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Architecting new diffraction-resistant light structures and their possible applications in atom guidance

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Abstract: In this work we extend the so called frozen wave method in order to obtain new diffraction resistant light structures that can be shaped on demand, with possible applications in atom guidance. The resulting beams and the corresponding optical dipole potentials exhibit a strong resistance to diffraction effects and their longitudinal and transverse intensity patterns can be chosen *a priori*. Besides the theoretical development, we also present the experimental confirmation of our approach; specifically, by generating three different beam profiles using a spatial light modulator that is addressed by a computer-generated hologram. In addition to its many potential applications in atom guiding, the method developed here can also lead to many new developments in optics and photonics in general.

OCIS codes: (260.0260) Physical optics; (260.1960) Diffraction theory; (140.3300) Laser beam shaping; (020.1335) Atom optics; (020.7010) Laser trapping.

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1. Introduction

The most common interactions between laser light and atoms, molecules, or dielectric particles are represented by the scattering and dipole forces [1–3]. More specifically, the dipole force on an atomic system is given by the interaction between the induced atomic dipole moment and the intensity gradient of the optical field [4]. This conservative force (i.e., obtained from an optical potential) can be used to guide atoms along hollow optical beams, which has significant

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advantages over atom guiding through a hollow optical fiber due to the absence of Van der Waals forces and the QED cavities effects [5].

Initially, hollow Gaussian beams, such as the Laguerre-Gauss ones, were considered and used for atom guiding [6, 7]. However, such beams posses limitations due to the diffraction effects, which corrupt their transverse intensity profiles with propagation. Later, an important improvement was achieved by using Bessel beams with order higher than zero [7]; the later belong to the class of nondiffracting waves and can thus overcome the limitations presented by the ordinary Gaussian-type beams.

In spite of their outstanding characteristics, the ideal nondiffrating beams, due to their mathematical structure of the type $\psi = A(x, y)\exp(ik_z z)\exp(-i\omega t)$, do not allow any kind of modelling over their longitudinal intensity pattern (i.e. along the z axis). Such longitudinal spatial modelling can be very interesting for atom guiding and may provide many new degrees of freedom to be exploited. It turns out that, during the development of the Localized Wave theory, new kind of diffraction-resistant beams were introduced, which allow the modelling of their longitudinal intensity profiles on demand [8,9]. Such beams, named frozen waves (FWs), consist of suitable superposition of co-propagating equal order Bessel beams and their experimental confirmation can be found in [10, 11].

This paper is intended to give two contributions: First, taking into account that, despite allowing a strong longitudinal intensity modelling, the FW method is rather restrictive with respect to the transverse intensity shaping (only allowing us to choose the transverse dimensions of the desired beam), we have extended this method proposing, as new beam solutions, superpositions of FWs of different orders, so that the resulting beams can also be transversally modelled in a more efficient way. Second, we propose the use of these new optical beams for atom guiding, giving some theoretical examples, obtaining the corresponding optical dipole potentials and creating the beams through computer generated holograms reproduced by a spatial light modulator.

The next section is devoted to a very brief overview of the optical dipole potential and also of the traditional frozen wave method. In section 3 we present the extension of the FW method, applying it to atom guiding purposes and generating experimentally some of the new optical beams. Section 4 is devoted to the conclusions.

2. Brief overview on the optical potential and on the frozen wave method

Considering an atomic system with two levels, the optical dipole potential created by an optical field $\Psi(\rho, \phi, z)$ is given by [7]

$$U(\rho, \phi, z) = \frac{\hbar\Delta}{2}\ln\left[1 + \frac{I(\rho, \phi, z)/I_0}{1 + 4(\Delta/\Gamma)^2}\right], \tag{1}$$

where $I(\rho, \phi, z) = |\Psi(\rho, \phi, z)|^2$ is the field intensity, Γ is the natural linewidth, Δ is the laser frequency detuning from the doppler-shifted atomic resonance and I_0 is the saturation intensity. Here, we are going to consider the ^{85}Rb, line D_2, where $\Gamma = 2\pi \times 6.1$MHz, $\Delta = 30\Gamma$, I_0=16W/m^2 and the laser angular frequency ($\omega = 2.42 \times 10^{15}$rad/s, i.e., λ =780.2nm) has been located above the atomic transition frequency considering the blue-detuned guiding.

