



Estimation of Exchange Rate Volatility using APARCH-type Models: A Case Study of Indonesia (2010–2015)

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Abstract

Volatility measurement and modeling is an important aspect in many areas of finance. The main purpose of this study is to apply seven APARCH-type models with (1,1) lags to investigate the behavior of exchange rate volatility for the EUR, JPY, and USD selling exchange rates to IDR for the duration from January 2010 to December 2015. The competing models include ARCH, GARCH, TARARCH, TS-ARCH, GJR-GARCH, NARCH, and APARCH used with Gaussian normal distribution. In order to estimate the model parameters, this study applies the Bayesian inference using the adaptive random walk Metropolis method in the MCMC algorithm. Empirical results based on the deviance information criterion indicate that the GARCH (1,1), APARCH (1,1), and TARARCH (1,1) models provide the best fit for the EUR, JPY, and USD data, respectively. In those models, both the JPY and USD data have significant negative leverage effect at the 99% credible level. Moreover, the JPY returns also have significant Taylor effect in return volatility at the 99% credible level.

Keywords: APARCH, ARWM, IDR exchange rate, MCMC, volatility

JEL Classification: C30, F31

INTRODUCTION

Volatility of asset returns has been widely used in financial market. Volatility, of stock prices or exchange rates, can be considered as a measurement of the risk for the securities based on the fluctuation of the asset returns. Abdalla & Winker (2012) defined the volatility as a statistical measure of the spread of returns on a market index or a specific security, and it is usually measured by using the standard deviation of returns. As shown in many studies, the high volatility of exchange rate returns leads to higher risk for investors, traders, and policy makers.

In general, high frequency financial returns –weekly, daily, or minutes– is heteroskedastic, i.e. the value of volatility is changing over times. On the basis of the fact, Engle (1982) proposed ARCH (*autoregressive conditional heteroscedasticity*) model that has been well studied in financial literature. This model was extended to GARCH (*generalized ARCH*) model by Bollerslev (1986). The ARCH/GARCH models can capture the volatility clustering and fat tail (leptokurtic) successfully, but they both fail to capture the leverage effect, which is common phenomenon in financial markets. An extension of the GARCH model accommodating the above three effects is the asymmetric power GARCH

(APARCH) model from Ding *et al.* (1993). Furthermore, Danielsson (2011) states that the APARCH model combines two effects: (1) leverage effect: asymmetry in the impact of positive and negative lagged returns, and (2) allowing power in the volatility calculation to be flexible.

In this study, a family of APARCH model is fitted to the Euro (EUR), Japanese yen (JPY), and US dollar (USD) selling exchange rates to the Indonesian rupiah (IDR) for the duration from January 2010 to December 2015. We construct Markov chain Monte Carlo (MCMC) algorithm based on the adaptive random walk Metropolis method to estimate the APARCH-type models. Hereafter, their estimation results are compared to obtain the best fitting model for each data on the basis of the deviance information criterion (DIC).

LITERATURE REVIEW

There is a large number of literature on the modeling and forecasting of return volatility. In recent years, Nugroho & Morimoto (2014, 2015, 2016) applied the stochastic volatility models to stock markets. In the case of Indonesia foreign exchange markets, however, there is relatively very few literature and research applying the ARCH/GARCH models. For example, Safrudin *et al.* (2015); Salim *et al.* (2016), and Saputri *et al.* (2016) studied the behavior of EUR and JPY selling exchange rates to the IDR during the period 2009–2014 using ARCH/GARCH models with non-normal distributions for returns errors. Their results confirm that a non-normal distribution for returns errors in ARCH/GARCH models is appropriate for the data. This study, on the other hand, will consider only the use of normal distribution for returns errors and focus on the generalization of the GARCH models to APARCH-type models.

