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Artigos Gerais

Quantum "Ghosts"

"Fantasmas" na Mecânica Quântica

Gabriela M. Amaral, David Q. Aruquipa, Ludwing F. M. Camacho, Luiz F. C. Faria, Sofía I. C. Guzmán, Damaris T. Maimone, Melissa Mendes, Marco A. P. Lima*

Instituto de Física "Gleb Wataghin", Universidade Estadual de Campinas, 13083-859, Campinas, São Paulo, Brazil

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Can you pick a complex subject in quantum mechanics and discuss it with a minimum number of equations, in a simplified form that the general scientific public could understand? This was a question presented to graduate students of the one-year Quantum Mechanics course based on the text book *Modern Quantum Mechanics* by J. J. Sakurai and Jim Napolitano, at the State University of Campinas (UNICAMP), Brazil. The first seven authors of this paper are graduate students (alphabetical order) that accepted to try it. The chosen subject was "delocalized quantum states", and it will be discussed using colloquial terms like quantum *ghosts*, spooky action, splitting beings and invisibility cloak.
Keywords: delocalized state, interference effects, double slit experiment.

Pode-se escolher um tópico complexo em mecânica quântica e discuti-lo com um número mínimo de equações, e de forma simplificada para que um público com apenas conhecimento básico em física possa entender? Essa foi a pergunta apresentada aos alunos de pós-graduação das disciplinas de um ano de Mecânica Quântica I e II da Universidade Estadual de Campinas (UNICAMP), baseadas no livro "Quantum Mechanics" de J. J. Sakurai e Jim Napolitano. Os primeiros sete autores desse artigo são os alunos de pós-graduação (em ordem alfabética) que aceitaram o desafio. O tópico escolhido foi estados quânticos delocalizados, e será discutido utilizando termos coloquiais como fantasmas quânticos, ações fantasmagóricas, entidades divididas e capa de invisibilidade.
Palavras-chave: estados delocalizados, efeitos de interferência, experimento de dupla fenda.

1. Introduction

Quantum mechanics [1] is one of the most tested and well-established theories for the description of the microscopic world. The knowledge of quantum mechanics allowed the understanding and the consequent controlled manipulation of the nanoworld, giving birth to the largest technological revolution in human history. However, it has many aspects that are mystifying and puzzling, mainly because our classical intuition does not work in this microscopic world. The wave function, the description of a quantum mechanical system, seems much like a strange *ghost* [2]: it can split into pieces and be in several places at the same time and although unable to be perceived directly, it "commands" all possible results observed in experiments. It even reveals "spooky action at distance" in Einstein's terms. Roughly speaking, a measurement in the system that finds a particle in one place can cause an immediate effect on all other pieces of the wave function. This strange and immediate collapse is even faster than the light traveling between two parts, causing an apparent break of local realism (apparent because it does not transfer information, mass or energy, faster than light). Numerous scientists such as Albert Einstein, Aharonov, and many others considered this topic perplexing, and after 100 years since its first announcement, it is still hard to accept and understand several fundamental properties of the wave functions' nature. Through a revision of experiments, where the wave packet is divided, causing its parts to retain information

*Endereço de correspondência: maplima@ifi.unicamp.br.

about the whole system, we attempt to describe in this paper some curious aspects of this non-intuitive theory that are sometimes passed through unperceived. Thereby, we start with some basic concepts and then discuss the set of experiments [1] concerning interference patterns due to gravitation phase shift [3], the Aharamov-Bohm effect [4], spin precession in magnetic fields [5], and spin correlation [6], all due to the division of the wave packet in the mysterious world of quantum mechanics. In the conclusions, we summarize the most important findings and try to address the naturally induced question: if this strange *ghost* is affected by the environment, can the environment be affected by the *ghost*?

2. Basic concepts

The dynamic description of a particle in classical mechanics is given by a set of observable quantities and the future (the trajectory) of the particle can be completely established if enough about its surroundings is known. The initial state can be characterized by the position and velocity, for example. If you know the environment of the particle (all forces acting on it) you know its future (position and velocity at each moment, *i.e.* its trajectory). The time dependence of the system is given by its Newtonian equations of motion.

Unlike classical mechanics, quantum mechanics does not define the particle's trajectory deterministically, but only probabilities of finding the particle in space. At the very beginning in this field, de Broglie proposed we should associate a wavelength to particles, similar to the Einstein's idea of giving momentum to waves in the description of the photoelectric phenomenon [7] (electromagnetic waves consisting of photons capable of ionizing materials by photon impact). de Broglie [8] established that the wavelength associated to particles is proportional to the inverse of its momentum, $p = mv$. Later, Schrödinger [9] proposed to associate a wave function to the particle, interpreted by Born [10] as a probability amplitude, such that the probability of finding the particle is its squared modulus. This wave function is our *ghost* or if you want it in a more mysterious way, the particle's soul. The interpretation says: (1) the particle may be only where the amplitude is different from zero; (2) the particle with well defined momentum has a well defined wavelength. This wave function oscillates harmonically in the momentum direction from $-\infty$ to $+\infty$, and the probability (squared modulus of a plane wave) of finding the particle has the same value everywhere [11]; (3) if you want to trap a particle, you must trap its *ghost*. This concept is borrowed from classical wave mechanics, where for example only specific waves resonate in the violin box. This idea is the origin of the energy quantization. An electron bound to a proton means its wave function (its *ghost*) is a prisoner of the proton (only specific energy values satisfy these conditions - as also for waves resonating in violin boxes); the particle trajectory can not be defined but the future of the wave function can be deterministically found. All your knowledge about the particle is in the wave function and in this sense, quantum mechanics also has an equation, the Schrödinger's equation, that gives the future of a particle (indeed, the future of its wave function, *i.e.* the future of its *ghost*). The success of the theory is because we have learned how to interpret the results.

