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ORIGINAL PAPER

Experiences Situating Mathematical Problem Solving at the Core of Early Childhood Classrooms

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Abstract Our goal in this article is to discuss the importance of problems in early childhood education for the child's development and engagement with the mathematics existing in childhood culture. Our assumption is that an important task for young children's education is to create a democratic and critical environment, in which multiplicity of perspectives is celebrated, along with diversity of concepts and practices, with movement between imaginary and real worlds. In light of this, the goal of this article is to defend a perspective for curriculum and for the role of the mathematics educator, promoting the learning of mathematics through problem solving in early childhood years. In order to discuss and illustrate this perspective we describe the pedagogical practices of two teachers who teach 4- and 5-years-olds, who create for their students an environment rich in problem solving and investigations. In both classrooms, all children individually succeeded in sharing their unique solutions and new knowledge constructed as a result of their inquiries. The experience provides evidence that problem solving affords children the opportunity to raise conjectures, to discuss possibilities and to draw conclusions, even if partial ones, that are then vetted by the group as the authors share their solutions. In this way, the work with problem solving nurtures cooperative learning and promotes the exploration of a diversity of ideas.

Keywords Problem solving · Mathematics education · Early childhood education · Curriculum

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Introduction

The goal of the present article is to discuss a curriculum perspective, and the role of the mathematics educator, in fostering the learning of mathematics through problem solving in early childhood. Curricular issues seem to be an important aspect to emphasize in the discussion of problem solving, as the curriculum needs to reflect what happens in society, where problems naturally emerge. We have examined the work developed by two teachers with students between the ages of 4 and 5, as a way to examine the process of mathematical education in a problem-solving environment.

The underlying assumption of this work is that children should grow and develop in a democratic and inquisitive environment. The goal is for children to experience situations arising from multiple perspectives, involving a variety of concepts, procedures and approaches, and leading them to navigate between reality and imagination.

The children's culture is crucial for their development, and is also the means through which most of the experiences of their lives are reflected, related and interpreted. Such culture must be considered in the planning of educational activities for children: it is important to take their interests, curiosity and playful interactions into account.

Under a Vygotskian perspective, "children will begin to operate with concepts, and employ conceptual thought before they are clearly aware of the nature of such operations" (Vygotsky 1998, p. 86). Therefore, we believe the development of children must take place in dialogical teaching/learning spaces that foster knowledge, awareness, and not just the transfer of information. These are spaces in which knowledge can be socially produced, and new hypotheses for understanding the world can be elaborated by children in a investigative environment created by the teacher.

In shaping our study, we considered the concept of investigative scenarios proposed by Skovsmose (2000): learning spaces in which the students can "mathematize", that is, they can formulate, critique, and develop mathematical ways to understand the world (Skovsmose 2000, p. 51). Such *matematizing* occurs with children in early childhood, as they are already able to create different ways of expressing mathematical thought. There is a misconception that only formal and rigorous institutionalized expression can be identified as mathematics. However, it is important to understand that thought and language are linked, and that there is a manifestation of mathematics that is possible for young children, which leads to a more formal and institutionalized sort of mathematics. The question that arises is: which type of mathematics makes more sense to children?

Children witness processes through which adults sell, buy, swap, measure quantities, evaluate increases, decreases or alterations, use maps, calculate and estimate measurements and distances, etc. Children are embedded in a social and cultural context in which mathematics is ever-present. They experience it by manipulating objects, placing one inside the other, drawing, estimating the duration of enjoyable activities, and understanding quantities. This mathematical knowledge exerts a certain fascination in children and stimulates their epistemological curiosity, thus, arousing the desire to understand the universe around them. The phrase "epistemological curiosity" is used by Freire (1996) and describes the curiosity of children, youngsters and adults about knowledge, the means through which knowledge is produced and the reasons that lead to such production.

In view of that, we advocate the use of problem solving in early childhood, because we consider it to be the foundation of learning since children develop their intelligence as a result of their intentional actions, "[…] however incipient they may be; […] egocentric speech progressively becomes appropriate for planning and solving problems, as the activities of the child become more complex" (Vygotsky 1998, p. 27).

