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## Subtractive Renormalization of the NJL model \*

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### Abstract.

In this work, we apply a subtractive renormalization method to the  $SU(2)$  Nambu–Jona-Lasinio (NJL) model in Born approximation and compare the results obtained here with those achieved by the standard momentum cutoff regularization. We have computed the dynamical quark mass, the chiral condensate, the pion mass and the quark-antiquark T-matrix as a function of the cutoff  $\Lambda$  and the subtraction scale  $\xi$ . We have shown that both approaches give similar results and the dependence of the physical quantities on the subtraction point  $\xi$  is much weaker than the dependence on the cutoff  $\Lambda$ .

### 1. Introduction

The Nambu–Jona-Lasinio (NJL) model has originally been proposed to describe the nucleon-nucleon ( $NN$ ) interaction via contact forces [1]. Later, this model was reinterpreted as an effective field theory for quantum chromodynamics (QCD), the fundamental theory of strong interactions between quarks and gluons. Moreover, the NJL model became the most popular model for QCD and inspired a whole program of applying it to hadron structure [2] and quark matter problems [3]. More recently, the model has been used to study inhomogeneous quark condensates [4] and quark matter under strong magnetic fields [5, 6].

Since the NJL model is based on fermion contact interactions, it seems reasonable that renormalization methods which are applied to the nucleon-nucleon interaction may also be applied to the NJL model for quarks. We have shown that the  $NN$  interaction at leading order in chiral perturbation theory can be renormalized by performing one subtraction in the kernel of the  $NN$  scattering equation [7], since in this case the divergency comes from a pure contact interaction which is just a constant in momentum space.

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When higher order interactions are included in the chiral expansion additional subtractions are required [8] so that NLO interactions need three subtractions [9], N2LO interactions need four subtractions [10, 11] and N3LO interactions need five subtractions [12]. In this work, we apply one subtraction to the NJL model in the Born approximation and compare the results to those is obtained in the standard cutoff approach.

## 2. The NJL model

The Lagrangian for the NJL model is written as

$$\mathcal{L}_{\text{NJL}} = \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - m_0)\psi + G \left[ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2 \right]. \quad (1)$$

Linearizing the interaction in a mean field approximation yields to the well known gap equation of the NJL model

$$m = -2G \langle \bar{\psi}\psi \rangle, \quad (2)$$

where  $\langle \bar{\psi}\psi \rangle$  is the scalar density corresponding to the quark loop,

$$\langle \bar{\psi}\psi \rangle = -i \text{tr} S_F(0), \quad (3)$$

with

$$S_F(x-y) = -i \langle T [\psi(x)\bar{\psi}(y)] \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip \cdot (x-y)}}{\not{p} - m + i\epsilon}. \quad (4)$$

Evaluating the trace of equation (3) we have

$$\begin{aligned} \text{tr} S_F(0) &= \int \frac{d^4p}{(2\pi)^4} \frac{4m}{p_0^2 - p^2 - m^2}, \\ &= - \int \frac{d^3p}{(2\pi)^3} \frac{4}{(2\pi)} \frac{\pi}{2} \frac{i m}{\sqrt{p^2 + m^2}}, \\ &= - \frac{4i}{8\pi^2} \int dp p^2 \frac{m}{\sqrt{p^2 + m^2}}. \end{aligned} \quad (5)$$

## 3. The cutoff scheme

At this point, a momentum cutoff  $p_{\text{max}} = \Lambda$  is usually applied to the integral in order to ensure that the gap equation, equation (2), has a finite solution. In figure 1 we show the solutions of the gap equation (upper left panel) and the result obtained for the chiral condensate (upper right panel), both as a function of the cutoff  $\Lambda$ .

