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Gluon Schwinger-Dyson equation in the PT-BFM scheme

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Abstract. Schwinger-Dyson equations provide an appropriate framework for tackling non-perturbative QCD phenomena requiring a continuum treatment. However, an inadequate truncation of this tower of integral equations can compromise the symmetries underlying the theory in question. The synthesis of the Pinch Technique and the Background Field method provides a framework where it is possible to devise a self-consistent truncation scheme, exploiting the Ward identities satisfied by the effective Green's functions that emerge. In this work we review how this truncation scheme is implemented, and show that the new series of dressed diagrams for the background gluon propagator organizes itself in characteristic subsets that are *individually transverse*. In addition, we discuss how the Background Quantum identity connects the background gluon propagator with the conventional one, computed in the lattice simulations.

1. Introduction

A variety of interesting QCD phenomena, such as chiral symmetry breaking, formation of bound states, and dynamical mass generation for quarks and gluons occur in the non-perturbative regime of the theory [1, 2, 3]. In particular, it is well know that the symmetries of the QCD Lagrangian prohibit a gluon mass term, and therefore its appearance can only happen through purely dynamical effects of non-perturbative nature [4]. Those effects must be triggered by a subtle mechanism which will guarantee that the local gauge invariance remains intact [3].

The most widely used continuum approach for the study of non-perturbative QCD is based on the Schwinger-Dyson equations (SDEs). The SDEs consist in an infinite system of coupled integral equations connecting all Green's functions of the theory. A major difficulty in dealing with this system is related to the fact that it is necessary to truncate the series at some level. In general, a good truncation scheme is considered to be one where a tractable subset of these equations is selected and, at the same time, the underlying symmetries of the theory are not compromised. Most importantly, the gauge invariance must be kept intact at all stages of any truncated treatment [5, 6, 7].

Here we are particularly interested in the non-perturbative behavior of the gluon self-energy. The advent of a self-consistent truncation scheme that preserves the transversality of the gluon self-energy is a highly non-trivial task, mainly because the fully dressed vertices appearing in the standard gluon SDE satisfy complicated Slavnov-Taylor identities (STIs). As a result, the transversality of the gluon self-energy emerges only after the contributions of all diagrams appearing in Fig. 1 have been included [3].

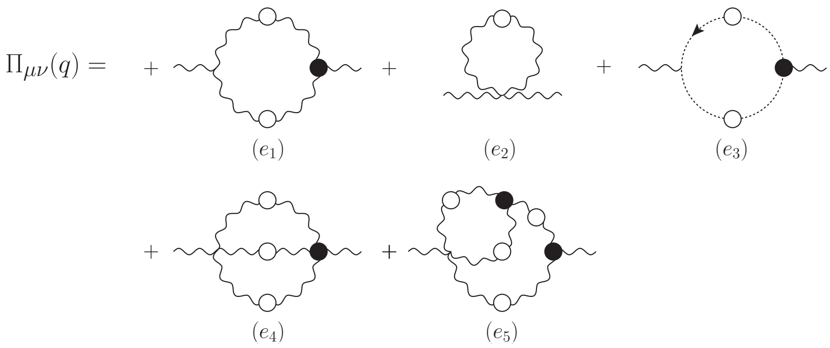

Figure 1. The conventional gluon self-energy. Wavy lines with white blobs represent full gluon propagators, while dashed lines with white blobs are full quantum ghost ones. The black blobs represent full vertices.

The formulation of the SDE based on the synthesis of the Pinch Technique (PT) [4, 5, 8] and the Background Field method (BFM) [9], denoted simply as PT-BFM scheme [3, 6, 7], provides a suitable framework for eliminating this drawback. Essentially, this special framework allows the construction of a new SDE series for the gluon self-energy, where the dressed diagrams are organized in a natural way into independently transverse subgroups, thus leading to a transversality preserving truncation scheme [5, 6, 7]. The reason why such a truncation becomes possible may be traced back to the fact that the PT-BFM Green's functions satisfy Abelian Ward identities (WIs), in contradistinction to the ghost-infested STIs satisfied by the conventional functions; for more details see Ref. [5].

