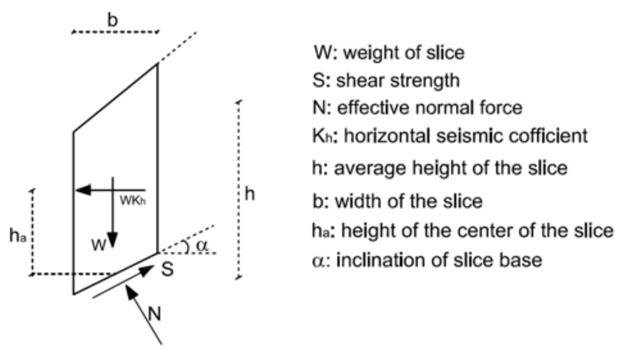

Fig.1 Forces acting on a typical slice in Bishop's method

The probability of failure of the slope can be expressed in terms of the performance function by the following integral:

$$P_f = P[g(X \leq 0)] \tag{4}$$

The most effective applications of probability theory to the analysis of slope stability have stated the uncertainties in the form of a reliability index (β). The reliability index provides more information and is a better indication of the stability of a slope than the factor of safety alone because it incorporates information of the uncertainty in the values of the performance function. It also provides a good comparative measure of safety; slopes with higher β are considered safer than slopes with lower β.

Depend on the form of the performance function several definitions of the reliability index exist. Hasofer and Lind [6] proposed an invariant definition of the reliability index as the minimum distance from the origin in the standard normal space to the limit state surface. This distance is defined as β_{HL}, can be described as and Fig. 2:

$$\beta = \min_{X \in F}(\boldsymbol{U}^T.\boldsymbol{U})^{1/2} \tag{5}$$

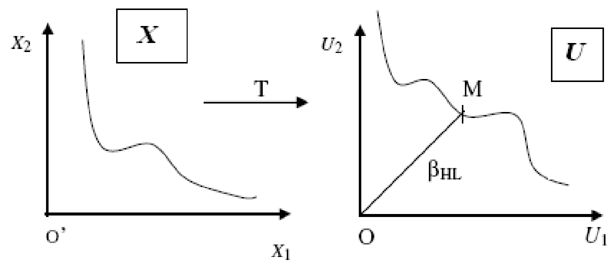

Fig.2 The geometrical representation of the definition of the reliability index

where $\boldsymbol{X}$ is a vector representing the set of random variables x_i, F is the failure domain. To determine the H-L reliability index (β_{HL}), all the random variables $\boldsymbol{X}$ should be transformed into a standard normal space $\boldsymbol{U}$, by an orthogonal transformation such that:

$$u_i = \frac{x_i - \mu_i}{\sigma_i} \tag{6}$$

As mentioned before, the H-L reliability index (β_{HL}) is defined as the minimum distance from the origin of the axis in the standard normal space to the limit state surface. To evaluate β_{HL} the following constrained optimization problem should be solved:

$$\begin{aligned} &Minimize\ \beta_{HL} \\ &Subject\ to\ g(\boldsymbol{U}) = 0 \end{aligned} \tag{7}$$

Solve Eq. (7) is equivalent to solve the relaxed form obtained by penalty method as:

$$Minimize\ \beta_{HL} + r\left|g(\boldsymbol{U})\right|^l \tag{8}$$

The parameters r and l are problem dependent, and r should be a suitably large positive constant. In the present study, the values set for r and l were 1000 and 2, respectively.

The solution of the above optimization problem is the design point or MPP in the standardized normal variables space. Several algorithms have been recommended for the solution of optimization problem in Eq. (8). In the current study, a particle swarm optimization is proposed for the solution.

III. PARTICLE SWARM OPTIMIZATION

The original particle swarm optimization algorithm introduced by Kennedy and Eberhart in 1995 [7]. The PSO is derived from a simplified version of the flock simulation. It also has features that are based upon human social behaviour.

PSO contains a number of particles which called the swarm. The particles are initialized randomly in the multi dimensional search space of an objective function. Each particle represents a potential solution of the optimization problem. The particles fly through the search space and their positions are updated based on each particle's personal best position as well as the best position found by the swarm. The objective function is evaluated for each particle during iterations, and the fitness value is used to determine which position in the search space is better than the others.