METHOD

Data Used for the Analysis

As Salvatore (2013) explains, there are five dominant international currencies in the world economy in 2010, i.e. USD, EUR, JPY, GBP, and CHF. The data analyzed in this study are the selling exchange rates for the EUR, JPY, and GBP to the IDR. The data sets were obtained from Bank Indonesia's internet website (<http://www.bi.go.id>) covering the six years from January 2010 to December 2015 on a daily basis, excluding weekends and holidays. Hereafter, we denote the Y selling exchange rate to the X by X/Y , meaning that the number of X s are required by buyer to purchase one Y from trader.

Models Used in This Study

On the basis of the assumption that an asset price follows geometric Brownian motion, returns can be expressed by (Tsay, 2005):

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right) = \mu + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_t^2),$$

where μ is the average of returns and S_t is the asset price at time t . Furthermore, this study assumes that $\mu = 0$ and the returns are expressed in percentage as follows:

$$R_t = 100 \times \ln\left(\frac{S_t}{S_{t-1}}\right).$$

In practice it is assumed that the returns are not serially correlated.

Models that offers more flexibility than GARCH models are APARCH-type models, where σ_t^2 in the GARCH model is replaced by σ_t^δ , with $\delta > 0$. The conditional volatility dynamics of these models are expressed by

$$\sigma_t^\delta = \omega + \sum_{i=1}^p \alpha_i (|R_{t-i}| - \gamma_i R_{t-i})^\delta + \sum_{j=1}^q \beta_j \sigma_{t-j}^\delta,$$

where $t = 1, 2, \dots, T$, $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, $\delta > 0$, and $-1 < \gamma_i < 1$. Here, α and β are the ARCH and GARCH coefficients respectively, γ is the leverage effect between volatility and returns when $\gamma \neq 0$, and δ is the Taylor (power) effect after Taylor (1986), when $\delta \neq 1$. Taylor effect accommodates the fact that absolute returns sometimes have stronger autocorrelation than squared returns. The idea behind the allowing δ to take the form of a free parameter arose from the fact that, the assumption of normality in modeling financial data, which restricts δ to either 1 or 2, is often unrealistic due to significant skewness and kurtosis (Longmore and Robinson, 2004).

The effect of R_{t-i} upon σ_t is through the function $k_{\gamma_i}(R_{t-i})$, where $k_{\gamma_i}(R_{t-i}) = |R_{t-i}| - \gamma_i R_{t-i}$. If $\gamma > 0$, then $k_{\gamma_i}(-R_{t-i}) > k_{\gamma_i}(R_{t-i})$ for any $R_{t-i} > 0$, so there is a leverage effect, meaning that past “bad news” (negative returns) have a stronger impact on current volatility than past “good news” (positive returns). If $\gamma < 0$, then $k_{\gamma_i}(R_{t-i}) > k_{\gamma_i}(-R_{t-i})$, so there is a leverage effect in the opposite direction to what is expected, meaning that past positive returns increase current volatility than past negative returns.

The APARCH model is a nested model including as special cases (Laurent, 2003):

When $\delta = 2$, $\beta = 0$, and $\gamma = 0$, the model is ARCH.

When $\delta = 2$ and $\gamma = 0$, the model is GARCH.

When $\delta = 1$ and $\gamma = 0$, the model is TS-GARCH.

When $\delta = 2$, the model is GJR-GARCH.

When $\delta = 1$, the model is TARCH.

When $\beta = 0$ and $\gamma = 0$, the model is NARCH.

This study applies the above models to real data and compares their modeling performance. By using the GARCH (1,1) model, Ruppert (2011) empirically showed that autocorrelation decays slowly decrease after one lag. This capability appears to be the main reason that the GARCH model with autocorrelation lag-1 fits so many financial time series. Therefore, this study takes $p = 1$ and $q \leq 1$ in the APARCH-type models.

MCMC Method for APARCH(1,1)

Nugroho & Morimoto (2014) pointed out that the implementation of MCMC method involves two steps. The first step is to construct Markov chain that has as an equilibrium distribution which matches the conditional posterior distribution. The second step is to summarize posterior distribution of the parameter as MCMC output, using the Monte Carlo method.