When dealing with atom guidance, an important quantity is the transverse penetration depth into the potential barriers as a function of the propagation distance. The maximum penetration [6] is given by $r_{Apd}(z) = m\rho_c v_{atom}^2/U(\rho_c, z)$, where $m = 85m_a$ is the atomic mass (m_a is the atomic mass unit), v_{atom} is the average atomic velocity ($v_{atom} < 0.07$m/s, $T = 100\mu$K) and ρ_c is the transverse distance, for each value of z, where U is maximum.

Concerning the frozen waves [8,9], they are very special exact solutions to the wave equation consisting in diffraction-resistant beams whose longitudinal intensity pattern can be chosen on demand within a prefixed range $0 \leq z \leq L$ of the propagation axis. This longitudinal intensity pattern can occur over a cylindrical surface of radius $\rho_l \geq 0$. Mathematically, we can construct a

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beam ψ such that $|\psi(\rho = \rho_l, \phi, z, t)|^2 \approx |F(z)|^2$, within $0 \le z \le L$ and with $F(z)$ and ρ_l of our choice.

To obtain these results, a FW beam is composed of a superposition of equal frequency co-propagating Bessel beams of order l:

$$\psi(\rho, \phi, z, t) = \mathcal{M}_l \, e^{-i\omega t} \sum_{n=-N}^{N} A_n J_l(h_n \rho) e^{il\phi} e^{\beta_n z} \;, \tag{2}$$

where $\mathcal{M}_l = 1/[J_l(.)]_{max}$ and $[J_l(.)]_{max}$ is the maximum value of the lth-order Bessel function of the first kind, A_n are constant coefficients, h_n and β_n are the transverse and longitudinal wave numbers of the n-th Bessel beam in the superposition, satisfying the relation $h_n^2 = k^2 - \beta_n^2$, with $k = \omega/c$. The FW method requires the longitudinal wavenumbers to be $\beta_n = Q + 2\pi n/L$, Q being a constant to be selected according to the desired transverse dimensions of the beam, and the coefficients $A_n = (1/L) \int_0^L F(z) e^{-i\frac{2\pi}{L}nz} \mathrm{d}z$.

This is sufficient to obtain Diffraction-Resistant beams with the required longitudinal intensity pattern, concentrated: (i) either along the propagation axis ($\rho = 0$), when zero-order ($l = 0$) Bessel beams are chosen in solution (2); it is then possible to determine the spot radius, r_0, of the resulting beam from the parameter Q via the relation $Q = (k^2 - 2.4^2/r_0^2)^{1/2}$; (ii) or over a cylindrical surface, if a non-null integer is adopted for l in (2); in which case, the cylinder radius can be approximately evaluated as the first non-null root of the equation $[(\mathrm{d}/\mathrm{d}\rho) J_l(\rho\sqrt{k^2 - Q^2})]\Big|_{\rho=\rho_l} = 0$. More details about the FW method can be found in [8,9].

3. Extending the frozen wave method: new structures of diffraction resistant beams and their use for atom guiding

As we have seen, the FW method is very effective for the longitudinal modeling of the beam intensity, but it is somewhat limited in modeling the transverse pattern, allowing to choose only the radius of the spot or of the cylinder where there will be the field concentration; more specifically, in regions where the beam has no neglegible intensity, the beam cross-section is uniform along z. It would be interesting to have more possibilities on the choice of the transverse intensity pattern, considering that such nondiffracting light structures could open new and interesting application possibilities involving optical guiding of atoms (not only for atom guidance, but also for optical tweezers and optical beam manipulation in general). To achieve this, we now propose an extension of the FW method, where the resulting beam is given by a superposition of FWs of different orders. That is, we consider beams of the type:

$$\Psi(\rho, \phi, z, t) \;=\; e^{-i\omega t} \sum_{l=-\infty}^{\infty} \mathcal{M}_l \sum_{n=-N_l}^{N_l} A_{nl} J_l(h_{nl}\rho) e^{il\phi} e^{\beta_{nl} z} \tag{3}$$

with β_{nl}, h_{nl} and $A_{n\,l}$ given by

$$\beta_{nl} \;=\; Q_l + \frac{2\pi}{L} n \;, \quad h_{nl} \;=\; \sqrt{k^2 - \beta_{nl}^2} \quad and \quad A_{nl} \;=\; \frac{1}{L} \int_0^L F_l(z) e^{-i\frac{2\pi}{L}nz} dz \tag{4}$$