Vygotsky's premise is that spontaneous and non-spontaneous concepts, particularly scientific ones, are related and constantly influence each other, and are parts of a single process, which is the development of concepts occurring in school-age children. Children's development process is unique, in that their different lived experiences shape how they perceive and interpret both scientific and spontaneous concepts. This highlights the importance of problem solving in the daily lives of children as a means for learning. This way, teachers can immerse children in social practice situations, and, thus, explore their cognitive ability, imaginary movements and emotional experiences. Understanding the children will allow teachers to better nurture and promote their problem solving and other strategies that are useful for their life-long learning. Giving children such opportunities promotes systematic knowledge as, under a Vygotskian perspective, we teach the children many things that they could not see or experiment with by themselves.

In recent years, early childhood education has faced challenges in the recognition of its pedagogical function. It is necessary to unveil the complex relationship between learning and the development of scientific concepts, since the thought process of children is not deliberate and fully conscious of itself (Vygotsky 1998). On the one hand, this has raised doubts regarding the preparation of a curriculum, on the other hand, it has brought more clarity about the goals related to children's education, which encompass not only the acquisition of communication, expression, logic and operational skills and competencies, but also their cognitive, affective, social and moral development (Ramani and Brownell 2014).

Given this scenario, in early childhood education it is necessary to create an environment that is democratic and promotes critical thinking, with the goal of celebrating the multiplicity of perspectives, diversity of concepts and practices and the contestability of all knowledge and truth-claims (Moss 2002).

A more meaningful education based on inquiry can result from the recognition of the richness of the culture of the daily lives of children. In a Vygotskian perspective, culture shapes intelligence, and the games and activities we use with children should favor the creation of imaginary situations and reorganize experiences.

According to McLennan (2010, p. 84): "when children are encouraged on a regular basis to explore the process, and not necessarily focus on the creation of a product, they become empowered to create personal, invested understandings about themselves and their places within the world." In this sense, learning should start with games and activities, in which one learns to create meaning, communicate with each other, decode rules, express language, make decisions, and socialize. Creating educational space for such experiences for children requires the adoption of curriculum guidelines that promote mathematical learning through problem solving in early childhood.

Curricular Guidelines for Mathematical Education in Early Childhood

If we believe that the curriculum must reflect what happens in society and, at the same time, enable some sort of intervention in the world, then in order to learn mathematics it is important to use problem solving and questioning beginning in early childhood. This perspective may lead to the inclusion—or exclusion—of themes from the curriculum. It is up to the teachers, within their educational institution, to make this decision. This increases the responsibility of the teachers to constantly update the curriculum. Oliveira (2002) emphasizes that educators working with children must not only be competent but also take into consideration the social and historical scenario of a complex and contradictory world. They must acquire an ethical background and base their teaching actions on a reflective process. This demands emotional investment, commitment to the development of the children, while simultaneously drawing on technical and pedagogical knowledge.

Teaching mathematics in early childhood education means understanding that mathematics is about having your own ideas, listening to the ideas of others, and then formulating and communicating procedures for solving problems. It further suggests an understanding that mathematical activity involves challenging, questioning, problematizing and seeking missing data to solve problems, explore space, elaborate mental images, and produce and organize data. Moreover, it is necessary to involve learners in analyzing the mathematical processes used, discussing mistaken procedures and analyzing what did not work, that is, examining mistakes and proposing new solution paths and strategies.

In addition the intentional effort on the part of the teacher to promote the learning of mathematics by the child cannot be isolated from other areas of knowledge nor defined by steps and stages. Phases or steps do not characterize the acquisition of mathematical language, concepts or forms of registry. For instance, it is a common belief that it is not possible to work with the decimal system before the child acquires the concept of number. As a consequence, excessive focus is given to activities involving ordering, classifying, and sorting in hopes that the children will be able to conserve quantities, and then start working with the concept of number. This is a vision that fragments the acquisition of mathematical knowledge and defines stages of comprehension of numbers by opting to work first with quantities up to 10 and then 20 and 100, etc. The idea of number is constructed in real-life situations through the interactions among children when they face the need to control the variability of quantities (e.g. the score of a game) or even register quantities or numbers in a numeric sequence (e.g. when playing hopscotch).

Childhood play, games, and problem-solving activities enable the development of mathematical concepts that create opportunities for children to construct meaning and establish relationships. Childhood play has also been shown to enhance children's metacognitive and self-regulatory behaviors, considered essential for their development as creative problem solvers (Whitebread et al. 2009).