2.1. The wave packet and its splitting

The fundamental quantity in such theory is the wave function $\psi(\vec{r},t)$ and it can be found by solving Schrödinger's equation (written here only to point out that the derivatives with respect to the space and time coordinates characterize this equation as a wave equation):

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi(\vec{r},t) + V(\vec{r})\psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t), \quad (1)$$

where $\hbar$ is the reduced Planck constant, m is the mass of the particle, and $V(\vec{r})$ is a potential, which describes the environment of the particle. Later, Dirac [12] realized that the wave function was just a representation for a more abstract thing called a ket, a powerful mathematical description of the state of the particle. Here we just need to know that the above equation is linear and therefore, if $\psi_1(\vec{r},t)$ and $\psi_2(\vec{r},t)$ are two solutions of Eq. 1, any combination of these solutions $\psi(\vec{r},t) = a\psi_1(\vec{r},t) + b\psi_2(\vec{r},t)$ will also be a solution. This is a key property to understand interference effects. Physically, the information concerning the wave function is the probability of finding a particle in a determined position $\vec{r}$, inside the infinitesimal volume dv, and is given by its square modulus times the volume dv, *i.e.*, $|\psi(\vec{r},t)|^2 dv$. Particularly, when you have the above combined wave functions, the joint probability of finding a particle is determined by:

$$|\psi(\vec{r},t)|^2 dv = [|a\psi_1(\vec{r},t)|^2 + |b\psi_2(\vec{r},t)|^2 + (a\psi_1(\vec{r},t))^*(b\psi_2(\vec{r},t)) + (a\psi_1(\vec{r},t))(b\psi_2(\vec{r},t))^*]dv. \quad (2)$$

This quantity can be bigger (constructive interference) or smaller (destructive interference) than $|a\psi_1|^2 + |b\psi_2|^2$. It could even be zero (where the last two terms cancel out the first two terms), but it will never be negative. This set of properties explains the results of the double slit experiment, where a beam of particles, particle by particle, passing through two slits, mark a film, collision by collision, and build up an interference pattern (constructive and destructive fringes) just like a wave would do. Before discussing that experiment in more detail, let us define a wave packet.

To do so, let us set the potential to be zero and find the solution of the Schrödinger's equation for a free-particle ($V = 0$, in Eq. 1). First, let us suppose that the linear momentum is well defined and given by $\vec{p}$. In this case the energy of the particle is also well defined and equal to the classical one, $E = \frac{|\vec{p}|^2}{2m}$. It is easy to show (exercise for the reader) that the solution of Eq. 1 with well defined momentum and energy is a plane wave, is given by

$$\psi(\vec{r},t) = Ae^{\frac{i}{\hbar}(\vec{p}.\vec{r}-Et)}, \quad (3)$$

where A is a normalization constant. As mentioned before, if we calculate the probability of finding the particle in $\vec{r}$ within the volume dv, through the expression $|\psi(\vec{r},t)|^2 dv$, we obtain the same value ($|A|^2 dv$) everywhere. The free solution, the plane wave, puts the particle everywhere with the same probability! If we want to be sure that the particle is in one specific region of the space (for instance, an electron in a beam coming from your left side moving towards a double slit), we have to mix free solutions. By mixing an infinite number of free solutions (a continuum number of different values of $\vec{p}$ varying around $\vec{p}_0$) the interference summation process, explained above, can cause a constructive effect in a particular region around $\vec{r}_0$ and cause a fully destructive effect everywhere else. By losing the knowledge of the velocity (momentum) of the particle (we have mixed solutions with different $\vec{p}$'s), we gained knowledge about its whereabouts. This is the definition of a wave packet. This packet is nothing less than a compromising mixture of plane waves that allows us to place a particle in $\vec{r}$ within a small volume around $\vec{r}_0$, knowing that the particle will have momentum $\vec{p}$, within a small "volume" around $\vec{p}_0$. This mixture is also a solution of Eq. 1 (with $V = 0$) and allows the description of a particle moving in a beam of particles. In a regular quantum mechanics course, it is possible to show that the center of the packet travels according to the classical motion of a free particle with momentum $\vec{p}_0$ and the width of the packet increases with time (expected, if you consider that the components with larger values of $|\vec{p}|$ runs faster than the components with smaller values of $|\vec{p}|$).