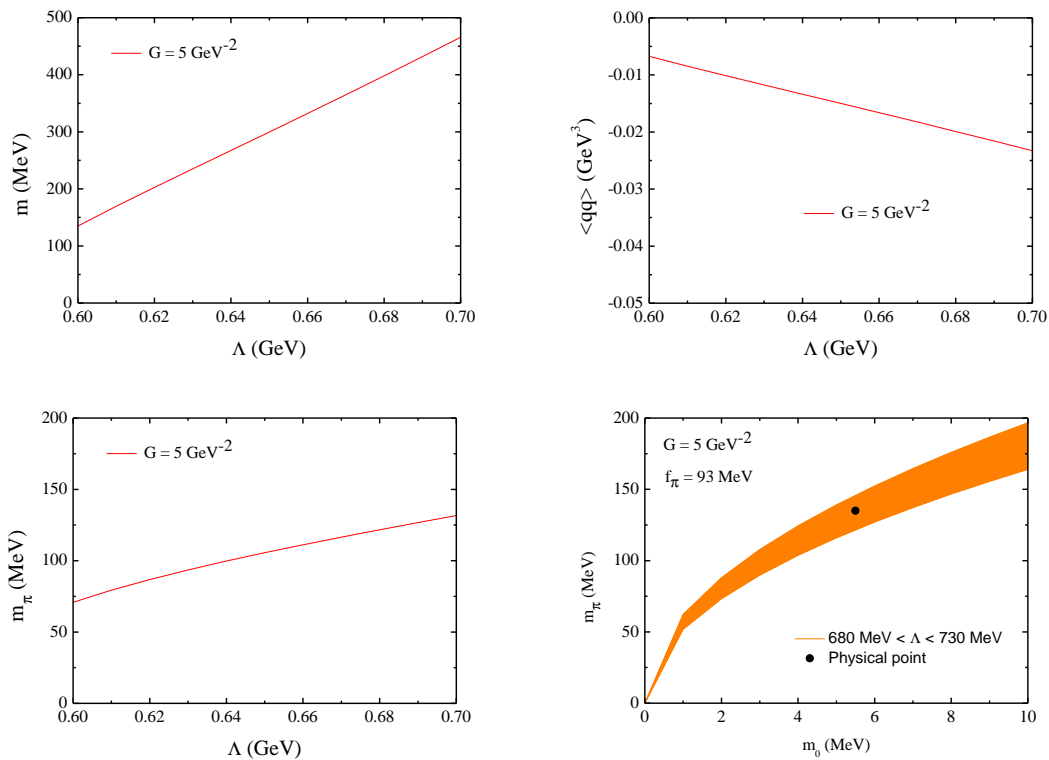
Both the mass and the condensate depend strongly on the cutoff and they grow very rapidly as  $\Lambda$  increases so that this scale is normally kept in the range  $0.6 < \Lambda < 0.7$  GeV. In figure 1 we also show the dependence of the pion mass on the cutoff as obtained from the Gell-Mann–Oakes–Renner (GOR) relation (lower left panel) and its dependence on the current quark mass within the usual cutoff range (lower right panel), for a current quark mass of 5.5 MeV and a pion decay constant of 93 MeV.

The quark-antiquark  $T$ -matrix is given by

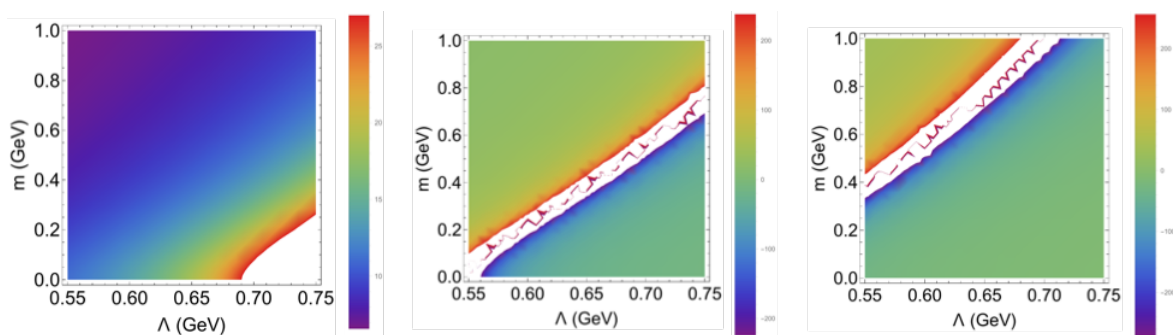
$$\begin{aligned} T &= K + KJK + \dots, \\ &= \frac{K}{1 - KJ}, \end{aligned} \quad (6)$$

where

$$J(q^2) = i \text{tr} \int \frac{d^4p}{(2\pi)^4} \left( i\gamma_5 \frac{1}{\not{p} + \frac{\not{q}}{2} - m + i\epsilon} i\gamma_5 \frac{1}{\not{p} - \frac{\not{q}}{2} - m + i\epsilon} \right), \quad (7)$$