The work with mathematics in early childhood education prioritizes the mathematical literacy process taking advantage of the intuitive ideas of children that emerge from both social and cultural mathematical experiences. Children's language and their developmental needs are the compass for the exploration of ideas about numbers and the decimal system, space, shapes, measurements, combinatorics, probability, and statistics. Therefore, the teaching/learning process must allow children to develop understanding while simultaneously nurturing the enjoyment and curiosity related to mathematics.

This requires that the plans for early childhood education incorporate contexts and experiences from the universe of the lives of children. It must also be based on their natural language in the development of mathematical notions. The goal is to push beyond what they seem to know or are able to experience physically in order to help them understand their own thoughts and actions.

Such considerations lead to a concept of early childhood education focused on an integrated curriculum that allows the children to learn and develop in order to establish relationships with their universe. They perceive the world holistically, i.e. according to a vision of the human being as an indivisible whole without assigning meaning to isolated knowledge (Lopes 2003).

The premises and concepts presented above make it possible to advocate for working with mathematical content in early childhood education, respecting the social and cultural environment, as well as the learning and developmental possibilities of the children. In consonance with that, children should be involved in investigative scenarios consisting of games, problem solving and playful activities.

In this sense the curriculum for early childhood education, as well as the activities developed for the children, should be primarily interdisciplinary. It is not possible to treat the mathematics embedded in games and other playful activities without linking it to the physical motor development of children or to their mother tongue. Thus, the development of interdisciplinary educational projects is not only possible but also desirable. Mathematics can be present in several themes of the project contributing to a mathematical approach to real-life and practical situations.

Mathematical literacy enables a mathematical view of the world where the children are proficient to analyze the same problem situation under different viewpoints. The perspectives they take may be emotional, social, kinetic, scientific, linguistic, and, dare we say, mathematical.

Thus, a curriculum plan for early childhood must foster artistic, musical, logical-scientific, and pictorial experiences in diversified spaces in contexts appropriate for children. With this multiplicity of ways of operating children may develop several abilities that will result in a comprehensive and balanced education. In the following section, we will analyze the concept of learning mathematics through problem solving.

Problem Solving and Learning Mathematics

Problem solving, used as a means to teach mathematics, points towards the design of a plan for mathematics education that encompasses the social experiences of students. Teaching through problem solving starts with an investigation of students' social interactions and invites them to formulate problems derived from such situations.

The classroom becomes a place for questioning, contextualizing and formulating problems, instead of dealing with ready-made questions and predictable answers. School activities focused on problem solving enable the development of citizens who are equipped to deal with uncertainty, possibilities, and decision-making, thus contributing to their independence and autonomy. All this can start at a very early age, with a problem-solving approach in early childhood education. The question that frequently arises is: how can very young children, who, for the most part, cannot read or write, solve mathematical problems? This type of question reveals the misconception – which must be overcome – that solving mathematical problems means calculating, or employing a set of rules (or an algorithm).

While exploring social relations, manipulating objects and interacting with people, children are able to formulate ideas, test them, and accept or reject what they learn. The construction of knowledge by trial and error is part of the problem-solving process. Through exploration and experimentation one can analyze hypotheses and explore solutions. With this approach, learning becomes personal and meaningful for children. Children construct meaning from their efforts to discover or invent. When the teacher discusses various situations and creates a landscape of investigations marked by time, space and manipulable materials, the children are encouraged to construct their own knowledge.

A pedagogical plan for early childhood education must prioritize social interactions. It must also take into account the children's experiences, along with their emotional, psychological and cognitive needs, enabling each child to gain an understanding of themselves as human beings and of the world in which they live. Another fundamental aspect to be considered is the interaction with others. When children work collectively, they build a sense of cooperation, solidarity, critical judgment and sensibility, perceiving themselves as individuals who can transform society.

Questioning the children's simple daily-life situations can be an interesting pedagogical practice, as it engages the children in mathematical thinking. Day-to-day situations can be the source of interesting problems for children to solve. Examples might include: "How many children are here today? How many are absent? If a monster came into the room, what would you do? Why did you lose the game? How many points would you need to get a draw? What can we take out of this box so that it closes? How do you get in or out of a huge box? How can you know if you still have a chance of winning a game?", among others.