Suppose now we have a packet, the *ghost* of a particle, moving towards a double slit, two small holes, near to each other, opened in an "impenetrable" wall. Suppose the packet is large enough that one piece goes through slit 1 and the other piece goes through slit 2. At the instant t_0, defined as the time that the packet collides with the double slit wall, we could call the piece of the wave function coming out of slit 1 by $\psi_1(\vec{r},t_0)$ and the piece of wave function coming out of slit 2 by $\psi_2(\vec{r},t_0)$. As time goes by, both pieces will evolve according to Eq. 1 (with $V = 0$) and both hit the film later on. At instant t, in a particular position $\vec{r}$ of the film, we will have contributions from slit 1 and slit 2. The overall contribution is given by Eq. 2. Although delocalized, $\psi_1(\vec{r},t_0)$ and $\psi_2(\vec{r},t_0)$, are part of the same *ghost* (the packet arriving against the double slit). The two pieces exist separately in t_0 and the interference phenomenon will take place only if they continue existing in instant t, as they arrive and overlap against each other nearby the film. Note that if you close slit 1, $\psi_1(\vec{r},t_0) = 0$ and Eq. 2 shows that only the second term ($|b\psi_2(\vec{r},t)|^2$ will survive (in other words, if we know that the particle passed through slit 2 for sure, the interference curve is destroyed). A similar thing happens if you close slit 2 [13]. All these situations are represented in Fig. 1. Note also that the interference happens on the film, at $\vec{r}$, at instant t, only if both $\psi_1(\vec{r},t)$ and $\psi_2(\vec{r},t)$ are different from zero at these position and instant. If you consider only one event of a particle colliding with the film, you cannot tell that it was due to interference because it could happen also if

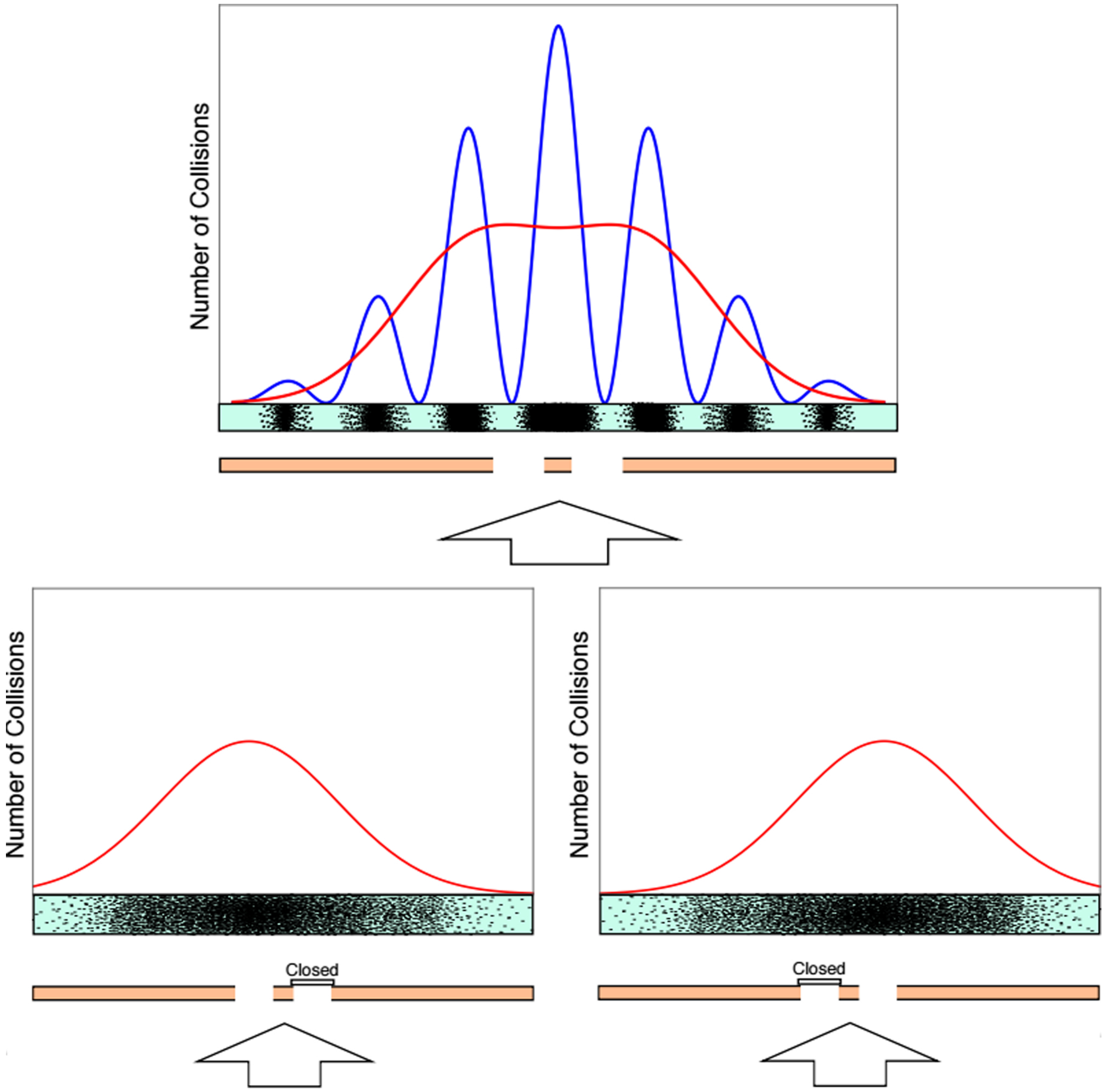

Figure 1: Double slit experiment results. The bottom panel represents the collision count process, considering slit 1 open and slit 2 closed (left side) for 10 minutes and then slit 1 closed and slit 2 open for another 10 minutes. The top panel has two curves, red line is just the sum of the counting process above and the blue line is for slit 1 and 2 open at the same time for 10 minutes.

one of the slits were closed (see Fig. 1). One mark is not a measurement of the wave character, only the net effect of many marks will give you the wave signature, as shown in Fig. 1.