The solution given by Eq. (3) is a summation of FW beams of order l, with $-\infty < l < \infty$, each one having its own predefined longitudinal intensity pattern $|F_l(z)|^2$, modelled within $0 \le z \le L$, where the transverse structure of each FW in superposition (3) is obtained in the usual way, detailed in items (i) and (ii) of the preceding section. A judicious superposition of these beams can be employed to create diffraction resistant light structures with very interesting and unusual geometries as discussed in the following subsection.

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3.1. Applying the extended method: theoretical examples and their experimental implementations

Here we will apply the extended method to design some interesting diffraction resistant light beams with quite unusual spatial forms. Once the beam is theoretically derived, the corresponding optical dipole potential can be calculated by using Eq. (1). To confirm the validity of our approach, the new beams are experimentally generated through computer generated holograms reproduced optically by a spatial light modulator at a wavelength $\lambda = 532$nm, which is different from that considered here for the atom guiding ($\lambda = 780.2$nm), the latter due to the availability of the laser sources in our laboratory. The experimental setup is shown in Fig.1 and consists of a 532 nm laser expanded and collimated on a (amplitude) spatial light modulator (Holoeye LC2012 SLM), where it gets the information from a computer generated hologram [10, 11] constructed from the theoretical solution given by Eq. (3). The emanated optical beam goes through a 4 f optical system where a iris filters out the desired FW beam, which is then captured with a CCD camera with 1 cm resolution steps along the longitudinal direction of the beam. It is worth noting that in the following examples the theoretical beam figures appear normalized with respect to the intensity $I = 0.2\ \mu W/\mu m^2$.

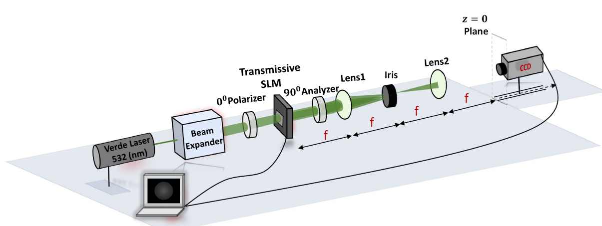

Fig. 1. Experimental setup for the generation of the new optical beams.

3.1.1. Light beam and optical potential shaped as a cylinder with a stopper

Here we will construct a diffraction resistant beam and the corresponding optical dipole potential shaped as a cylinder with a stopper. For this purpose, we use the solution given by Eq. (3) with just two FW beams in the superposition, one of order zero ($l = 0$) and the other of order two ($l = 2$), where the former will be responsible for the "optical stopper" and the latter for the "optical cylinder". The desired spatial structure for the intensities of the beams can be achieved through the functions $F_l(z)$ and the parameters L and Q_l, the latter obtained from the radii r_0 and ρ_2 intended to the stopper and to the cylinder, respectively. From $F_l(z)$, L and Q_l, the coefficients A_{nl} and the longitudinal wavenumbers β_{nl} can be calculated via Eqs.(4), so that the solution (3) is finally fixed. In this example, for $0 \le z \le L$, we make the following choices:

$$F_l(z) = \begin{cases} \delta_{l0} + \delta_{l2} & \text{for } z_{il} \le z \le z_{fl} \\ 0 & \text{elsewhere,} \end{cases} \tag{5}$$

where δ_{pq} is the Kronecker delta, $L = 1$m, $z_{i0} = 6$ cm and $z_{f0} = 25$ cm for the initial and final longitudinal coordinates of the "optical stopper", $z_{i2} = 21$ cm and $z_{f2} = 42$ cm for the initial and final longitudinal coordinates of the "optical cylinder". We also choose the radii $r_0 \approx 54\mu$m and $\rho_2 \approx 69\mu$m, which implies that $Q_0 = Q_2 = 0.999985\omega/c$ when $\lambda = 780.2$ nm. In this case we use $N_0 = N_2 = 12$.