Under this perspective Lopes and Grando (2012) contend that problem solving as a teaching method for early childhood education entails the following:

- Diversity in the manner through which problems are presented (orally, with children's stories; role playing with images; through games and playful interactions; using daily situations; or through physical experiences).
- Elaboration and re-formulation of open-ended problems (problems that admit more than one solution, problems with missing data, or that are unsolvable) with the possibility of attributing different meanings and interpretations to the context of the problem.
- Genuine mathematical reasoning (creating hypotheses, arguments, validation, documentation- writing and rewriting).

Such ideas and considerations can guide the process of learning and teaching mathematics in early childhood education.

Actions of the Mathematics Teacher in Early Childhood Education

Contemporary society is in continuous, rapid and complex process of change, which requires schools to reflect constantly on the educational practices adopted. This puts teachers in the challenging position of elaborating activities for their classes, which foster a thirst for knowledge in the children. Early childhood education teachers need to take charge of the construction of their own professional knowledge; focus on their practice, and rethink their educational plans; examine success in light of contemporary society and evaluate the constraints that such educational plans pose; and consider the influences of the cultural environment in which they teach.

Early childhood education teachers need to recognize the social and psychological competencies of the children, as well as their social fragility, evidenced by their dependence on adults to have their basic needs met. These professionals are required to conduct several tasks that require a robust understanding of developmental issues that are of physical, emotional and cognitive nature.

For Bujes (2001) the experience of children in early childhood education is complex, as it is there that the child develops ways of thinking and dealing with their feelings. It is in the early childhood years that children develop sensibility towards other children and preferences towards certain cultural manifestations rather than others. These are not results that can be overlooked when examining the time and experiences that the children have during their early childhood education.

Mathematics teachers who work with this age group must consider such complexity and respect the developmental process that the child is undergoing. Logical reasoning and the construction of scientific concepts cannot be the main goal at this educational stage, although they must not be ignored when there are signs of curiosity and desire for acquiring knowledge.

In order to examine such considerations in practice, we will give the example of the work developed by Ms. Katia Gabriela Moreira, which was described in her undergraduate research project report (Moreira 2009). Katia Gabriela worked for 5 years in early childhood education teaching kindergarten, where she conducted a research study titled "Records produced by young children in problem-solving situations with unconventional problems: research possibilities about mathematical thinking of children". Nowadays, Ms. Moreira is a teacher at a public elementary school.

She posed the following problem to her kindergarten children (4 to 5 years of age):

> "Mr. Brown's three hens lay eggs every day. There are hens that lay only one egg, and there are hens that lay two eggs a day. One day Mr. Brown decided to bake a cake, as his daughter was coming home from a trip and she loved cake. To make the cake he needed five eggs. How many eggs did he find in the chicken pen?"
> Source: Adapted from Grando et al. 2008, p. 57.

After presenting the problem statement, she gave the children pieces of paper so that each one could record: "How many eggs did he find?"

During this activity she walked around the room in order to observe the student's production without interfering. Then, as a scribe for the children she started to write on each paper the meaning attributed and expressed orally by each child about their drawing. This collaborative effort between the students and the teacher was characterized by the teacher "listening" to the thoughts expressed by the children and documenting those thoughts for them.

After that Ms. Moreira proposed that the records be shared, but before the presentations began she revisited the problem situation, asking questions such as: *"What was happening to Mr. Brown? Why?"*, etc. While explaining the number of eggs the hens lay, Victor said: *"one hen lays one, the other two, and the other one"* and the teacher realized that was the solution he had found for the problem. For Victor, when Mr. Brown went to the pen he found four eggs.

During the presentations each student told their classmates about their solution and, as a result of the issues raised by the teacher, they described their answers. The first to present was a student named Yasmin who claimed to have drawn the eggs in the nest. The teacher asked: *"How many eggs did Mr. Brown find in the pen?"* and the answer was: *"Fifteen eggs!"* At that moment, some classmates said that Yasmin was correct while others thought she was not. The teacher asked: "*Would Mr. Brown be able to make the cake with fifteen eggs?*" All agreed that he would. Ms. Moreira: *"Did he really find fifteen eggs?"* Carlos Eduardo, with certitude, said: *"No!"* But when Katia asked him to justify his answer he changed his mind and said: *"Yes, Mr. Brown did find fifteen eggs!"* At this moment the teacher thought that the change in Carlos Eduardo's answer was due to the fact that he did not know how to explain his answer, which led him to agree with Yasmin.