At every iteration, the update moves a particle by adding a change velocity V_i^{k+1} to the current position $X_i^{\,k}$ as illustrated in the following equation [9]:

$$X_i^{k+1} = X_i^{\,k} + V_i^{k+1} \qquad i = 1,2,3,...,N \tag{9}$$

The velocity is a combination of three contributing factors: (1) previous velocity V_i^k, (2) movement in the direction of the local best P_i^k, and (3) movement in the direction of the global best $P_g^{\,k}$. The mathematical formulation is expressed as [4]:

$$V_i^{k+1} = w \times V_i^k + c_1 \times r_1 \times (P_i^k - X_i^k) + c_2 \times r_2 \times (P^k{}_g - X_i^k) \tag{10}$$

where w is an inertia weight to control the influence of the previous velocity; r_1 and r_2 are two random numbers uniformly distributed in the range of (0, 1); c_1 and c_2 are two acceleration constants usually considered equal 2; P_i^k is the best position of the i^{th} particle up to iteration k and $P_g^{\,k}$ is the best position among all particles in the swarm up to iteration k. The inertia weighting function in Eq. (10) is usually calculated using following equation:

$$w = w_{max} - (w_{max} - w_{min}) \times k / G \tag{11}$$

where w_{max} and w_{min} are maximum and minimum values of w, G is the maximum number of iterations and k is the current iteration number. Figure 3 shows the position update of a particle in PSO.

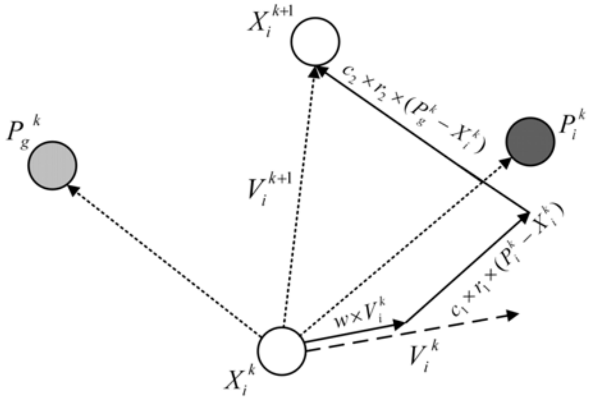

Fig. 3 Position update of particle in PSO

IV. NUMERICAL EXAMPLES

This section investigates the validity and effectiveness of the proposed algorithm to probabilistic slope stability analysis. To verify and assess the applicability of the proposed two benchmark problems were selected from the literature. The procedure has been carried out using a computer program was developed by MATLAB. The program searches for the most critical deterministic and probabilistic slip surface. Based on above explanation, the implementation procedure of the proposed method for the reliability analysis of the earth slope is constructed as follows:

1. Initialize a set of particles positions and velocities randomly distributed throughout the design space bounded by specified limits.
2. Evaluate the objective function values using Eq. (8) for each particle in the swarm.
3. Update the optimum particle position at current iteration and global optimum particle position.
4. Update the velocity vector as specified in Eq. (10) and update the position of each particle according to Eq. (9).
5. Repeat steps 2–4 until the stopping criteria is met.

To calculate the minimum value of β_{HL} using PSO the parameters of the algorithm should be adopted accurately. In our study, proper fine tuning of these parameters was obtained utilizing several experimental studies examining the effect of each parameter on the final solution and convergence of the algorithm. As a result, a population of 40 individuals was used; w_{max} and w_{min} were chosen as 0.95 and 0.45 respectively; and the values of the acceleration constants (c_1 and c_2) were selected equal to 2.Finally, a fixed number of maximum iteration (G) of 3000 was applied. The optimization procedure was terminated when one of the following stopping criteria was met: (i) the maximum number of generations is reached; (ii) after a given number of iterations, there is no significant improvement of the solution.

A. Example 1

Figure 4 shows the geometry of a slope in homogeneous soil. The parameters considered as random variables in the probabilistic analysis are: the effective friction angle, effective cohesion, unit weight and pore water pressure ratio. Table I presents the mean values and standard deviation associated with each random variable.

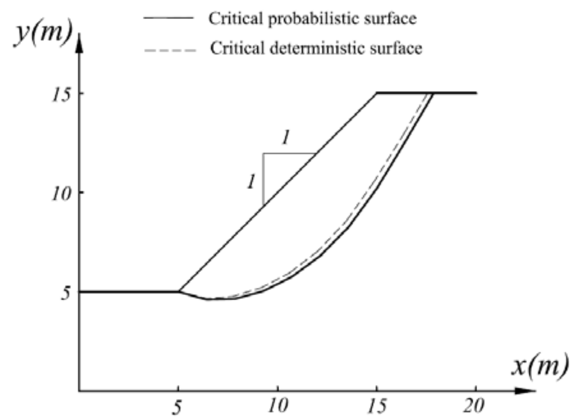

Fig. 4 Cross section of homogeneous slope-example 1

TABLE I
STATISTICAL PROPERTIES OF SOIL PARAMETERS- EXAMPLE 1

Random variable	Mean	Standard deviation	Distribution
c'	18.0kN/m^2	3.6kN/m^2	Log-normal
$tan\ \varphi'$	tan 30	0.0577	Log-normal
γ	18.0kN/m^3	0.9kN/m^3	Log-normal
r_u	0.2	0.02	Log-normal

The problem was previously solved by Li and Lumb [10], Hassan and Wolff [1] and Bhattacharya et al. [2]. The results of the proposed method and previous studies are summarized in Table II.