In this work we will review the main results obtained in Ref. [6], where the PT-BFM truncation scheme for the gluon SDE was first presented. To do that, in the Section 2 we introduce all the WIs satisfied by the vertices appearing in the PT-BFM gluon SDE. Then, in Section 3 we show how the transversality in blocks emerges within this formalism. In Section 4 we discuss how the background quantum identity (BQI) [10] connects the two distinct gluon propagators: *i.e.* the conventional and the PT-BFM one, appearing in the resulting gluon SDE [5, 6, 7]. Finally, in Section 5 we summarize our conclusions.

2. The PT-BFM formalism and the WIs

In the PT-BFM framework one distinguishes between three types of gluon propagators, depending on the nature of the the external gluon legs [5]. The legs can be formed either by a background (B) or by a quantum (Q) gluon. More specifically, the BB gluon propagator, to be denoted $\widehat{\Delta}_{\mu\nu}(q)$, is formed by two B gluons. On the other hand, the BQ propagator, $\widetilde{\Delta}_{\mu\nu}(q)$, is composed by one Q and other B gluon fields. Finally, the conventional propagator, $\Delta_{\mu\nu}(q)$, is formed by QQ gluons.

In general covariant R_ξ gauges, the BB gluon propagator is defined as

$$\widehat{\Delta}_{\mu\nu}(q) = -i\left[P_{\mu\nu}(q)\widehat{\Delta}(q^2) + \xi\frac{q_\mu q_\nu}{q^4}\right], \tag{1}$$

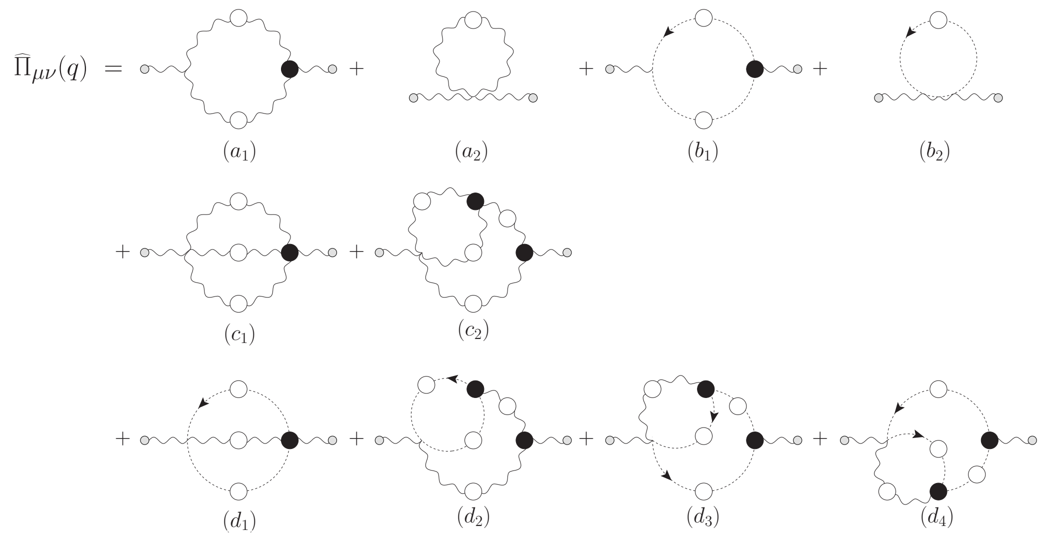

Figure 2. The PT-BFM gluon self-energy. Wavy lines with white blobs represent full quantum gluon propagators, while dashed lines with white blobs are full quantum ghost ones. External lines ending with a small gray circle are background gluons. The black blobs represent full vertices.

where the transverse projector reads

$$P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}. \tag{2}$$

The scalar co-factor $\widehat{\Delta}(q^2)$ is related to the all-order BB self-energy $\widehat{\Pi}_{\mu\nu}(q)$ by

$$\widehat{\Pi}_{\mu\nu}(q) = P_{\mu\nu}(q)\widehat{\Pi}(q^2); \qquad\qquad \widehat{\Delta}^{-1}(q^2) = q^2 + i\widehat{\Pi}(q^2), \tag{3}$$

where the diagrammatic representation of $\widehat{\Pi}_{\mu\nu}(q)$ is given in Fig. 2.