Let $\mathbf{R} = (R_1, R_2, \dots, R_T)$ denotes the observation vector, and $\theta = (\omega, \alpha, \beta, \gamma, \delta)$ the parameter vector. By applying Bayes' rule, the posterior distribution of the parameters given the observed data can be written as:

$$p(\theta | \sigma, \mathbf{R}) = L(\mathbf{R} | \theta) p(\theta),$$

where

$$L(\mathbf{R}|\theta) = \prod_{t=1}^T \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{R_t^2}{2\sigma_t^2}\right)$$

and $p(\theta)$ is a prior distribution on θ .

MCMC methods have been widely used for estimation purposes in practical finance applications. One of the most common MCMC methods in practical use is the random walk Metropolis (RWM) which has been extended to the Adaptive RWM (ARWM) method to improve the efficiency of the method. Suppose y_n is a real sequence, the ARWM scheme is employed as follows (Atachade and Rosenthal, 2005):

- (i) Initialize θ_0 dan g_0 .
- (ii) Let $n \geq 0$, given θ_n and g_n .
 - a. Draw a proposal $y_{n+1} = \theta_n + \eta_t$ and $x \sim U(0,1)$, where $\eta_t \sim N(g_n, 1)$.
 - b. Calculate the Metropolis ratio:

$$r(\theta_n, \theta_{n+1}) = \frac{p(\theta_{n+1}|\mathbf{R})}{p(\theta_n|\mathbf{R})}$$

and the acceptance probability $\alpha(\theta_n, \theta_{n+1}) = \min\{1, r(\theta_n, \theta_{n+1})\}$.

- c. If $x \leq \alpha(\theta_n, \theta_{n+1})$, then $\theta_{n+1} = y_{n+1}$; otherwise, $\theta_{n+1} = \theta_n$.
- (iii) Let $g_i \in [g_{\min}, g_{\max}]$ and calculate:

$$v = \max\left\{g_{\min}, g_n + \frac{\frac{m(\theta)}{n+1} - \bar{\tau}}{(n+1)^\lambda}\right\},$$

where $m(\theta)$ denotes the number of accepted proposals up to iteration- $(n+1)$ th.

If $v > g_{\max}$, then $g_{n+1} = g_{\max}$; otherwise, $g_{n+1} = g_n$.

We set:

$$g_{\min} = 10^{-5}, g_{\max} = 10, \bar{\tau} = 0,44, \lambda = 0,6$$

where $\bar{\tau}$ was chosen to make the acceptance rate of proposals as close as possible to 0.44 (Roberts & Rosenthal, 2009). As Atachade & Rosenthal (2005) explain, the choice of the scaling parameter g_i has a large effect on the algorithm's mixing time. Intuitively, if g_i is too small, the resulting algorithm will make very small moves, resulting in a poor mixing time. On the other hand, if g_i is too large, then large moves will usually be proposed, and these are likely to be rejected so the algorithm will again mix poorly.

RESULT AND DISCUSSION

Descriptive Statistics of the Data

In this study, the proposed models are fitted to daily returns of IDR/EUR, IDR/JPY, and IDR/USD during the six years period from January 2010 to December 2015. The dataset consists of 1472 observations, excluding weekends and holidays. Figure 1 displays the time series plots of daily exchange rate and its returns (in percent). Meanwhile, the descriptive statistics for returns, such as mean and standard deviation (SD), Jarque–Bera(JB) normality test, and Ljung–Box (LB) autocorrelation test are summarized in Table 1. Normality and autocorrelation tests show that all returns are neither normally distributed nor

serially correlated for 8 lags. In order to provide a principal empirical structure for developing more general models, the non-normality distribution are avoided.