What we have done so far was to learn how to construct a wave packet, a *ghost* that commands the motion of a particle with momentum $\vec{p}$ around $\vec{p}_0$. Then we have learned how to split it into two (same particle with a split *ghost*) through the double slit interference experiment. What we do in the next sections is to submit the split *ghost*, $\psi_1(\vec{r},t)$ and $\psi_2(\vec{r},t)$, to different potentials (*i.e.* they will be placed in different environments - here represented by different magnetic fields, different electric fields and different gravitational fields) and we will see what happens to the interference pattern. Pictorially [14], we present this situation in Fig. 2. That figure shows our *ghost* being split into two by a double slit

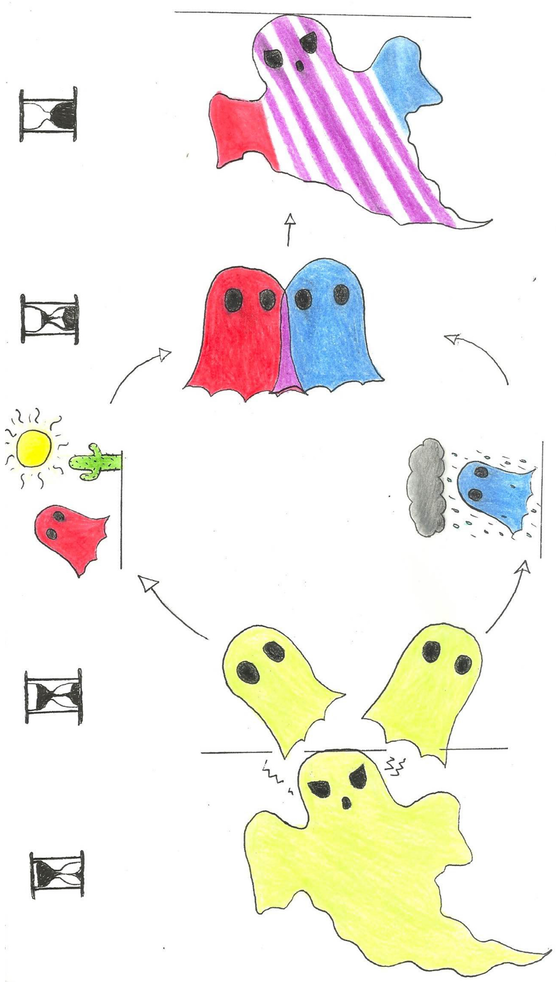

Figure 2: Pictorial figure for the double slit experiment. It represents a *ghost* (wave function) being split into two pieces, where each each piece is submitted to a different environment. The pieces are then put together and the compounded *ghost* commands the probability of the particle to mark the film in a determined position. The trick is to compare the resulting interference figure with one where the pieces traveled under the same environmental conditions.

experiment. Each *ghost* piece then travels through different environments and they are put together just before collision against the film. Before discussing these split *ghost* experiments (so named as the *ghost* pieces belong to one particle), we need to introduce the concept of spin $\frac{1}{2}$, a mysterious intrinsic angular momentum of the electron.

2.2. Angular momentum, Spin and Rotations

In classical mechanics [15] the orbital angular momentum, defined as $\vec{L} = \vec{r} \times \vec{p}$, has a very important role in the solution of problems with spherical symmetry (for instance, finding the trajectory of planet Earth moving around the sun, ignoring all other planets, satellites, and external forces). Spherical potentials give rise to forces incapable of changing the angular momentum. Therefore this quantity is conserved. A similar thing happens in quantum mechanics with important differences. In a classical mechanics course, it is possible to show that angular momentum is responsible for rotations of the system. In quantum mechanics, it has a similar role. We say that its component (projection of the vector $\vec{L}$) along an axis is responsible for the rotation of the system around that axis. The problem is, as you rotate the system in 360^o, we return to the same point in the space, where we expect the wave function to have the same value as before the rotation. This repeated value for every rotation is in a certain way similar to trapping the wave function in a box, and as we explained above for the energy case, this procedure results in quantization.

In a basic quantum mechanics course, we learn that the possible values of the squared modulus, $|\vec{L}|^2$, of the angular momentum are $\ell(\ell+1)\hbar^2$, with $\ell \geq 0$ and an integer value. The intriguing property is that the projected angular momentum on any direction $\hat{n}$, *i.e.* $L_n = \vec{L}.\hat{n}$, if measured, will have a value equal to $m\hbar$ with m an integer and $-\ell \leq m \leq \ell$. The quantization, now of the orbital angular momentum, is in some sense (again) due to wave trapping (*i.e.* a *ghost* in a prison). If you think that an atom possesses angular momentum $|\vec{L}|^2 = \ell(\ell+1)\hbar^2$, you could imagine that it corresponds to a charge rotating around a particular axis. This current loop would give birth to a magnetic moment. In fact this simple model reflects the reality. The quantization of angular momentum is responsible for the quantization of magnetic moments of an atom. If a particular atom has an overall angular momentum given, for instance, by $\ell = 1$, we obtain 3 possible values of the projected magnetic moment of the atom, corresponding to $m = -1, 0, +1$. These "projected" values are the same for any axis of your choice. Classically, if you "shoot" an atom against a strongly varying magnetic field in a particular direction (perpendicular to the motion) the beam would spread depending on the projection of the magnetic moment of the atom along the magnetic field. In quantum mechanics the number of possibilities is restricted by the quantization of the angular momentum. For a beam of randomly oriented atoms going through a strongly varying magnetic field, an atom with $\ell = 1$ would split its wave function (*ghost*) into 3 pieces (one, $m = +1$, displaced along the field direction, one, $m = 0$, unchanged, following the original beam direction, and one, $m = -1$, displaced against the field direction). On the other hand, if you have a homogeneous magnetic field pointing along a particular direction, the atom with magnetic momentum will precess around the field (as in classical electromagnetism [16]). If the atom has an angular momentum different from zero, say $\ell = 1$ for instance, it will rotate and as it completes 360^o it will have the same value of the wave function as in 0^o.