Figures 2(a), 2(b) and 2(c) show, respectively, the theoretically obtained intensity of the beam, the corresponding optical dipole potential (for atom guiding purposes) and the experimentally generated beam. As mentioned before, the experimental generation was performed at $\lambda = 532$

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nm and due to this, for obtaining the same values for the radii r_0 and ρ_2, we have used there $Q_0 = Q_2 = 0.999993\omega/c$. In the figures, the portion above $\rho = 0$ corresponds to $\phi = 0$ and that below corresponds to $\phi = \pi$. Figure 2(j) shows the transverse penetration depth into the barriers as a function of the propagation distance.

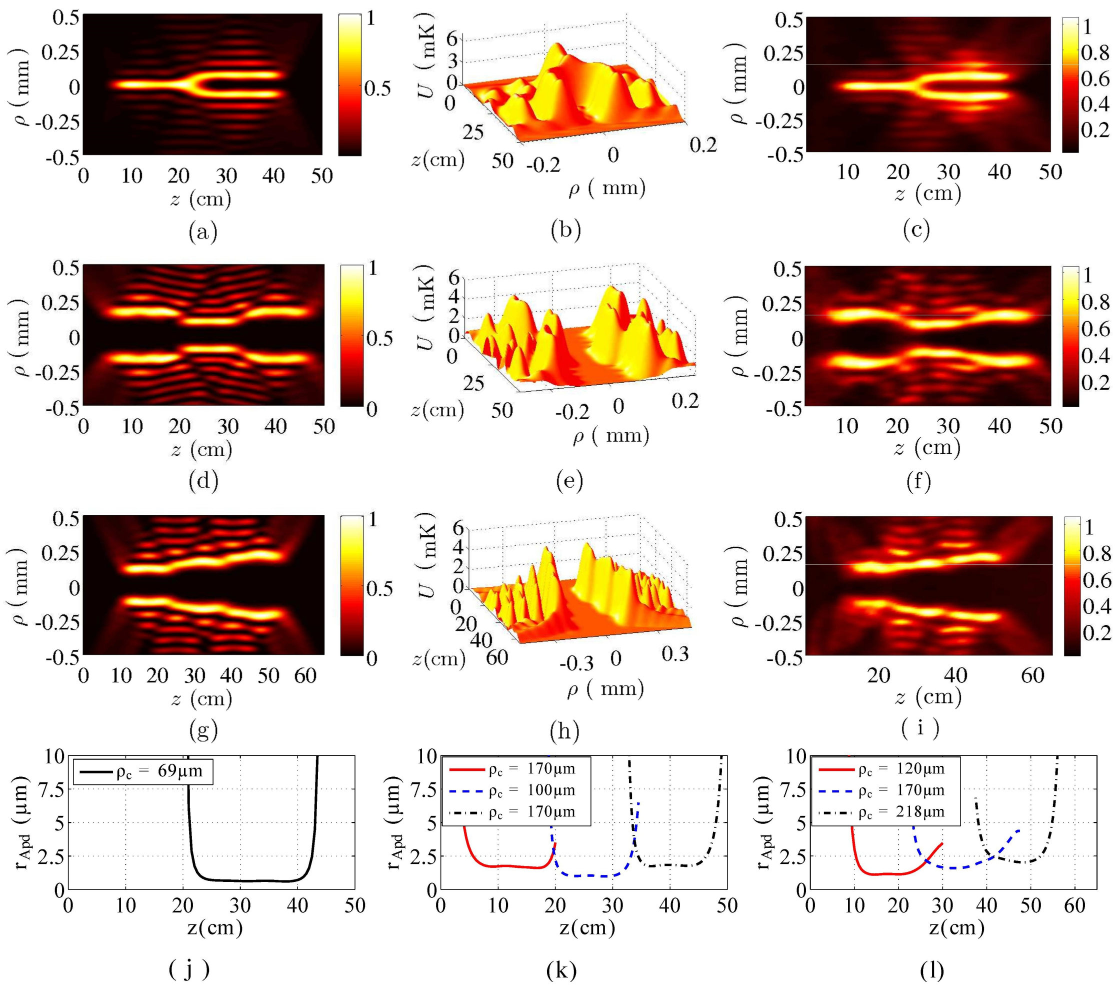

Fig. 2. The first three rows show the three examples considered. In each of these rows the three columns show, respectively, the theoretical beam (normalized with respect to the intensity $I = 0.2\ \mu W/\mu m^2$), the corresponding optical potential and the beam experimentally generated. The fourth row shows the transverse penetration depth for each optical dipole potential.