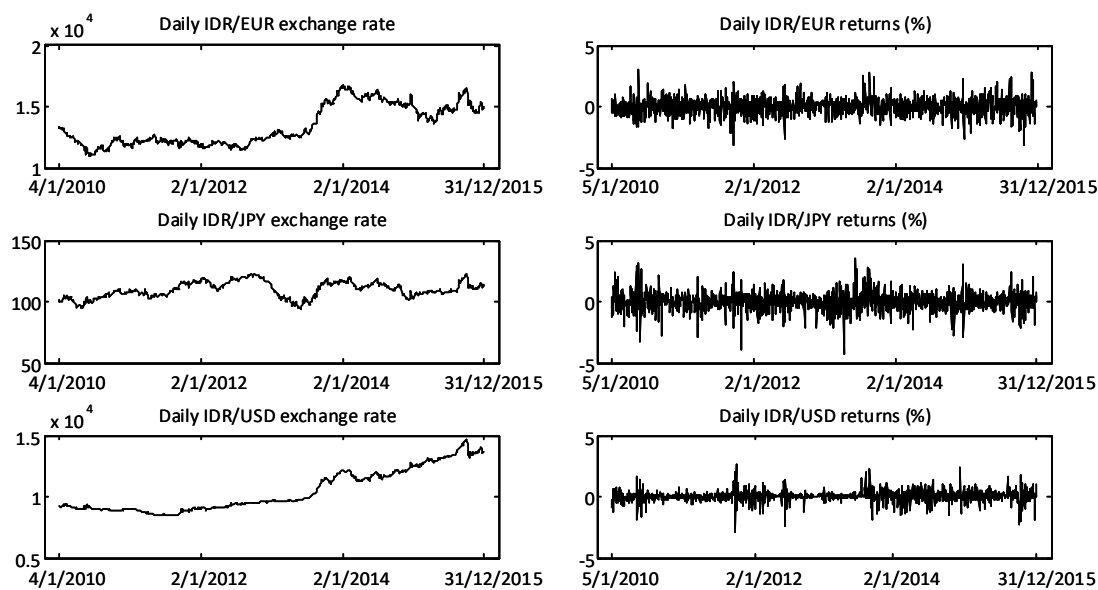


Figure 1. Time series plots for daily selling exchange rate and their returns (in percent).

Source: Authors' calculation.

Table 1. Descriptive statistics of the daily returns of the IDR/EUR, IDR/JPY, and IDR/USD (January 2010 to December 2015).

Returns	Number of observation	Mean	SD	Min.	Max.	JB stat. (normality)	LB stat. (lag-8 autocorr.)
IDR/EUR	1471	0.008	0.680	-3.199	3.092	184.2 (no)	6.72 (no)
IDR/JPY	1469	0.009	0.746	-4.284	3.499	666.4 (no)	9.97(no)
IDR/USD	1415	0.028	0.461	-2.858	2.713	2237.4 (no)	7.99 (no)

Source: Authors' calculation

Stationarity Test

This study has applied the Augmented Dickey–Fuller (ADF) test (Dickey dan Fuller, 1981) to investigate whether the exchange rate and its returns series are stationary. The results reported in Table 2 indicate that the test does not reject the null hypothesis of a unit root at all confidence levels for all exchange rate series, but the test allows rejecting the null hypothesis for the returns series. It means that the returns are stationary series.

Table 2. ADF unit root test output for the exchange rate series and their returns series.

Series	IDR/EUR	IDR/JPY	IDR/USD
Selling exchange rate	-0.87	-2.50	0.87
Returns	-36.55	-39.37	-37.83

Source: Authors' calculation.

Notes: 1. H_0 is a unit root, H_A is stationarity.

2. ADF test include a constant term without trend.

3. Maximum lag length was set to 22.

4. Critical values for unit root tests at 99%, 95%, and 90% confidence intervals are respectively -3.45, -2.87, and -2.57.

Heteroscedasticity Test

Since the purpose of this study is to analyze volatility of returns series, it is necessary to perform a test for heteroscedasticity of residuals. The test was implemented in MATLAB with “archtest” function for the return residuals, i.e the difference between returns and its average. The results, as shown in Table 3, can be summarized as follows: $H=1$, the p-value is less than 0.05, the ARCH test statistics is greater than the critical value at 95% confidence level. These imply that there is a strong evidence for rejecting the null hypothesis of no ARCH effect. The rejection indicates the existence of ARCH effects in the return residual series and therefore the variance of all observed returns is not constant.