In Table II, FS_{min} and β_{FS} are the minimum factor of safety and the reliability index associated with the critical deterministic slip surface, respectively, and β_{min} is the minimum reliability index corresponding to the critical probabilistic slip surface.

According to analysing the results of this table, it can be observed that, the minimum reliability index calculated using presented method is 2.212, which is lower than the values reported by Li and Lumb (2.5) Hassan and Wolff (2.293), Bhattacharya et al. (2.239). Further, the minimum factor of safety calculated from a deterministic analysis based on the mean values of the soil properties obtained by PSO is 1.309, which is lower than 1.326 reported by Bhattacharya et al. [2].

The corresponding critical deterministic and the critical probabilistic slip surfaces are also presented in Fig. 4. As it can be seen, two surfaces are located reasonably close to each other as expected in a homogeneous slope. It's because of the proximity of the values of β_{FS} and β_{min} presented in Table II. The failure surfaces reported by previous researchers are also similarly located.

TABLE II
RESULTS COMPARISON- EXAMPLE 1

Method	β_{FS}	β_{min}	FS_{min}
Li and Lumb [10]	-	2.5	-
Hassan and Wolff [1]	2.336	2.293	-
Bhattacharya et al [2]	2.306	2.239	1.326
Present study (PSO)	2.295	2.212	1.309

B. Example 2

Figure 5 shows the cross section and geometry of a two layered slope in clay bounded by a hard layer below and parallel to the ground surface. The soil strength parameters that are related to the stability of slope, including friction angle φ, and cohesion c, are considered as random variables. The statistical moments (mean value and standard deviation) of the parameters are summarized in Table III.

This example was also solved previously by Hassan and Wolff [1] and Bhattacharya et al [2] in terms of FS_{min}, β_{FS} and β_{min}. The results obtained from current study together with a comparison of those reported by previous researchers are summarized in Table IV. For the results shown in this table, it can be considered that the minimum reliability index evaluated using PSO is 2.771, which is almost lower than those reported by Hassan and Wolff [1] and Bhattacharya et al. [2]. Besides, the minimum factor of safety obtained by PSO is found to be smaller than the others. The corresponding critical deterministic and the critical probabilistic slip surfaces are presented in Fig. 5. In accordance with the difference in the values of β_{FS} and β_{min} presented in Table IV, the two surfaces are located significantly separate.

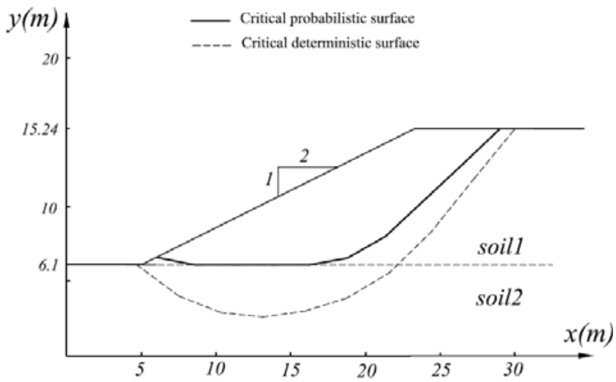

Fig. 5 Cross section of non homogeneous slope-example 2

TABLE III
STATISTICAL PROPERTIES OF SOIL PARAMETERS- EXAMPLE 2

Material	Parameter	Mean	Standard deviation	Distribution
Soil 1	c_1	38.31kN/m^2	7.662kN/m^2	Log-normal
	φ_1	0	-	Log-normal
Soil 2	c_2	23.94kN/m^2	4.788kN/m^2	Log-normal
	φ_2	12	1.2	Log-normal

TABLE IV
RESULTS COMPARISON- EXAMPLE 2

Method	β_{FS}	β_{min}	FS_{min}
Hassan and Wolff [1]	4.442	2.869	1.663
Bhattacharya et al [2]	5.064	2.861	1.665
Present study (PSO)	4.545	2.771	1.655

V. Conclusions

This paper outlines a procedure of probabilistic analysis of earth slope. The Hasofer-Lind reliability index (β_{HL}) is used instead of the conventional reliability index β. The problem of searching the critical probabilistic surface with the minimum reliability index, β_{min}, can be formulated as an optimization problem and a modified particle swarm optimization is proposed for the solution. The described framework has been coded in MATLAB and used to carry out parametric studies for the numerical problems. The applicability of the proposed methodology developed herein, has been examined on two slope stability problems from the literature. A comparison of results show that, the results obtained in the present study using MPSO has evaluated values of minimum reliability index that are reasonably lower than those reported in the literature and is capable to identify the failure sequence. Further as illustrated trough the test problems; the critical probabilistic and deterministic slip surface is almost close for slope in a homogenous soil whereas these surfaces are located quite separate for non homogenous slopes.

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