On the other hand, the definition for the conventional propagator $\Delta_{\mu\nu}(q)$ can be obtained by just omitting the hats in the above equations. However, in this case the diagrammatic representation of the self-energy $\Pi_{\mu\nu}(q)$ is given in Fig. 1.

Let us now concentrate on the structure of the PT-BFM self-energy given in Fig. 2. We notice that $\widehat{\Pi}_{\mu\nu}(q)$ is built out of ten dressed diagrams, to be grouped into the following four categories: (a) one-loop dressed gluonic diagrams containing gluons only [diagrams (a_1) and (a_2)], (b) one-loop dressed diagrams with internal ghost lines [(b_1) and (b_2)], (c) two-loop dressed gluonic diagrams [(c_1) and (c_2)], and finally, (d) the two-loop dressed ghost contribution [(d_1), (d_2), (d_3), (d_4)].

Here we will show that the aforementioned groups of diagrams are separately transverse [6]. In other words, we will demonstrate that for each subgroup

$$q^\mu \, \widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(i)} = 0, \tag{4}$$

where q^μ is the gluon momentum, and $i = a$, b, c or d refers to each group of diagrams appearing in Fig. 2.

Notice that $\widehat{\Pi}_{\mu\nu}(q)$ of Fig. 2 contains vertices of the type BQQ, $B\bar{c}c$, $BQQQ$, and $BQ\bar{c}c$, to be denoted $\widetilde{\mathbf{\Gamma}}_{\mu\alpha\beta}$, $\widetilde{\mathbf{\Gamma}}_{\mu}$, $\widetilde{\mathbf{\Gamma}}_{\mu\nu\alpha\beta}$ and $\mathbf{\Gamma}_{\nu\alpha\beta}$, respectively. The trilinear vertices satisfy the following

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WIs [3, 5, 6]

$$
\begin{aligned}
q_1^{\mu}\widetilde{\mathbf{\Gamma}}^{abc}_{\mu\alpha\beta}(q_1,q_2,q_3) &= gf^{abc}\left[\Delta^{-1}_{\alpha\beta}(q_2)-\Delta^{-1}_{\alpha\beta}(q_3)\right],\\
q_1^{\mu}\widetilde{\mathbf{\Gamma}}^{acb}_{\mu}(q_1,q_2,q_3) &= gf^{abc}\left[D^{-1}(q_2)-D^{-1}(q_3)\right],
\end{aligned} \tag{5}
$$

where in the RHS of these expressions appear the inverse of the all-order QQ gluon, $\Delta_{\alpha\beta}(q)$, and ghost, $D(q)$, propagators.

Similarly, the four-point Green's functions, $\widetilde{\mathbf{\Gamma}}_{\mu\nu\alpha\beta}$ and $\widetilde{\mathbf{\Gamma}}_{\mu\nu}$, satisfy the WIs [3, 5, 6]

$$
\begin{aligned}
q_1^{\mu}\widetilde{\mathbf{\Gamma}}^{abcd}_{\mu\nu\alpha\beta}(q_1,q_2,q_3,q_4) &= igf^{abx}\mathbf{\Gamma}^{cdx}_{\alpha\beta\nu}(q_3,q_4,q_1+q_2)\\
&\quad +igf^{acx}\mathbf{\Gamma}^{dbx}_{\beta\nu\alpha}(q_4,q_2,q_1+q_3)\\
&\quad +igf^{adx}\mathbf{\Gamma}^{bcx}_{\nu\alpha\beta}(q_2,q_3,q_1+q_4),
\end{aligned} \tag{6}
$$