Table 3. MATLAB output for the ARCH effect test of return residuals.

Lag	IDR/EUR			IDR/JPY			IDR/USD		
	10	15	20	10	15	20	10	15	20
H	1	1	1	1	1	1	1	1	1
P-value	0	0	0	0	0	0	0	0	0
ARCH statistics	48.4	51.7	51.9	59.9	62.6	63.0	239.4	252.2	264.5
Critical value	18.3	25.0	31.4	18.3	25.0	31.4	18.3	25.0	31.4

Source: Authors' calculation.

Notes: 1. H_0 : no ARCH effect is present.

2. $H = 1$ indicates rejection of H_0 .

3. Critical values are computed at 95% confidence interval.

Empirical Results

All empirical results were obtained via implementation of our own scripts in Matlab. For all cases, this study employed the MCMC simulation for 15000 iterations. In this process, we discard the first 5000 iterations and store the remaining 10000 iterations to contribute to the required statistics, such as the mean, standard deviation, 95% highest posterior density interval, and the integrated autocorrelation time (IACT) as an indicator of inefficiency sampler. IACT can be roughly interpreted as the number of iterations required so as to produce independent draws. A value of one indicates that the sampler delivers uncorrelated draws set, while large values deliver slowly decaying correlations as well as slow convergence. Tables 4–6 summarize the empirical results for the IDR/EUR, IDR/JPY, and IDR/USD returns data sets, respectively. Overall, the IACT values indicate that the ARWM method provides sufficiently efficient estimation for all applied models.

Table 4. Results of MCMC estimation for the models adopting IDR/EUR returns, the conditional DIC estimates, and the pD (effective number of parameters) estimates.

Para- meter	Statistics	Model							
		ARCH	GARCH	TARCH	TS- GARCH	GJR- GARCH	NARCH	APARCH	
ω	Mean	0.402	0.027	0.100	0.162	0.037	0.346	0.277	
	SD	0.018	0.012	0.019	0.026	0.014	0.115	0.045	
	LB	0.368	0.006	0.065	0.112	0.009	0.143	0.193	
	UB	0.439	0.050	0.133	0.208	0.062	0.557	0.363	
	IACT	14.7	315.0	368.7	385.0	305.4	319.3	467.7	
α	Mean	0.135	0.056	0.086	0.060	0.062	0.115	0.106	
	SD	0.034	0.013	0.017	0.012	0.015	0.040	0.025	

Para-meter	Statistics	Model						
		ARCH	GARCH	TARCH	TS-GARCH	GJR-GARCH	NARCH	APARCH
β	LB	0.072	0.030	0.057	0.038	0.036	0.038	0.054
	UB	0.205	0.083	0.124	0.083	0.094	0.192	0.154
	IACT	15.0	187.5	150.9	78.4	146.3	131.2	95.2
	Mean	0	0.884	0.785	0.717	0.856	0	0.526
	SD		0.035	0.033	0.041	0.039		0.109
	LB		0.815	0.731	0.640	0.788		0.313
γ	UB		0.943	0.846	0.794	0.935		0.705
	IACT		342.9	380.4	394.1	324.4		452.9
	Mean	0	0	-0.143	0	-0.063	0	-0.027
	SD			0.097		0.066		0.122
	LB			-0.338		-0.199		-0.278
	UB			0.040		0.068		0.207
δ	IACT			4.8		7.7		7.7
	Mean	2	2	1	1	2	2.528	1.472
	SD						0.895	0.230
	LB						1.119	1.041
	UB						4.310	1.926
	IACT						339.5	245.0
Persistence		-	0.941	0.801	0.778	0.887	-	0.638
Conditional		3012.1	2996.7	3004.6	2998.7	2998.6	3014.9	3009.8
DIC (SD)		(0.03)	(0.58)	(1.52)	(0.79)	(0.68)	(0.07)	(0.57)
pD		2	5	6	4	7	3	7
Rank		6	1	4	3	2	7	5

Source: Authors' calculation from IDR/EUR returns.