Based on this idea a very revealing experiment was carried out by Stern and Gerlach [17], where a beam of silver atoms passed through a strongly varying magnetic field. The silver atom has 47 electrons, and it was known that the net combination of the individual orbital angular momenta would be zero. So according to what we described above, the atoms should go through the magnetic field without spreading along the field direction. The surprise was that the experiment showed 2 peaks (one displaced along the direction of the magnetic field and the other away the opposite direction). This experiment demonstrated the existence of the intrinsic angular momentum of the electron (its origin is not orbital, $\vec{L} = \vec{r} \times \vec{p}$) giving rise to its magnetic moment. Because there are only two peaks, the associated quantum number ℓ (we will call it s, for spin, in this case to remind us of its different origin [18]) should be $\frac{1}{2}$, with the measured values along any axis orientation given by $m_s\hbar = -\frac{1}{2}\hbar$ or $+\frac{1}{2}\hbar$. In a basic quantum mechanics course, we learn that the net combination of magnetic moment contributions from all 47 electrons of the silver atom is indeed the contribution of the outmost one (the internal electron contributions cancel each other out). So, the authors concluded that the presence of only two peaks in this experiment demonstrates the existence of the intrinsic spin, $s = \frac{1}{2}$, in electrons.

Contrary to the orbital angular momentum, spin has no classical analog. Its origin comes from a necessary conciliation proposed by Dirac [18], between quantum mechanics and relativity, that will not be discussed here. Its existence, however, imposes that our *ghost* description needs an extension, in case of particles with spin. The wave function $\psi(\vec{r}, t)$ is not enough, we have to specify also the spin χ_+ or χ_-. Therefore, in a general form, our *ghost* description becomes $\psi(\vec{r}, t)\chi$, with $\chi = c\chi_+ + d\chi_-$, where $\psi(\vec{r}, t)$ tells us where the particle can be in space and χ carries the information about its spin (up or down and with which probability). In a quantum mechanics course we learn why spin is indeed an angular momentum, and that any composition of particles with orbital and spin angular momenta are restricted to the rule: the possible values for the overall squared modulus $|\vec{J}|^2$ are $j(j+1)\hbar^2$, with $j \geq 0$, and being an integer or semi-integer, and its component along any axis $\hat{n}$, *i.e.* $J_n = \vec{J}.\hat{n}$, if measured, will have a value equal to $m\hbar$ with $-j \leq m \leq j$, with the m value jumping one by one from $-j$ to $+j$. We also learn a very weird property of χ: if we rotate the system by 360^o we get $-\chi$ and not χ. This property will not be demonstrated here, but will be part of one of the "split *ghost*" experiments that we comment on below.

3. The Split "Ghost" Experiments

The general idea of the split *ghost* experiments that will be discussed in this section is represented in Fig. 3. In this figure, a wave packet is split into two parts with the help of a double slit at point A. Each piece of our *ghost* travels through independent branches, with similar dimensions, up to its exit slit point towards the film F. The arrangement is such that ψ_1 comes from slit 1 of A, travels to C, suffers the influence of V_1, in region 1, between C and E, and exits to meet ψ_2 in F. On the other hand, ψ_2 comes from slit 2 of A, travels to B, suffers the influence of V_2, in region 2, between B and D, and exits to meet ψ_1 in F. In F our split *ghost* has its pieces reencountered and the resulting *ghost* commands the odds of where in the film the particle will cause a mark. Repeated collisions produce an interference pattern similar to the one described above (Fig. 1) for a simple double slit experiment.

The idea is to compare cases where $V_2 = V_1$ with cases where $V_2 \neq V_1$, and answer the question: can our strange split *ghost* be affected by the environment of its parts? Besides interference patterns obtained with different scalar potentials, we will use the same scheme given by Fig. 3 to report what happens when one of the split *ghost* pieces of a particle, with spin, passes in a region where the magnetic field is zero but there is a vector potential different from zero acting on the *ghost*, and, as a third

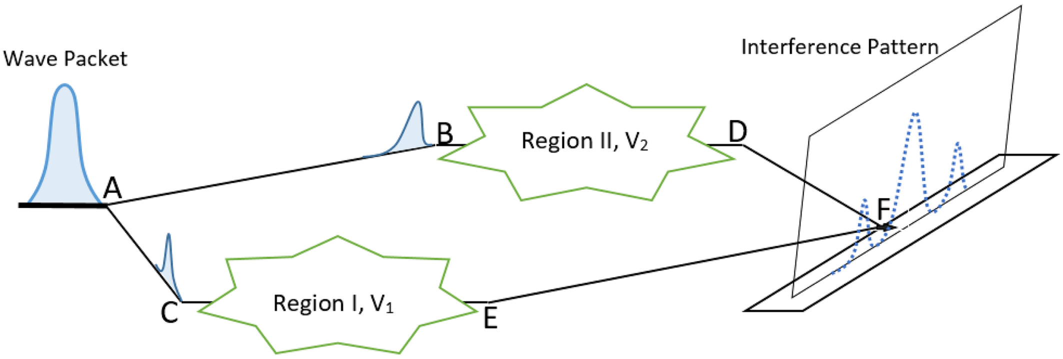

Figure 3: Conceptual interference experiment where A, B, C, D and E are suitable devices to divide a wave packet (*ghost* splitting), of a particle arriving in A, and make each piece experience a different potential, V_1 or V_2. F is the meeting region of the split *ghost* which contains the film for measuring the particle arrival.

case, when one of the pieces goes through a constant magnetic field.