and

$$
\begin{aligned}
q_1^{\mu}\widetilde{\mathbf{\Gamma}}^{cdba}_{\mu\nu}(q_1,q_2,q_3,q_4) &= -igf^{cdx}\mathbf{\Gamma}^{axb}_{\nu}(q_4,q_2+q_1,q_3)\\
&\quad -igf^{cbx}\mathbf{\Gamma}^{adx}_{\nu}(q_4,q_2,q_3+q_1)\\
&\quad -igf^{cax}\mathbf{\Gamma}^{xdb}_{\nu}(q_4+q_1,q_2,q_3),
\end{aligned} \tag{7}
$$

where on the RHS we have the all-order conventional QQQ ($\Gamma_{\nu\alpha\beta}$) and $Q\bar{c}c$ (Γ_{ν}) vertices.

3. Transversality Property of the gluon SDE

Now we are in position to show how the separation of the diagrams into independently transverse groups emerges. To that end, we will exploit the rearrangements that the above WIs enforce, when the vertices in the diagrams are contracted by the external gluon momentum [6].

Let us start with the contraction of diagram (a_1) by q^{ν}; with the help of the first line of Eq. (5) we can write

$$
\begin{aligned}
q^{\nu}\,\widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(a_1)} &= \frac{1}{2}\int_k \widetilde{\Gamma}^{aex}_{\mu\alpha\beta}\Delta^{\alpha\rho}_{ee'}(k)\left[q^{\nu}\widetilde{\mathbf{\Gamma}}^{be'x'}_{\nu\rho\sigma}\right]\Delta^{\beta\sigma}_{xx'}(k+q)\\
&= \frac{1}{2}\int_k \widetilde{\Gamma}^{aex}_{\mu\alpha\beta}\Delta^{\alpha\rho}_{ee'}(k)gf^{be'x'}\left[\Delta^{-1}_{\rho\sigma}(k+q)-\Delta^{-1}_{\rho\sigma}(k)\right]\Delta^{\beta\sigma}_{xx'}(k+q),
\end{aligned} \tag{8}
$$

where the brackets have been employed to emphasize where the use of the WI takes place. In addition, we have introduced the compact notation $\int_k \equiv \int \mathrm{d}^d k/(2\pi)^d$, where $d=4-\epsilon$ is the dimension of space-time, appearing in the dimensional regularization.

To proceed further, we use into Eq. (8) the Feynman rule for the bare PT-BFM three gluon vertex, and recalling that $f^{aex}f^{bex}=C_A\delta^{ab}$ we obtain

$$
\begin{aligned}
q^{\nu}\,\widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(a_1)} &= \frac{1}{2}g^2C_A\delta^{ab}\int_k\left\{\left[k-q-\frac{1}{\xi_Q}(k+q)\right]_{\gamma}g_{\alpha\mu}+\left[q+(k+q)-\frac{k}{\xi_Q}\right]_{\alpha}g_{\gamma\mu}\right.\\
&\quad \left.+\left[-(k+q)-k\right]_{\mu}g_{\gamma\alpha}\right\}\times\left[\Delta^{\gamma\alpha}(k)-\Delta^{\gamma\alpha}(k+q)\right].
\end{aligned} \tag{9}
$$

Performing the shift $k\to k+q$, it is straightforward to show that $\int_{k+q}\Delta^{\gamma\alpha}(k+q)=\int_k\Delta^{\gamma\alpha}(k)$, which leads to

$$
q^{\nu}\,\widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(a_1)} = C_Ag^2\delta^{ab}q_{\mu}\int_k\Delta^{\rho}_{\rho}(k). \tag{10}
$$

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Then, a direct calculation of the contraction of diagram (a_2) by q^ν yields

$$
\begin{aligned}
q^\nu\, \widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(a_2)} &= \frac{1}{2} q^\nu \int_k \widetilde{\Gamma}^{abex}_{\mu\nu\alpha\beta} \Delta^{\alpha\beta}_{ex}(k) \\
&= -C_A g^2 \delta^{ab} q_\mu \int_k \Delta^\rho_\rho(k). \qquad (11)
\end{aligned}
$$

Thus, from Eqs. (10) and (11) follows that the subgroup (a) obeys

$$
q^\nu\, \widehat{\Pi}^{ab}_{\mu\nu}(q)\Big|_{(a)} = q^\nu \left(\widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(a_1)} + \widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(a_2)} \right) = 0, \qquad (12)
$$

which implies that the sum of the diagrams (a_1) and (a_2) is independently transverse.