But, Mariana explained gesturing with her hands: *"Because one hen lays one egg and the other two"* and Victor retorted: *"and the other one, one"*. Mariana said: *"there was a hen that laid one egg and the other two!"* Again Ms. Moreira asked: *"Was it possible to find fifteen eggs in the pen?*" to which Sophia answered: *"No, because there were only three hens, and one laid one egg, the other one and the other … three* (gesturing)*, so there can only be three eggs!"*. In the end they all agreed that Mr. Brown could not have found fifteen eggs in the pen, as this was too many.

On the other hand, a student called Suhayb claimed that Mr. Brown had found five eggs, however, during the presentations, when questioned, he found that his record showed three eggs. That was when the teacher asked, in fact, which was his answer., Suhayb responded: *"It is true, I forgot, it is really five!"* And the teacher questioned the other students: *"Could Mr. Brown have found five eggs in the pen?*" And the answer was unanimous: *"No!"* and Victor explained again: *"One hen lays one, the other two, and the other one"*.

In view of Kaique's report, that showed a picture with three eggs, the classmates claimed that it was not possible, as there was one hen that laid two eggs, and that not all of them laid only one egg.

When beginning her presentation, a student named Sophia said: *"I drew five eggs, that were enough to make the cake, but he found only three eggs in the pen!"*, that is, she explained that she did not report the solution as asked, but she showed the information given by the problem—the number of eggs needed for the cake. A student named Victor agreed with his classmate, and said: *"I think there were three, because one hen laid one egg and the other two!"* and the teacher reminded him that there was yet another hen. Nonetheless, Sophia insisted: *"Oh, Miss, one hen lays one and the other lays two eggs* (using her hands to represent the quantities) *so, we have three"*, and the teacher asked: *"But, what about the other hen?"*. And the answer was: *"It did not lay any!"* At this time, the teacher understood that Sophia had found a solution to the problem occurring in her record. When she realized that she had forgotten to register a hen, Sophia said that the chicken had not laid any eggs, therefore, there was no need to include it, and her record would be complete.

This experience of the teacher with her students demonstrates that the questioning led to the restructuring of mathematical thought, and allowed them to assign meaning to the quantities, fostering the construction of the concept of number.

In this discussion, the children presented several possibilities, which they had envisioned. One solution indicated the impossibility of having only three eggs, since one of the hens had laid two eggs. Therefore, there should be more than three eggs. Similarly, there could not be six eggs, because in the context of the problem statement it had been made clear that there is at least one hen that lays one egg. At that moment, a new problem arose and a new solution: four eggs are insufficient to make the cake, but with five, it would be possible.

What we see are children elaborating solutions, within the scope of the statement of a problem situation linked to a story that does not consider the real possibility that the hens may not lay any eggs. Under this perspective, the answers provided by the children were plausible. Despite the fact that the real context (to lay eggs or not) is not considered in the problem statement, still, it influences the way children think mathematically.

The children's movement of negotiation between the mathematical context and the real world led to the attribution of meaning to the quantities and numbers, and created opportunities for the construction of the concept of number and mathematical operations.

We were able to observe that the actions of the teacher changed, as she paid more attention to the thoughts and actions of the children than to the details of her own actions and practices (Goldsmith and Schifter 1997). The teacher started to "listen" to her students giving them voice and allowing them to act on their own learning (D'Ambrosio and D'Ambrosio 2006). Her actions were guided by the reasoning expressed by the children, which ultimately directed her curricular and methodological objectives and decisions. These actions of listening, questioning and observing consisted of the developmentally appropriate assessment practices used by the teacher (Charlesworth and Leali 2012). The willingness of the teacher to listen to the students, so that they could have a voice, and to foster the questioning and socialization of several procedures created the possibility of a process for learning mathematics through which they attributed enhanced meaning to what they were learning.

Letting the children take ownership of tasks and problems, i.e. letting them use their own approaches and strategies is important to help them understand how to deal with mathematics. Their ideas almost always make sense to them, and thus contribute to their learning (Andrews and Trafton 2002). This emphasizes the importance of abandoning the idea that we must teach the children according to our perspectives, or to seek or provide answers before they can understand, for this may interfere in the pursuit of meaning.