We first consider the feature of leverage effect in the TARCH, GJR-GARCH, and APARCH models. In the case of applying the returns of the EUR selling exchange rate to the IDR, the 95% HPD intervals of γ include 0, indicating that the data do not provide statistically significant evidence in support of the leverage effect. Meanwhile, in the case of the IDR/JPY and IDR/USD returns data, the leverage parameter is found to be statistically negative significant in the APARCH model, which implies the presence of leverage effect. Although not reported, we find that even 99% HPD interval of γ excludes 0. It indicates that the IDR/JPY and IDR/USD returns volatilities are greater after bad news (negative return) than after good news (positive return).

Table 5. Results of MCMC estimation for the models adopting IDR/JPY returns, the conditional DIC estimates, and the pD (effective number of parameters) estimates.

Para-meter	Statistics	Model						
		ARCH	GARCH	TARCH	TS-GARCH	GJR-GARCH	NARCH	APARCH
ω	Mean	0.432	0.034	0.073	0.084	0.041	0.655	0.127
	SD	0.022	0.015	0.013	0.015	0.010	0.062	0.024
	LB	0.390	0.010	0.049	0.057	0.022	0.531	0.081
	UB	0.476	0.066	0.101	0.124	0.060	0.773	0.169
	IACT	8.4	263.9	294.2	384.0	122.6	229.8	433.1
	α	Mean	0.255	0.097	0.144	0.058	0.105	0.233
SD		0.048	0.024	0.018	0.009	0.019	0.039	0.016
LB		0.166	0.052	0.109	0.041	0.067	0.154	0.091
UB		0.352	0.148	0.182	0.078	0.142	0.302	0.156

Para- Statistics meter	Model							
	ARCH	GARCH	TARCH	TS- GARCH	GJR- GARCH	NARCH	APARCH	
β	IACT	9.968	212.5	128.8	171.9	79.5	157.2	123.6
	Mean	0	0.844	0.792	0.841	0.817	0	0.753
	SD		0.045	0.023	0.026	0.028		0.032
	LB		0.748	0.748	0.780	0.768		0.693
	UB		0.921	0.834	0.890	0.872		0.812
γ	IACT		281.7	319.4	390.5	136.7		460.2
	Mean	0	0	-0.329	0	-0.324	0	-0.363
	SD			0.090		0.076		0.085
	LB			-0.502		-0.477		-0.544
	UB			-0.153		-0.178		-0.208
δ	IACT			12.9		12.5		18.2
	Mean	2	2	1	1	2	0.755	1.124
	SD						0.257	0.052
	LB						0.274	1.025
	UB						1.289	1.221
	IACT						199.5	317.1
Persistence	-	0.942	0.772	0.900	0.760	-		0.945
Conditional	3244.0	3201.9	3178.6	3210.8	3181.3	3235.9		3176.3
DIC (SD)	(0.04)	(0.28)	(0.12)	(0.57)	(0.12)	(0.19)		(0.24)
pD	2	3	4	3	4	3		5
Rank	7	4	2	5	3	6		1

Source: Authors' calculation from IDR/JPY returns.

Regarding the Taylor effect, the NARCH and APARCH models are considered. In the IDR/EUR case, the Taylor parameter (δ) is significantly different from 1 (even in terms of the 99% HPD interval) but not from 2 in the NARCH model and significantly different from 1 or 2 in the APARCH model. In the IDR/JPY case, the Taylor parameter is significantly different from 2 but not from 1 in the NARCH model and significantly different from 1 or 2 in the APARCH model. We even found that the 99% HPD interval of δ does not include 1 or 2 for the APARCH model adopting the JPY returns (results not shown). Meanwhile, in the IDR/USD case, the Taylor parameter is significantly different from 1 or 2 in both NARCH and APARCH models. In general, these deviations imply the presence of Taylor effect in all observed data.