Now let us turn our attention to the diagrams of the group (b). In this case the proof of the transversality uses the second line of Eq. (5). Following the same steps performed in the previous case, one finds that

$$
\begin{aligned}
q^\nu\, \widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(b_1)} &= -\int_k \widetilde{\Gamma}^{aex}_{\mu} D_{ee'}(k) \left[q^\nu \widetilde{\mathbf{\Gamma}}^{be'x'}_{\nu} \right] D_{xx'}(k+q) \\
&= 2C_A g^2 \delta^{ab} q_\mu \int_k D(k), \qquad (13)
\end{aligned}
$$

and

$$
q^\nu\, \widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(b_2)} = -q^\nu \int_k \widetilde{\Gamma}^{abex}_{\mu\nu} D_{ex} = -2C_A g^2 \delta^{ab} q_\mu \int_k D(k). \qquad (14)
$$

Then, we find that the combination of diagrams (b) vanishes, *i.e*

$$
q^\nu\, \widehat{\Pi}^{ab}_{\mu\nu}(q)\Big|_{(b)} = q^\nu \left(\widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(b_1)} + \widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(b_2)} \right) = 0, \qquad (15)
$$

which proves the transversality of the subgroup (b).

The analysis of the two-loop dressed diagrams is slightly more difficult. We begin with group (c). The contraction of the diagram (c_1) by q^ν triggers the WIs of the Eq. (6), where $q_1 = -q$, $q_2 = l + q$, $q_3 = -k - l$ and $q_4 = k$, yieldings the result

$$
\begin{aligned}
q^\nu\, \widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(c_1)} &= \frac{1}{6} \int_k \int_l \widetilde{\Gamma}^{acex}_{\mu\alpha\beta\gamma} \Delta^{\alpha\alpha'}_{cc'}(k) \Delta^{\beta\beta'}_{ee'}(k+l) \Delta^{\gamma\gamma'}_{xx'}(l+q) \left[q^\nu \widetilde{\mathbf{\Gamma}}^{bx'e'c'}_{\nu\gamma'\beta'\alpha'} \right] \\
&= \frac{ig}{6} \int_k \int_l \widetilde{\Gamma}^{acex}_{\mu\alpha\beta\gamma} \Delta^{\alpha\alpha'}(k) \Delta^{\beta\beta'}(k+l) \Delta^{\gamma\gamma'}(l+q) \Big[f^{bxi} \mathbf{\Gamma}^{eci}_{\beta'\alpha'\gamma'}(-k-l, k, l) \\
&\quad + f^{bei} \mathbf{\Gamma}^{cxi}_{\alpha'\gamma'\beta'}(k, l+q, -q-k-l) + f^{bci} \mathbf{\Gamma}^{xei}_{\gamma'\beta'\alpha'}(l+q, -k-l, k-q) \Big]. \qquad (16)
\end{aligned}
$$

At this point, since $\widetilde{\Gamma}^{acex}_{\mu\alpha\beta\gamma}$, does not depend on the momenta, we can shift the integration momenta for the different terms as in the derivations above. Then, with an appropriate relabeling of the dummy indices, it is possible to show that all three terms contribute equally [6], leading to

$$
q^\nu\, \widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(c_1)} = \frac{1}{2} i g f^{bxi} \int_k \int_l \widetilde{\Gamma}^{acex}_{\mu\alpha\beta\gamma} \Delta^{\alpha\alpha'}(k) \Delta^{\beta\beta'}(k+l) \Delta^{\gamma\gamma'}(l+q) \mathbf{\Gamma}^{eci}_{\beta'\alpha'\gamma'}(-k-l, k, l). \qquad (17)
$$