Ms. Moreira's work shows an adequate form of pedagogical intervention. By questioning the number of eggs that might be in the pen, she promoted a series of mobilizations of numerical thought, far beyond mere counting. It is possible to observe the various modes of thought of the children, which were all respected by the teacher. She gave voice and listened to the ideas expressed by the children, introducing questions that enabled them to reexamine their reasoning, without imposing the "correct" way of interpreting and solving the problem. Ms. Moreira made it evident that she was clear about her objective, which was to induce the construction of the concept of number, by the children, through problem solving.

In this case the teacher acquired a significant understanding of how the children were thinking, and was clear about the concepts and ideas that could be mobilized in the construction of their knowledge. In order to do so, she listened attentively to what the students were saying during the unfolding of the activity, this form of active "listening" is what guided the actions of the teacher.

In the activities conducted by another teacher, Ms. Selene Coleti, we could also observe the problem-solving process in kindergarten classes as we examine her work with 17 students aged 4–5. For the last 30 years, Ms. Coleti has been a kindergarten and elementary school teacher in the public school system of the city of Itatiba, state of São Paulo in Brazil.

Ms. Coleti developed an activity called "DROP" with her students, which consists of drawing a geometrical shape on the floor, with chalk, and placing marbles inside that contour. The players, who have a marble in their hands, will take turns and stand over the shape and drop that marble. The marbles that roll out of the drawing will then belong to that player. The game ends when there are no more marbles inside the contour. The winner will be the player who has the most marbles in the end. In this activity Ms. Coleti asked the students to record the scores on a table. Initially, this was only meant as a warm-up activity for the fieldtrip to *Estação Ciências*, the town's science museum. Ms. Coleti wanted to test the children's knowledge about Physics—specifically about the force exerted on the first marble, which then moves the others—thus working with the idea of motion and conservation of energy.

After the visit to the science museum, she proposed that they play a cultural marble-shooting game, with traditional rules, by drawing the ring, measuring the distance for the taw line, and introducing the idea of the shooter (the first marble to be launched, also called boss or taw). The children made the ring by counting 7 feet, and marking the taw line, from which the shooter would be thrown. Each player received three marbles, and bet either one or two, by placing them inside the ring. Each child took turns behind the taw line, and threw the remaining marble—the shooter. The marbles that were knocked out of the ring would then belong to that player. The winner would be the player who had the most marbles at the end of the game. The children were very involved in this activity. The marble game was a strategy used by the teacher to work on mathematical concepts focusing on space, numbers and data processing.

After the presentation of the rules and after playing two games the children made drawings to represent the activity. The following week Ms. Coleti introduced a rule about the shooter: it could not be counted when figuring out the final score. This enabled the exploration of the idea of subtraction. The children then played four games and registered the scores on a table. In the first game they did not write the zero to represent that no points had been scored, they only left a blank space. However, in the last game the zero was used. The slots for the children that had been absent that day were also left blank. Then the teacher asked if there was a way to differentiate the records for the classmates that were absent from those of the players who scored no points. The children decided that they would make an X on the slots of the absentees, and a zero for those who had not scored any points. After this definition, the teacher started to introduce other questions: *"How many games have we played? Who scored the most points in each of the games? Who scored the least points? Who scored the most points altogether? Who was the winner?"* She guided the questioning towards a student called M, who had scored 33 points, and a student named G, who had scored 28 points. The children stated that M was the winner, and the teacher asked how many points would be necessary to tie the game.

This line of questioning reveals the intention of the teacher to make the children realize, through the game, the series of mathematical operations necessary to find the winner and make comparisons among players' scores.

After that, Ms. Coleti proposed that they make a bar graph called "Marble Game Winner." She told students to take a number of little strips[1] equivalent to their total score and, initially, asked the children which representation would make it easier to see the winner, whether in the graph or in the table. Some said that the table would be easier while others favored the graph. Two of the children, B and A, claimed that it would be easier to observe the graph, as "*all you need to do is look at the size*" In order to check how many little strips each of them needed, the children first looked at the table, to check their score. The teacher reported that this was a natural initiative of the students and that she did not tell them to use such strategy to solve this problem. It is possible that other numeracy practices in school had advanced the children's understanding of how to build bar graphs, as proposed.