3.1.2. Light beams and optical potentials shaped as diffraction resistant cylindrical structures with nonuniform cross sections

Let us construct two diffraction resistant light beams, with their respective optical dipole potentials, consisting of cylindrical structures of nonuniform cross sections. More specifically, the two new beams will consist of a sequence of three connected cylinders: in the first case, the first and third cylinders have the same length and radius and are connected by a second cylinder with the same length but with a smaller radius; in the seconde case, the three cylinders connected in sequence have the same length and radii possessing ascending values.

Again, we consider the proposed solution (3), using just two FW beams (of orders $l = 4$ and $l = 6$) for the first case and three FW beams (of orders $l = 4$, $l = 6$ and $l = 8$) for the second one. The desired spatial structure for the beams intensities can be achieved through the functions $F_l(z)$ and the parameters L and Q_l (obtained from the desired radii of the cylinders) of each case, as we detail below.

First case:

In this example, for $0 \leq z \leq L$, we make the following choices:

$$F_l(z) = \begin{cases} \delta_{l6} + \delta_{l4} & \text{for } z_{il} \leq z \leq z_{fl} \\ \delta_{l6} & \text{for } z'_{il} \leq z \leq z'_{fl} \\ 0 & \text{elsewhere,} \end{cases} \tag{6}$$

with $L = 1$ m, $z_{i6} = 4$ cm, $z_{f6} = 20$ cm, $z_{i4} = 19.5$ cm, $z_{f4} = 32.5$ cm, $z'_{i6} = 32$ cm and $z'_{f6} = 48$ cm, for the initial and final longitudinal coordinates of the three light cylinders connected in sequence. We also choose the radii of the first and third cylinder as $\rho_6 \approx 170\,\mu$m, which implies that $Q_6 = 0.999985\omega/c$ and the radius of the second cylinder as $\rho_4 \approx 100\,\mu$m, which results in $Q_4 = 0.999978\omega/c$ when $\lambda = 780.2$ nm. Here, we use $N_4 = N_6 = 12$.

Second case:

Here, for $0 \leq z \leq L$, we choose:

$$F_l(z) = \begin{cases} \delta_{l4} + \delta_{l6} + \delta_{l8} & \text{for } z_{il} \leq z \leq z_{fl} \\ 0 & \text{elsewhere,} \end{cases} \tag{7}$$

with $L = 1$ m, $z_{i4} = 10$ cm, $z_{f4} = 25$ cm, $z_{i6} = 25$ cm, $z_{f6} = 40$ cm, $z_{i8} = 40$ cm and $z_{f8} = 55$ cm, for the initial and final longitudinal coordinates of the three light cylinders connected in sequence. We also choose the radii for the three cylinders as $\rho_4 \approx 120\,\mu$m, $\rho_6 \approx 170\,\mu$m and $\rho_8 \approx 218\,\mu$m, which imply $Q_4 = Q_6 = Q_8 = 0.999985\omega/c$ when $\lambda = 780.2$ nm. Here, we set $N_4 = N_6 = N_8 = 12$.

The intensities of the theoretical beams of the first and second cases are shown in Figures 2(d) and 2(g), respectively, and their corresponding optical potentials are shown in Figures 2(e) and 2(h). The experimental generation results for the two beams are shown in Figures 2(f) and 2(i), where we have used $\lambda = 532$nm and so, for obtaining the same radius values of the theoretical beams, we have used $Q_6 = 0.999993\omega/c$ and $Q_4 = 0.99999\omega/c$ for the first case and $Q_4 = Q_6 = Q_8 = 0.999993\omega/c$ for the second one. Figures 2(k) and 2(l) show the transverse penetration depths into the barriers as function of the propagation distance for the two corresponding optical dipole potentials.

4. Conclusions

We have extended the frozen wave method proposing, as new beam solutions, superpositions of FWs of different orders. As a result, new diffraction-resistant light structures with very interesting spatial shapes can be constructed and we have proposed their use for atom guiding purposes. We have provided some theoretical examples, obtained the corresponding optical dipole potentials and created the beams through computer generated holograms reproduced by a spatial light modulator. The experimental results are in very good agreement with theory and we believe this new approach can open new perspectives for optical beam modelling and applications.

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