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On the other hand, for the diagram (c_2) we invoke again the first WI of Eq. (5), which lead us to

$$\begin{aligned}
q^\nu\,\widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(c_2)} &= \frac{1}{2}\int_k\int_l \widetilde{\Gamma}^{acex}_{\mu\alpha\beta\gamma}\Delta^{\alpha\alpha'}_{cc'}(k)\Delta^{\beta\beta'}_{ee'}(k+l)\mathbf{\Gamma}^{ne'c'}_{\sigma\beta'\alpha'}\Delta^{\sigma\sigma'}_{nn'}(l)\left[q^\nu\widetilde{\mathbf{\Gamma}}^{bx'n'}_{\nu\gamma'\sigma'}\right]\Delta^{\gamma\gamma'}_{xx'}(l+q)\\
&= -\frac{1}{2}igf^{bxi}\int_k\int_l \widetilde{\Gamma}^{acex}_{\mu\alpha\beta\gamma}\Delta^{\alpha\alpha'}(k)\Delta^{\beta\beta'}(k+l)\mathbf{\Gamma}^{iec}_{\gamma'\beta'\alpha'}(l,-k-l,k)\Delta^{\gamma\gamma'}(l+q)\\
&\quad+\frac{1}{2}igf^{bxi}\int_k\int_l \widetilde{\Gamma}^{acex}_{\mu\alpha\beta\gamma}\Delta^{\alpha\alpha'}(k)\Delta^{\beta\beta'}(k+l)\mathbf{\Gamma}^{iec}_{\gamma'\beta'\alpha'}(l,-k-l,k)\Delta^{\gamma\gamma'}(l). \qquad (18)
\end{aligned}$$

The last term is independent of q and must vanish identically, since the free Lorentz index μ cannot be saturated. Then, observing that the conventional three-gluon vertex inside the integral is Bose symmetric, $\mathbf{\Gamma}^{eci}_{\beta'\alpha'\gamma'}(-k-l,k,l) = \mathbf{\Gamma}^{iec}_{\gamma'\beta'\alpha'}(l,-k-l,k)$, we conclude that

$$q^\nu\,\widehat{\Pi}^{ab}_{\mu\nu}(q)\Big|_{(c)} = q^\nu\left(\widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(c_1)} + \widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(c_2)}\right) = 0. \qquad (19)$$

Finally, let us analyze the transversality of group (d). For the calculation of the divergence of the graph (d_1) we use the WI of Eq. (7), obtaining

$$\begin{aligned}
q^\nu\,\widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(d_1)} &= -\int_k\int_l \widetilde{\Gamma}^{acex}_{\mu\alpha}D_{cc'}(k+l)\Delta^{\alpha\beta}_{ee'}(k)D_{xx'}(l-q)\\
&\quad\times\left[q^\nu\widetilde{\mathbf{\Gamma}}^{bx'e'c'}_{\nu\beta}(-q,q-l,-k,l+k)\right]\\
&= ig\int_k\int_l \widetilde{\Gamma}^{acex}_{\mu\alpha}D(k+l)\Delta^{\alpha\beta}(k)D(l-q)\left[f^{ebi}\mathbf{\Gamma}^{bxi}_{\beta}(l+k,-l,-k)\right.\\
&\quad\left.+f^{bei}\mathbf{\Gamma}^{cxi}_{\beta}(l+k,q-l,-k-q)+f^{bci}\mathbf{\Gamma}^{ixe}_{\beta}(k+l-q,q-l,-k)\right]. \qquad (20)
\end{aligned}$$