Once the graph was finished, the teacher posed the following questions:

- Who is the winner?

Even though, for the teacher, this question seemed obvious, she asked if it would be easy for someone who did not play to know who was the winner. And the children agreed that it would be easy to find out, for the winner would be the person whose bar was highest.

- Who had the lowest score?

The teacher said that some of the students claimed Gui was the player with lowest score, only two points. Others said it was L and Ta, as they had not scored any points. The teacher reported that she held two cards in one hand and none in the other, and asked the class in which hand there was less. The students answered, it was the hand that held no cards. She then suggested that the children revisit the graph. And they agreed that the two girls had the lowest score. Let us examine the questions:

- If F had scored more points, what would have happened to his bar?
- How about N?
- If M had lost 5 points, what would happen to his bar?
- And if I (the teacher) had scored another 5 points? What would have happened to my bar? Would I have beaten M?

Ms. Coleti believed that such questioning would help the children make a more detailed analysis of the graph and the data obtained. After such discussions, she proposed the following problem situation:

> "Julia and John were playing with marbles. Their marbles ended up scattered on the floor and got mixed up. The two had 10 marbles altogether. Julia remembered that she had 4. How many marbles did John have?"

Ms. Coleti's goal was for the children to reason mathematically while, at the same time, for her to observe the strategies that they used to solve the problem as they represented each situation with the material (marbles). Many of the children could not, immediately, think of the number of marbles that belonged to Julia and John, so they insisted on

[1] Little strips, in this activity, were 3 cm by 1 cm rectangular-shaped strips of cardstock.

the total of ten. Several others made estimates, answering 5 and 5; however, when asked to show the number of marbles belonging to Julia, they realized that John had 6.

Ms. Coleti claimed that, at that time, she was not concerned with the way the children would represent the problem on paper, but rather with the thought process they used to find the solution using the materials. She also tried to push them forward, based on the ideas they each proposed.

Ms. Coleti's practice demonstrates the experience of mathematical work in early childhood, characterized by a problem-solving approach that involves a playful and meaningful way of engaging children in "matematizing" their world. This involves the establishment of relationships between the concepts involved, and the mobilization of skills related to perceptual-motor coordination, dexterity and precision of motor skills. At this stage in childhood, this problem-solving process enables the emergence of important inter-relationships between mathematical ideas, children's culture, the relationship with their bodies, and the different ways children feel and think.

Final Considerations

The curriculum guidelines for early childhood education indicate the need to consider the children's different levels of development, as well as individual differences. It is necessary to create activities that, when conducted in a scenario of investigation and questioning, explore the diversity of situations belonging to the universe of children.

It is necessary to give children the opportunity to elaborate their own problem-solving procedures, as well as encourage mathematical communication, and socialization of the strategies that they create. The acceptance of the personal character of the ideas and hypothesis presented by children does not mean that the teacher will disregard the curriculum guidelines that have been defined, but rather that she will respect her students' mental development and elaboration processes. Children are naturally motivated to learn, but they need engaging activities, as it is hard for them to stay focused on uninteresting activities.

Learning mathematics requires the attention of the child, which is earned through significant opportunities to problematize, and must be linked to the experiences of the child, as seen in the work of Ms. Moreira and Ms. Coleti.

In order to learn, children tend to follow the teacher's line of thought and, therefore, they must solve problems posed by the teacher, not only those that arise as a natural consequence of their own, intentional, activities.

The perspective adopted by the teachers leads to a form of cooperative learning, which enables the children to experience a constant reasoning process, to elaborate strategies, and to communicate several different ways of solving problems, as well as to take advantage of mathematical procedures different from their own. Engaging in share, meaningful experiences fosters the cognitive and emotional development of the children. This creates the possibility of bringing them closer to scientific knowledge, without discounting their individual learning strategies, which are marked by exploration, experimentation, manipulation, fun and games.

The work conducted by Ms. Moreira and Ms. Coleti revealed that the students were able to participate and communicate, and that their cognitive and social needs were being satisfied. Each of the children, at his or her individual level, and in their own individual manner, was successful in the experience of sharing solutions. The problem-solving activities required that the children formulate hypotheses and arguments, and present conclusions, however partial, which were scrutinized by peers at the moment of socialization.