**Figure 1.** Top panels: Dynamical mass for the quarks (left) and chiral condensate (right) as a function of the momentum cutoff. Bottom panels: Pion mass as a function of the momentum cutoff (left) and as a function of the current quark mass (right) for  $G = 5.5 \text{ GeV}^{-2}$  and  $f_\pi = 93 \text{ MeV}$ . The square represents the physical point  $(m_{u,d}, m_\pi) = (5.5, 140) \text{ MeV}$ .



**Figure 2.** The quark-antiquark  $T$ -matrix, in  $\text{GeV}^{-2}$ , as a function of the dynamical mass and the cutoff for  $G = 0.5 G_\Lambda$  (left),  $G = 1.0 G_\Lambda$  (middle) and  $G = 1.5 G_\Lambda$  (right). The reference coupling in the cutoff scheme is  $G_\Lambda = 5.0 \text{ GeV}^{-2}$ .

and  $K = 2G$  in the Born approximation. In the cutoff scheme we obtain

$$\begin{aligned} T &= \frac{2G}{1 - 2G J} , \\ &= \frac{2G}{1 - \frac{N_c N_f}{m\pi^2} \int_0^\Lambda dp p^2 \frac{2Gm}{\sqrt{p^2+m^2}}} , \\ &= \frac{2G}{1 - 2G \frac{N_c N_f}{\pi^2} \left[ \Lambda \sqrt{m^2 + \Lambda^2} - m^2 \operatorname{arcsinh} \left( \frac{m}{\Lambda} \right) \right]} . \end{aligned} \quad (8)$$

Note that the gap equation, equation (2), can be obtained by setting the denominator of equation (8) equal to zero, which corresponds to the poles of the  $T$ -matrix:

$$2G \frac{N_c N_f}{\pi^2} \left[ \Lambda \sqrt{m^2 + \Lambda^2} - m^2 \operatorname{arcsinh} \left( \frac{m}{\Lambda} \right) \right] = 1 . \quad (9)$$

For a given value of the coupling  $G$ , the pole structure can be observed in figure 2 where the  $T$ -matrix is shown in a density plot as a function of the dynamical mass and the cutoff.

#### 4. $q\bar{q}$ subtracted $T$ -matrix at zero temperature

We want to derive the subtracted  $T$ -matrix inspired in the two-nucleon case. This requires an approximation of equation (5) with the replacement  $\sqrt{p^2 + m^2} \approx m + \frac{p^2}{2m} + \dots$ , namely

$$\operatorname{tr} S_F(0) \sim -\frac{i}{2\pi^2} \int dp p^2 \frac{2m^2}{2m^2 + p^2} . \quad (10)$$

Now the fermion propagator has a non-relativistic form and we can derive a subtracted  $T$ -matrix for the NJL model with the exact same formalism that has been applied to the  $NN$  interaction [7]. In the Born approximation, the NJL model can be treated in the same way as a pionless effective field theory for the two-nucleon system at leading order.

The scattering amplitude for two nucleons is given by the Lippman-Schwinger (LS) equation

$$\begin{aligned} T_{NN} &= V_{NN} + V_{NN} G_0 T_{NN} , \\ &= \frac{V_{NN}}{1 - V_{NN} G_0} , \end{aligned} \quad (11)$$

where

$$G_0 \equiv G_0(q; k) = \frac{1}{k^2 - q^2} , \quad (12)$$

and  $V_{NN} = C_0$  is the two-nucleon potential at leading order in a pionless EFT ( $C_0$  is usually fitted to the  $NN$   $s$ -wave scattering length). Performing subtractions in the kernel of the LS equation has proven to be effective to renormalize the nuclear two-body force [9, 10, 11]. With only a pure contact interaction, one subtraction is enough to renormalize the two-body  $T$ -matrix and the kernel  $G_0$  is subtracted at a scale  $\xi$ :

$$\begin{aligned} T_{NN} &= V_{NN} + V_{NN} G_1 T_{NN} , \\ &= \frac{V_{NN}}{1 - V_{NN} G_1} , \end{aligned} \quad (13)$$

where  $G_1$  is the Green's function with one subtraction

$$G_1 \equiv G_1(q; k, \xi) = G_0(q; k) - G_0(q; -\xi^2) = \frac{\xi^2 + k^2}{\xi^2 + q^2} G_0(q; k) . \quad (14)$$