Each of these three terms cancels one of the remaining three diagrams. Here, we will show only the cancellation of diagram (d_2). Using once again the first WI of Eq. (5), one obtains

$$\begin{aligned}
q^\nu\,\widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(d_2)} &= -\int_k\int_l \widetilde{\Gamma}^{acex}_{\mu\alpha}D_{cc'}(k+l)D_{ee'}(l-q)\mathbf{\Gamma}^{e'nc'}_{\beta}(q-l,-k-q,k+l)\\
&\quad\times\Delta^{\beta\beta'}_{nn'}(k+q)\left[q^\nu\widetilde{\mathbf{\Gamma}}^{bx'n'}_{\nu\alpha'\beta'}(-q,-k,k+q)\right]\Delta^{\alpha\alpha'}_{xx'}(k)\\
&= -igf^{bxi}\int_k\int_l \widetilde{\Gamma}^{acex}_{\mu\alpha}D(k+l)D(l-q)\mathbf{\Gamma}^{eic}_{\beta}(q-l,-k-q,k+l)\\
&\quad\times\left[\Delta^{\alpha\beta}(k)-\Delta^{\alpha\beta}(k+q)\right]. \qquad (21)
\end{aligned}$$

Now, shifting the second integral by $k \to k+q$ and $l-q \to l$, we again obtain a q-independent integral with a free Lorentz index, which cannot be saturated. Then, the remaining integral cancel against the second term of the Eq. (20).

Similarly, one can show the cancellation of graphs (d_3) and (d_4) against the remaining terms of Eq. (20) [6]. Finally, this cancellation lead us to the conclusion that

$$q^\nu\,\widehat{\Pi}^{ab}_{\mu\nu}(q)\Big|_{(d)} = q^\nu\left(\widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(d_1)} + \widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(d_2)} + \widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(d_3)} + \widehat{\Pi}^{ab}_{\mu\nu}\Big|_{(d_4)}\right) = 0, \qquad (22)$$

i.e. the subgroup (d) is also transverse.

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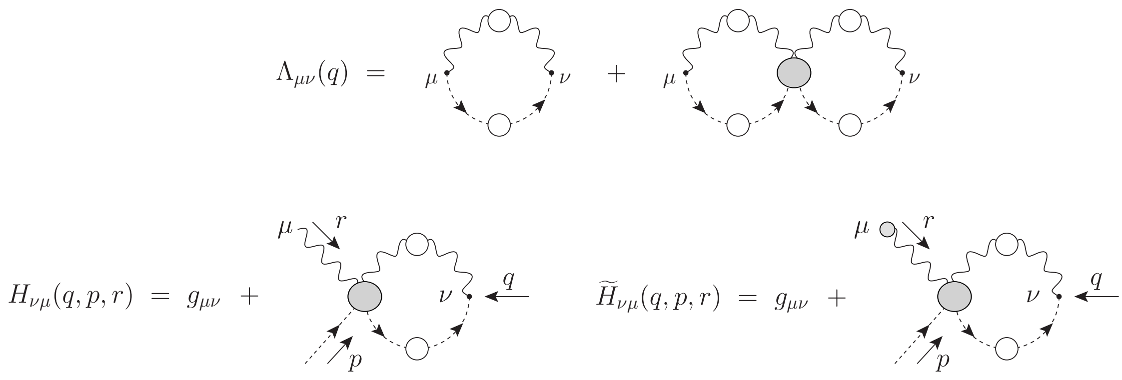

Figure 3. Diagrammatic representation of the auxiliary functions $\Lambda_{\mu\nu}$ and $H_{\mu\nu}$.

Then, it follows directly from Eqs. (12), (15), (19), and (22) that the full PT-BFM self-energy can be written as

$$\widehat{\Pi}(q^2)P_{\mu\nu}(q) = \left[\widehat{\Pi}(q^2)\Big|_{(a)} + \widehat{\Pi}(q^2)\Big|_{(b)} + \widehat{\Pi}(q^2)\Big|_{(c)} + \widehat{\Pi}(q^2)\Big|_{(d)}\right] P_{\mu\nu}(q)\,, \tag{23}$$

where we have omitted the color indices.