The work of the teachers revealed a learning/teaching perspective that allowed the children to develop problem-solving skills, which will lead them to resort to familiar and relevant experiences and interpretations, in order to build hypotheses about how to tackle new and unfamiliar problems. During problem-solving activities, children evaluate their choices and solutions and learn how to appreciate and justify their production, as well as the production of others.

Thus, problem solving constitutes a competency, which is most effectively learned with practice. Questioning, making choices, and validating solutions are actions promoted by the teacher, who, by observing, listening and questioning, encourages the students with questions such as: *"What if…?"* and *"What other ways can you think of to…?"*. For this type of mathematical education to become a reality, we must adopt pedagogical practices centered in an interactive process, which links the culture of the children, their time and the educational space to an investigative process. When early childhood educators adopt investigative processes in their classrooms, they allow the children to be active and able participants, who can integrate, interact and promote change.

References

Andrews, A. G., & Trafton, P. R. (2002). *Little kids-powerful problem solvers: Math stories from a kindergarten classroom*. Portsmouth, NH: Heinemann.

Bujes, M. I. E. (2001). Escola Infantil: para que te quero? In C. Craidy & G. Kaercher (Eds.), *Educação infantil: Pra que te quero?* (pp. 13–22). Porto Alegre: Artmed.

Charlesworth, R., & Leali, S. A. (2012). Using problem solving to assess young children's mathematics knowledge. *Early Childhood Education Journal, 39*, 373–382. doi:10.1007/s10643-011-0480-y.

D'Ambrosio, B., & D'Ambrosio, U. (2006). Formação de professores de matemática: Professor-pesquisador. *Atos de Pesquisa em Educação, 1*(1), 75–85.

Freire, P. (1996). *Pedagogia da autonomia*. São Paulo: Paz e Terra.

Goldsmith, L., & Schifter, D. (1997). Understanding teachers in transition: Characteristics of a model for developing teachers. In E. Fennema & B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 19–54). Mahwah, NJ: Lawrence Erlbaum Associates.

Grando, R., Toricelli, L., & Nacarato, A. M. (2008). *De professora para professora: Conversas sobre IniciAção Matemática*. São Carlos: Pedro & João Editores.

Lopes, C. E. (2003). O conhecimento profissional dos professores e suas relações com estatística e probabilidade na educação infantil. Doctoral thesis—Faculdade de Educação, Universidade Estadual de Campinas, Campinas, SP Brazil.

Lopes, C. E., & Grando, R. C. (2012). Resolução de problemas na educação matemática para a infância. In M. G. C. Tommasiello, et al. (Eds.), *Didática e práticas de ensino na realidade escolar contemporânea: Constatações, análise e proposições* (pp. 5247–5259). Campinas: Junqueira & Marin.

McLennan, D. M. P. (2010). Process or product? The argument for aesthetic exploration in the early years. *Early Childhood Education Journal, 38*, 81–85.

Moreira, K. G. (2009). *Registros produzidos por crianças pequenas em situações de resolução de problemas não convencionais: possibilidades de investigação sobre o pensamento matemático das crianças [Relatório de Pesquisa de Iniciação Científica]*. Itatiba: Universidade São Francisco.

Moss, P. (2002). Reconceitualizando a infância: Crianças, instituições e profissionais. In M. L. Machado (Ed.), *Encontros e desencontros em educação infantil* (pp. 235–248). São Paulo: Cortez.

Oliveira, Z. (2002). *Educação Infantil: Fundamentos e métodos*. São Paulo: Cortez.

Ramani, G., & Brownell, C. A. (2014). Preschoolers' cooperative problem solving: Integrating play and problem solving. *Journal of Early Childhood Research, 12*(1), 92–108.

Skovsmose, O. (2000). Cenários de investigação. *Bolema-Boletim de Educação Matemática, 14*, 66–91.

Vygotsky, L. S. (1998). *Pensamento e linguagem*. São Paulo: Martins Fontes.

Whitebread, D., Coltman, P., Jameson, H., & Lander, R. (2009). Play, cognition and self-regulation: What exactly are children learning when they learn through play? *Educational and Child Psychology, 26*(2), 40–52.