We then make the non-relativistic approximation on the quark propagator and introduce the subtraction in the same way as the subtracted Green's function  $G_1(q; k, \xi)$  was obtained in Ref. [7]. The result is

$$J(q^2 = 0) = \frac{1}{2m\pi^2} \int dpp^2 \frac{2m^2}{p^2 + 2m^2} \frac{\xi^2 + m^2}{\xi^2 + p^2}. \quad (15)$$

In the born approximation,  $K = 2G$  and one subtraction is enough to provide a finite amplitude

$$\begin{aligned} T &= \frac{2G}{1 - 2G J}, \\ &= \frac{2G}{1 - 2G \frac{N_c N_f}{2m\pi^2} \int dpp^2 \frac{2m^2}{p^2 + 2m^2} \frac{\xi^2 + m^2}{\xi^2 + p^2}}, \\ &= \frac{2G}{1 - \frac{2N_c N_f G m}{\pi} \frac{\xi^2 + m^2}{\xi + m\sqrt{2}}}. \end{aligned} \quad (16)$$

The gap equation for the mass is then

$$2G \frac{N_c N_f}{\pi} \left[ \frac{\xi^2 + m^2}{\xi/m + \sqrt{2}} \right] = 1. \quad (17)$$

In figure 3 the solutions of equation (17) (upper left panel) and the chiral condensate (upper right panel) are shown as functions of the subtraction point  $\xi$ .

Again, fixing the pion decay constant  $f_\pi$  we can obtain the pion mass from the GOR relation as a function of the subtraction point for fixed current quark mass or as a function of the current quark mass for fixed subtraction point. This is shown in the lower panels of figure 3. The pole structure of the  $T$ -matrix is shown in the density plot of figure 4, where  $T$  is shown as a function of the mass  $m$  and the subtraction point  $\xi$ .

The dynamical mass depends on both the coupling  $G$  and the renormalization scale  $\Lambda$  or  $\xi$ . Then, the running of the coupling  $G$  with the renormalization scales can be obtained by fixing the dynamical mass. Graphically, the running is given by the intersection of the  $m(G, \Lambda)$  surface with the physical mass plane. Setting the mass at  $m = 325$  MeV, the running of the coupling  $G$  with both  $\Lambda$  and  $\xi$  can be observed in figure 5.

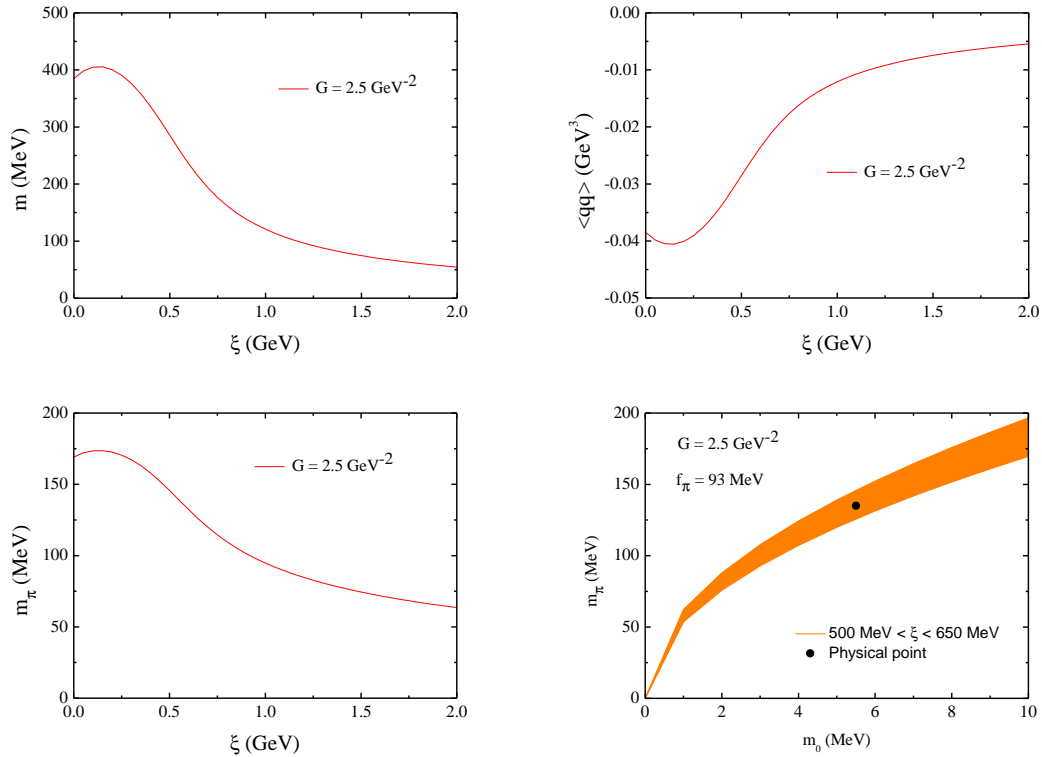
While the coupling  $G$  diverges at  $\Lambda = 0$ , it is finite at  $\xi = 0$ . Also, the mass doesn't increase with the subtraction point  $\xi$  as much as it grows with the cutoff  $\Lambda$ . The dependence on the renormalization scale is clearly much weaker in the subtraction scheme when compared to the standard cutoff approach.