As announced, the four groups of diagrams defining the PT-BFM gluon self-energy are independently transverse. In addition, we showed that the gluonic and ghost contributions are independently transverse. Moreover, we also demonstrated that the transversality properties of the one-loop and two-loop dressed diagrams do not mix.

4. A dynamical equation for the PT-BFM gluon propagator

As mentioned previously the PT-BFM SDE given in the Fig. 2, mixes the BB gluon propagator, $\widehat{\Delta}_{\mu\nu}(q)$, with the conventional gluon propagator, $\Delta_{\mu\nu}(q)$, appearing in the internal lines of the diagrams. In order to convert, this equation into a *dynamical* integral equation, it is necessary to translate $\Delta_{\mu\nu}$ into $\widehat{\Delta}_{\mu\nu}$ or vice-versa. As has been explained in the literature [3, 5, 6], we can accomplish this task by employing the so-called Background Quantum Identities (BQIs), which relate the PT-BFM Green's function with the conventional ones [10].

In the case of the gluon propagators, the corresponding BQI states that

$$\widehat{\Delta}(q) = \left[1 + G(q^2)\right]^2 \Delta(q). \tag{24}$$

The function $G(q^2)$ that appears in the above equation is defined as the scalar co-factor of $g_{\mu\nu}$ in the Lorentz decomposition of a special two-point function $\Lambda_{\mu\nu}(q)$, defined diagrammatically in the Fig. 4 [5, 6, 7]. More specifically, we can write

$$\begin{aligned} \Lambda_{\mu\nu}(q) &= \int_k H^{(0)}_{\mu\alpha} D(k)\Delta^{\alpha\beta}(k+q)H_{\nu\beta}(k+q,-k,-q) \\ &= g_{\mu\nu}G(q^2) + q_\mu q_\nu L(q^2), \end{aligned} \tag{25}$$

where $H^{(0)}_{\mu\nu} = -igg_{\mu\nu}$. Its fully-dressed $H_{\alpha\beta}$ is related to the conventional gluon-ghost vertex $\mathbf{\Gamma}_\beta$ by the following STI [11]

$$q^\alpha H_{\alpha\beta}(p,r,q) = \mathbf{\Gamma}_\beta(p,r,q). \tag{26}$$

Using Eqs. (3), (23) and (24), we can rewrite the SDE exclusively in terms of the conventional propagator in the following way [6]

$$\Delta^{-1}(q^2) = \frac{q^2 + i\widehat{\Pi}(q^2)}{[1+G]^2}\,, \tag{27}$$

since the diagrams that define $\widehat{\Pi}(q^2)$ involve only conventional propagators, $\Delta^{(}q^2)$.

Notice that the PT-BFM formalism provides a systematic way of truncating the gluon SDE without compromising the gauge invariance of the equation [3, 6, 7]. For example, the simplest truncation scheme would be one where we retain only the one-loop gluonic contribution, *i.e.* the diagrams of the group (a). A improved version of this truncation is the case where the ghost effects at one-loop dressed are also included, *i.e.* retain diagrams of the group (a) and (b). Then, we can improve further by including the two purely gluonic two-loop dressed diagrams of the group (c). Finally, the gluon SDE will be treated thoroughly when the group (d), is finally entirely included into the game.

5. Conclusion

In this presentation we have reviewed how the transversality properties of the gluon SDE emerge in blocks within the PT-BFM truncation scheme. In particular, we have shown that the formulation of this SDE in the context of this formalism furnishes considerable advantages, because it allows for a systematic truncation that respects manifestly, and at every step the gauge-invariance, expressed by the transversality of the gluon self-energy. We also reviewed that one may use the SDE for $\widehat{\Delta}(q)$, take advantage of its improved truncation properties, and then convert it to an equivalent equation for $\Delta(q)$ (the conventional gluon propagator simulated on the lattice) by means of the BQIs. It is important to mention that different versions of truncations, within this scheme, have been studied in the last few years [3, 6, 7, 12], and all results found so far corroborate the notion of an IR finite, massive gluon propagator.

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