Tables 4–6 also report the volatility persistence values computed as $\alpha + \beta$ for the GARCH and TS-GARCH models, $\alpha + 0,5\gamma + \beta$ for the TARCH and GJR-GARCH models, and $\alpha(1 - \gamma)^\delta + \beta$ for the APARCH model. According to Campbell et al. (1996), the volatility persistence measures how fast (or slow) the volatility reverts or decay towards its mean. A high persistence (greater than 0 but less than 1) implies slow reversion to the mean. In this study, the highest volatility persistences are found in the GARCH model adopting IDR/EUR returns and in the APARCH model adopting IDR/JPY and IDR/USD returns. Notice that the persistence measures fall close to one for the IDR/EUR returns in the GARCH model only, for the IDR/JPY returns in the GARCH, TS-GARCH, APARCH models, and for the IDR/USD returns in all models. We conclude that volatility in the USD returns is highly persistent.

Table 6. Results of MCMC estimation for the models adopting IDR/USD returns, the conditional DIC estimates, and the pD (effective number of parameters) estimates.

Para- meter	Statistics	Model						
		ARCH	GARCH	TARCH	TS- GARCH	GJR- GARCH	NARCH	APARCH
ω	Mean	0.112	0.007	0.023	0.054	0.006	0.213	0.016
	SD	0.006	0.001	0.004	0.007	0.001	0.041	0.010
	LB	0.100	0.004	0.015	0.041	0.004	0.142	0.001
	UB	0.124	0.010	0.032	0.067	0.008	0.296	0.038
	IACT	7.6	105.7	111.9	248.3	58.4	235.2	404.9
α	Mean	0.569	0.198	0.231	0.177	0.206	0.491	0.218
	SD	0.067	0.026	0.021	0.022	0.027	0.051	0.024
	LB	0.437	0.148	0.191	0.134	0.160	0.387	0.172
	UB	0.695	0.249	0.273	0.219	0.246	0.592	0.265
	IACT	6.9	160.7	133.1	227.1	64.7	71.9	476.0
β	Mean	0	0.784	0.782	0.787	0.791	0	0.750
	SD		0.026	0.020	0.024	0.022		0.028
	LB		0.730	0.746	0.738	0.756		0.690
	UB		0.833	0.825	0.830	0.826		0.794
	IACT		160.8	168.1	263.2	82.3		496.0
γ	Mean	0	0	-0.152	0	-0.129	0	-0.100
	SD			0.035		0.038		0.047
	LB			-0.225		-0.191		-0.193
	UB			-0.085		-0.065		-0.014
	IACT			7.6		7.4		129.6
δ	Mean	2	2	1	1	2	1.368	0.913
	SD						0.196	0.031
	LB						1.010	0.857
	UB						1.745	0.984
	IACT						229.1	217.1
Persistence		-	0.983	0.937	0.966	0.933	-	0.989
Conditional DIC (SD)		1520.4 (0.02)	1336.8 (0.16)	1322.4 (0.14)	1400.4 (0.18)	1326.9 (0.07)	1519.0 (0.25)	1336.4 (0.33)
pD		2	3	4	3	4	3	7
Rank		7	4	1	5	2	6	3

Source: Authors' calculation from IDR/USD returns.

In general, the estimation results related to the leverage and Taylor effects suggest that the APARCH model fits the IDR/JPY and IDR/USD returns. This results are confirmed by DIC value for the IDR/JPY returns only. Meanwhile, in the IDR/USD case, the DIC value suggests that the TARCH model provides the best fit among all competing models. It means that the leverage effect is only suggested to be accommodated into the IDR/USD returns. In the IDR/EUR case, the DIC value suggests that the GARCH model provides the best fit.