3.1. Interference effects due to gravity

The simplest situation would be to submit our split *ghost* to two different constant potentials (one negative and the other positive, for instance). A classical particle under the influence of a negative and constant potential V_1 (shallow-well potential) would be immediately accelerated in C, it would travel the region 1 (between C and E) with a constant velocity, faster than the $V_1 = 0$ case, and recover its original velocity in E. One particle under the influence of a positive and constant potential V_2 (low-potential barrier) would be immediately slowed down in B, it would travel the region 2 (between B and D) with a constant velocity, slower than the $V_2 = 0$ case, and recover its original velocity in D. Although both get to F with the same velocity, particle 1 would get there before particle 2.

In 1975 an experiment made by R. Colella, A. W. Overhauser and S. A. Werner [3] (known as the COW experiment) attempted to measure the gravitational effect on a quantum system. The experiment had a more elaborate scheme than the one given by our Fig. 3, but the essence of it can be obtained by rotating the apparatus of our Fig. 3 around the axis defined by the incoming beam of particles. So that, in this situation, we would have the $V_2 = mg\frac{H}{2}$ region in a higher position than the $V_1 = -mg\frac{H}{2}$ region, by considering that, before rotating, both sides were at the height $h = 0$, and after rotating, one potential region would ascend to $h = \frac{H}{2}$ and the other would descend to $h = -\frac{H}{2}$. All the momentum components of the half packet (split *ghost*) going up would decrease, and all momentum components going down would increase. By the time they arrive in F they all recover their original values, but the center (it travels like the classical particle) of the half *ghost* coming from above arrives later than the one coming from below. This is sufficient to cause a change in the interference pattern originated by gravity. This experiment is strong evidence that the split *ghost* interacts with the macroscopic environment of its parts. It is also evidence that the split pieces are kept along the whole process (otherwise the interference pattern would disappear).

3.2. Interference effects due to a magnetic flux

More elaborate scalar potentials could be used in our schematized experiment of Fig 3, including those that are not constant. The conclusion would be similar, as long as the potentials do not cause the collapse of the *ghost*. If the split *ghost* keeps its parts different from zero, the interference pattern would change with respect to the $V_1 = V_2 = 0$ case and the net result would carry the information that the split *ghost* was influenced by the potentials.

How about if the split *ghost* were submitted to a vector potential $\vec{A}(\vec{r}, t)$, defined in electromagnetism [16] to describe magnetic fields through $\vec{B} = \vec{\nabla} \times \vec{A}$. This experiment was made by Aharamov and Bohm [4] and it is illustrated in Fig. 4. The idea of the experiment is to make the split *ghost* circulate an infinite (very long) solenoid to achieve the film in F, one piece moving clockwise passing in B and D and the other moving counterclockwise passing in C and E. In an electromagnetism course [16], we learn that an infinite solenoid produces a constant

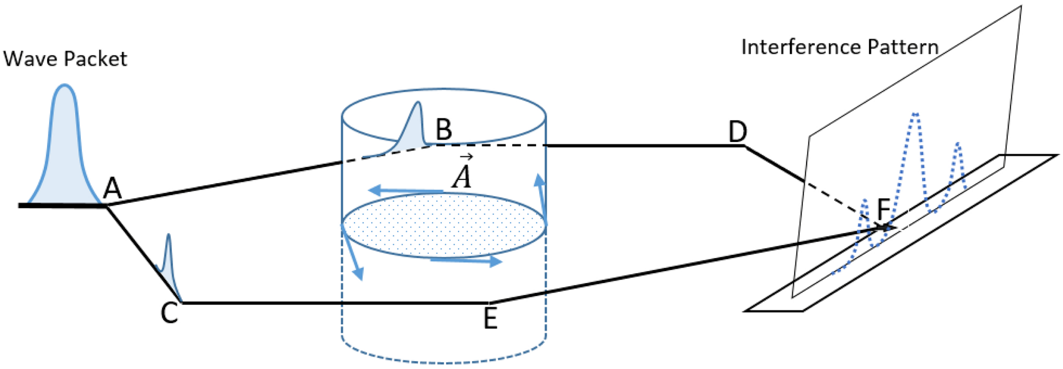

Figure 4: Observation of the Aharanov-Bohm effect for the vector potential. Dotted lines represent the deflected pattern [19].

magnetic field $\vec{B}$ parallel to its axis, which is different from zero inside it, but is zero outside it. We also learn that the vector potential that produces this magnetic field is not zero outside the solenoid. It is circular around the axis and decreases with the inverse of the distance from the solenoid axis [1]. Therefore, a piece of the split *ghost* is under the influence of a constant vector potential pointing in the direction against its motion and the other piece is under the influence of one in the same direction of its motion. This is sufficient to change the interference pattern and give birth to what is known as the Aharamov-Bohm effect. This also places the vector potential in a different perspective (in classical electromagnetism, it is only a mathematical tool): the split *ghost* passes only in regions where the magnetic field is zero ($\vec{B} = 0$ outside the solenoid) and in some way it is disturbed by $\vec{A}$, which is non-zero in these regions. In a quantum mechanics course [1] we also learn that the change in the interference pattern of the split *ghost* is related to $\frac{e}{c\hbar} \oint_C \vec{A} \cdot d\vec{x} = \frac{e}{c}\Phi$, where Φ is the flux of the magnetic field over a circular surface inside the solenoid. This experiment is also clear evidence that the split *ghost* interacts with the macroscopic environment of its parts.