## 5. Final remarks

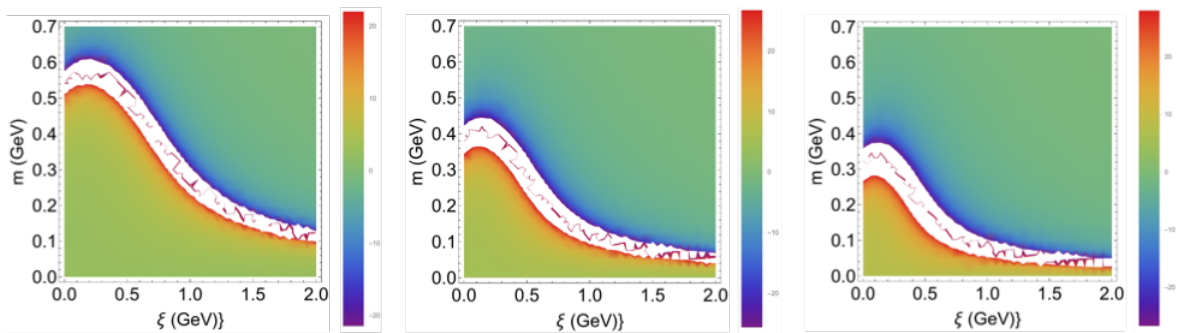
So far we have compared the cutoff and the subtractive renormalization procedures in the NJL model at zero temperature. The subtractive renormalization procedure has been applied to the  $NN$  interaction and here we extended it to quark degrees of freedom.

We found that the subtraction scheme gives good results when the strength of the coupling  $G_\xi$  is half the strength  $G_\Lambda$ . Furthermore, the dependence of the dynamical mass on the subtraction point is much weaker than in the cutoff approach.