On the basis of the best fitting models, volatility models for the IDR/EUR, IDR/JPY, and IDR/USD returns are now respectively expressed by

$$\begin{aligned} \sigma_t^2 &= 0.027 + 0.056R_{t-1} + 0.884\sigma_{t-1}^2, \\ \sigma_t^{1.124} &= 0.127 + 0.121(|R_{t-1}| + 0.363R_{t-1})^{1.124} + 0.753\sigma_{t-1}^{1.124}, \\ \sigma_t &= 0.023 + 0.231(|R_{t-1}| + 0.152R_{t-1}) + 0.782\sigma_t. \end{aligned}$$

Figure 2 presents the time series plots of the variance (squared volatility) for the above models. Comparing Figure 1 and Figure 2 shows that a large piece of returns (positive or negative) leads to a high volatility and a small piece of returns

leads to a low volatility, indicating volatility clustering. In particular, in the IDR/EUR case, the periods of high volatility occurred in May 2010, September and October 2011, June 2012, December 2014, and August to December 2015. For the IDR/JPY returns, volatility was high in January 2010, May 2010, June 2013, and December 2014. Finally, the IDR/USD returns have high volatility in September to October 2011 and October 2015.

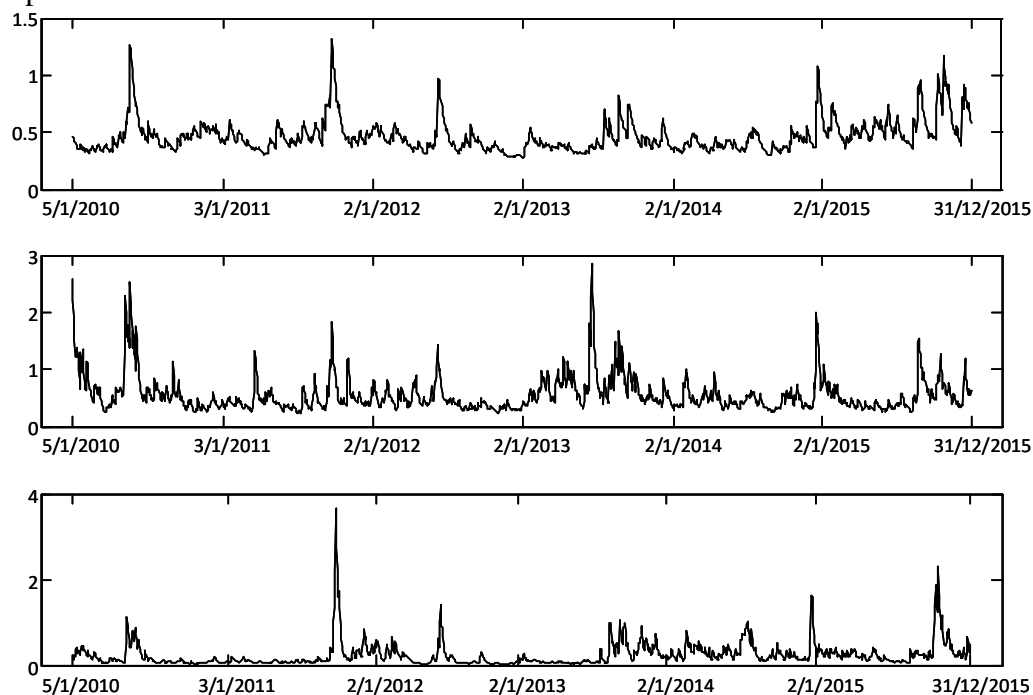


Figure 2. Time series plots for daily variance of the IDR/EUR (top), IDR/JPY (middle), and IDR/USD (bottom) returns estimated from the GARCH, APARCH, and TARCH models, respectively.

Source: Authors' calculation.

CONCLUSION

This study compared several APARCH (1,1)-type models, including the ARCH, GARCH, TARCH, TS-GARCH, GJR-GARCH, NARCH, and APARCH models. These models were applied to the daily returns of the EUR, JPY, and USD selling exchange rate to the IDR covering the period of January 2010 to 2015. The empirical results demonstrated that the GARCH, APARCH, and TARCH models best fitted to the EUR, JPY, and USD data.

FUTURE WORK

Future work may explore the non-normality of exchange rate returns as in many studies and reviews. Among the well-known ones are Student- t distribution and generalized Student- t distributions (e.g., see Nugroho et al. (2014, 2016)).

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