3.3. Interference effects due to a constant magnetic field

Another interesting split *ghost* experiment would be to pass one piece through a zero field region and the other through a constant magnetic field, pointing to any direction. Without the field we would have an interference pattern (meaning that we would be able to put together a double slit interference experiment). As we learned from section 2.2, the spin wave function would rotate around the magnetic field and return to its original value after 2 loops (4π).

In 1975, using neutron interferometry, Rauch and Zeilinger [5] showed a way to measure this phase difference which, theoretically, as we can learn in a quantum mechanics course [1], is given by $\Delta\alpha = \pm\frac{2\pi g_n \mu_n M\lambda B l}{\hbar^2}$. The signals $\pm$ are for the orientation of the spins, g_n is the neutron magnetic moment in nuclear magnetons (-1.91), μ_n is the nuclear magneton, M is the neutron mass and l is the distance the neutron wave packet travels in the field. This means that the piece of the *ghost* that passes through the magnetic field will rotate according to the intensity of the magnetic field, which creates a change in the interference pattern when both parts of the split *ghost* are put together to interfere. So, by either fixing the field magnitude and calibrating the length of the traveling split *ghost* trip, or fixing the length of the traveling split *ghost* trip and varying the magnetic field, we could confirm the 4π returning value hypothesis. In the experiment by Rauch and Zeilinger [5], they showed that the 4π (and not the 2π) rotation was the right one to obtain the original interference pattern (without the magnetic field). This experiment is further clear evidence that the split *ghost* interacts with the macroscopic environment of its parts.

4. Correlated quantum states of spin "ghosts"

Albert Einstein showed that his most famous equation, $E = mc^2$, means that we can create matter from electromagnetic waves. For instance, if you shoot two photons against each other you can create an electron-positron pair, two identical particles except for their charges (opposite sign). If you put them together they would turn back into light (two photons), but for a moment they could bind to each other forming positronium (a very light hydrogen atom) in a zero spin state. Electrons and positrons are spin $\frac{1}{2}$ particles but due to the angular momentum of the two photons before mass conversion, only overall spin zero for the electron-positron pair is allowed [20]. More precisely, in order to assure spin zero (on any direction of your choice - see section 2.2) of the positronium atom (made by the collision of the two photons), we must have: if you measure the spin of the electron to be up (down), the spin of the positron, if measured, must be down (up).

Accepting as true this very short review on positronium atom creation, let us separate its components without disturbing the spin. An electric field would split them apart, and you would know which one is the positron and which one is the electron. So, spatially speaking, they would not be a split *ghost* (just two *ghost*s, one for the positron and another for the electron). How about the spin? The only thing we know is: if you measure the positron spin to be up (down) the electron spin will be necessarily down (up), even if they are well apart from each other. This is true for any arbitrary chosen direction for measuring the spin of the pair. We say that the pair electron-positron is in a correlated quantum state (also known as entangled state). This kind of experiment "really disturbed Albert Einstein" and many other scientists [21]. In 1964, Bell [6] proposed an experiment that has shown that the quantum mechanics predictions are correct and it does not violate any of the relativity theory concepts (in this kind of experiment, no information, mass or energy travels with higher speed than the speed of light). Here we just point out that the spin correlated *ghost*, measuring a spin up (down) for the electron and, therefore, assuring that the spin of the positron is down (up), is very similar to the spatial split *ghost*, measuring that the particle is (particle is not) in region 1 and, therefore assuring that it is not (it is) in region 2. The concept of collapsing the wave function with a measurement applies for both cases and their weird split *ghost* properties seem to be related. A pair of photons produced by the annihilation process of a pair electron-positron in the spin zero state will also be in a spin zero state (meaning that any direction will give a sum zero for projected angular momentum of the photons). There are simpler ways of producing photons in a correlated state and the infinity number of possibilities for up's and downs (any direction) motivates a new application of quantum mechanics that will eventually give birth to quantum computers [22].

5. Can the environment be affected by the "ghost"?

Quantum mechanics allows split *ghosts* but it does not allow split beings. In other words, the *ghost* can be in two places at the same time but the particle itself cannot be in two places at the same time. Which means, by pursuing a measurement, if you find a particle in some place, it cannot be in any other place. All the other *ghost* pieces immediately disappear, *i.e.*, the probability amplitude for those pieces go to zero. An unexpected change in the field is a measurement of the presence of the particle and this causes the *ghost* collapse. The above assertions answer our question. If the environment of one piece of our split *ghost* is affected by it, the other piece cannot affect its environment, because, this would be an evidence that the particle could be in the two regions at the same time.