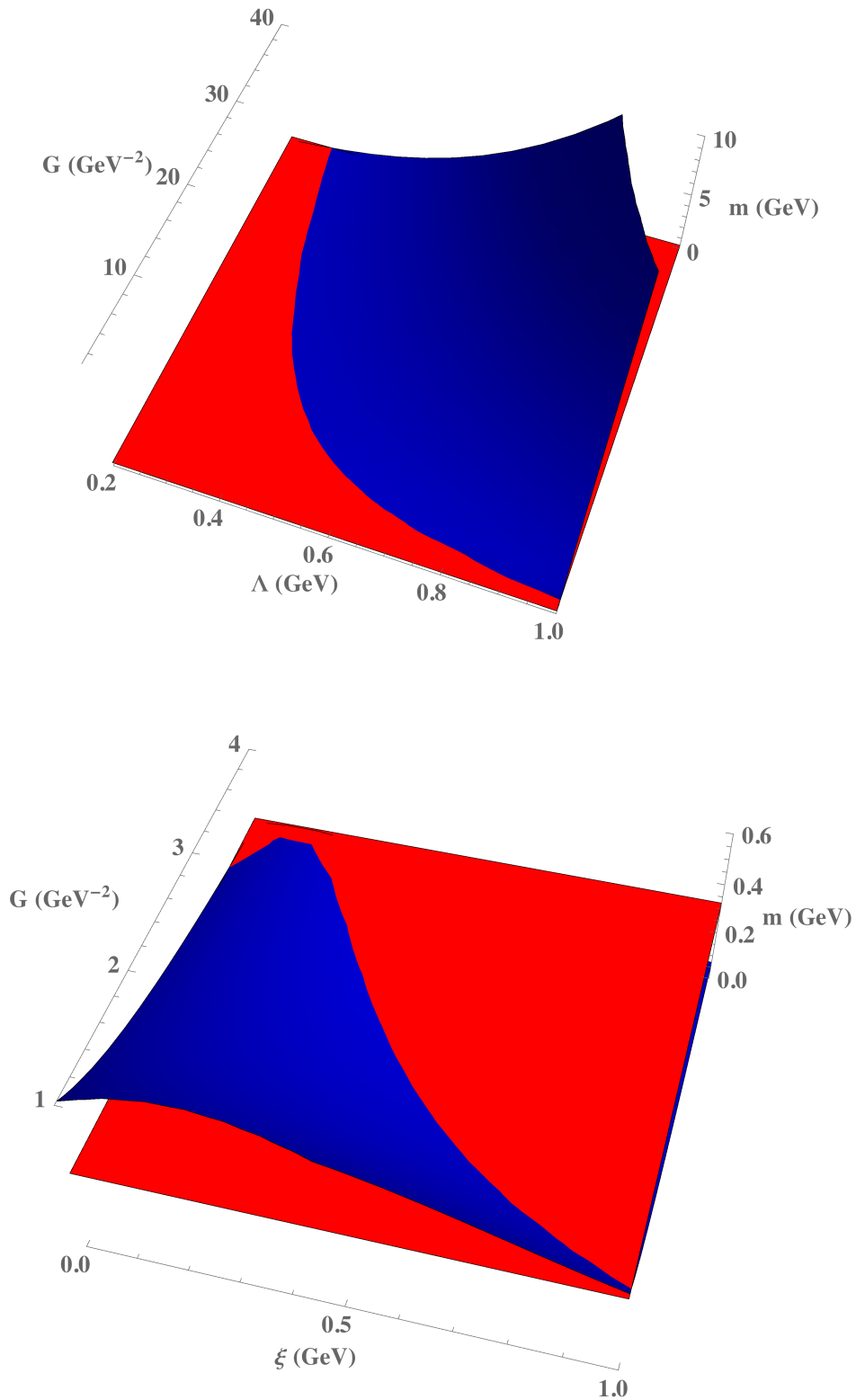
The subtractive renormalization may then be useful in the description of mesons with one heavy quark, where an additional (and larger) cutoff is required in order to describe the mass of the  $D$  meson [13].



**Figure 3.** Top panels: Dynamical mass for the quarks (left) and chiral condensate (right) as a function of the subtraction point  $\xi$ . Bottom panels: Pion mass as a function of the subtraction point (left) and of the current quark mass (right) for  $G = 2.5 \text{ GeV}^{-2}$  and  $f_\pi = 93 \text{ MeV}$ . The square represents the physical point  $(m_{u,d}, f_\pi) = (5.5, 140) \text{ MeV}$ .



**Figure 4.** The quark-antiquark  $T$ -matrix, in  $\text{GeV}^{-2}$ , as a function of the dynamical mass and the subtraction point for  $G = 0.5 G_\xi$  (left),  $G = 1.0 G_\xi$  (middle) and  $G = 1.5 G_\xi$  (right). The reference coupling in the subtraction scheme is  $G_\xi = 2.5 \text{ GeV}^{-2}$ .



**Figure 5.** Running of the NJL coupling  $G$  with the cutoff  $\Lambda$  and with the subtraction point  $\xi$  obtained by fixing the dynamical mass at  $m = 325$  MeV. The red plane corresponds to the fixed value of the mass and the blue surface corresponds to  $m(G, \Lambda)$  (top) and  $m(G, \xi)$  (bottom).



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