Considering that this interpretation is correct, we can conclude that in all the split *ghost* experiments discussed above the *ghost* pieces have not affected the macroscopic field (gravitation, the solenoid field, and the constant magnetic field), because if it had done so, by producing any measurable effect, it would identify the presence of the particle (you would be sure that it came from a particular slit) and this would cause the collapse of the *ghost*, destroying the interference pattern. Everything would take place as if the opposite slit were closed. In this case, if the collapse happened for all split *ghosts* of the beam, the resulting pattern would be just a combination of the patterns involving only slit one and only slit two being open (see Fig. 1).

In order to get a better insight of the situation, let us imagine another conceptual experiment, now submitting our split *ghost* to fields generated by other *ghosts* (*i.e.* a particle in the quantum mechanics regime). Suppose the regions V_1 and V_2 of Fig. 3 are replaced by vertical beams of particles, where each particle is described by its *ghost*. If our split *ghost* is of a particle with charge, and the vertical beams are also made up by charged particles, they may interact. Let us see what are the possibilities.

Suppose skillful experimentalists were able to put together a device for the vertical beams, where 100% of the particles would always arrive within a small ring shaped detector. Every time it arrives inside the ring detector, we hear a click. If those experimentalists were really clever, they could make two vertical apparatus, shooting simultaneous particles with their individual *ghosts* arriving in the region 1 and 2 at about the same time. If the horizontal split *ghost* is not present, these two particles will arrive at the detectors and produce two clicks.

What happens now, if we start shooting our horizontal split *ghost* in a calculated manner such that it can cross the two vertical beams of particles? If all the particles are charged there are some possibilities that we will explore for the interactions of the split *ghost* and the vertical beams. To simplify, let us imagine that in this hypothetical situation one (or at most one) collision would happen (the involved fluxes are very small). From what we have learned, we could state: (i) we will hear at least one click. Zero clicks are not possible, because that would put a particle of the horizontal beam in two regions at the same time, in order to deviate both particles of the vertical beams; (ii) we can hear two clicks, indicating that none of the vertical beam particles interacted with the split *ghost* in a measurable way. In this case, the split *ghost* interference pattern could change with respect to the pattern without the vertical beams (revealing that *ghosts* interact with *ghosts*). To better understand this last conclusion, let us remember that a particle of the vertical beam also obeys Schrödinger’s equation (Eq. 1). Without the split *ghost* it would be a free solution (a packet, as described in Sec. 2.1 for $V = 0$). With the presence of the split *ghost* the potential is no longer zero but a time dependent potential, indicating that if the split *ghost* piece is close enough a Coulomb interaction between the piece and the particle will take place. Same thing for the other vertical beam particle, which supposedly is near the other split *ghost* piece. The amazing part is, if one of the vertical beam particles is deviated from the ring detector, due to the split *ghost* piece presence, the other vertical beam particle needs to collapse to its free solution (the only way to hear at least one click). This would be like if it were wearing an invisibility cloak. For this to happen, the free solution ($V = 0$) must be a part of the general solution involving the split *ghost* potential ($V \neq 0$).

In the human invented quantum mechanics description, the *ghost* of a particle seems to be a way to assure that nature will explore all possibilities for the particle’s future. The *ghost* contains the information about the odds of incoming events. Every time the particle interacts and interferes (*i.e.* a measurement event) with its environment, a sudden collapse takes place, as if a new boundary condition were imposed to the particle. It happens almost as if the measurement triggers a sudden recoil of the split *ghost*, giving birth to a shrinkage *ghost* [23] that puts the particle, for sure, in the surroundings of the event. Among others, an intriguing mystery remains: in a very symmetric apparatus as we described above, if we hear one click, how has nature decided which one of the vertical beam particles has interacted with the split *ghost* particle? We only know that either collision is equally probable.

6. Final Remarks

Schrödinger’s Equation (Eq. 1) shows that the time dependent wave function is necessarily a complex function (it has real and imaginary parts). This complex function is a human invention and it is not a measurable entity. This is the reason we decided to make a supernatural joking analogy [2], and represent it by the *ghost* or the soul of the particle. The important message is that this mathematical formalism allows a precise description of the nanoworld reality, and as mentioned in the introduction it gave birth to our amount impressive technological progress. Much more is coming! The next revolution we believe is on information. We have started with the digital computer (binary codes - 2 letters in the alphabet), and today we understand the life “computer” (DNA - 4 letters in the alphabet). However the correlated states discussed above opens a new and very interesting area for information, the so called quantum computing (infinite letters in the alphabet) age. We need more people studying quantum mechanics to achieve that goal, and for this a real effort to introduce the subject in the earlier stages of our educational process is required. Writing this paper, aimed at people with an interest in science but with a minimum mathematical background, has shown how difficult this can be. But it must be done!

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[2] *Warning*: our playful choice of words is not meant to endorse baseless assumptions, but to bring attention to the quantum split wave function experiments. It must be understood as an analogy that aims to portray in a simple way these experiments and it must not be associated with any reality or property of nature. Care should be taken when trying to simplify a subject as complex as QM, and under no circumstance should it be applied outside of its realm, the nanoworld.

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[14] The time dependent wave function is necessarily a complex function. So, its simple representation in the real space is not possible. If we represent the complex wave function by its real part, the image would blink. As we learn in a quantum mechanics course, a stationary state can be written as $e^{-iEt/\hbar}\phi_E(\vec{r})$ and periodically the real part can disappear as long as the the imaginary part is different from zero (and vice-versa). A moving packet, a mixture of free states, is even more complicated (its parts would blink) and our pictorial representation is only the modulus of this complex wave function.

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