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<http://journals.aps.org/prd/abstract/10.1103/PhysRevD.93.053003>

DOI: 10.1103/PhysRevD.93.053003

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Hidden interactions of sterile neutrinos as a probe for new physicsZahra Tabrizi^{1,2,*} and O. L. G. Peres^{2,3,1,†}¹*School of Particles and Accelerators, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-1795, Tehran, Iran*²*Instituto de Física Gleb Wataghin—UNICAMP, 13083-859 Campinas, São Paulo, Brazil*³*Abdus Salam International Centre for Theoretical Physics, ICTP, I-34010 Trieste, Italy*

(Received 23 July 2015; revised manuscript received 22 November 2015; published 3 March 2016)

Recent results from neutrino experiments show evidence for light sterile neutrinos which do not have any Standard Model interactions. In this work, we study the hidden interaction of sterile neutrinos with an “MeV-scale” gauge boson (the ν_s HI model) with mass M_X and leptonic coupling g'_l . By performing an analysis on the ν_s HI model using the data of the MINOS neutrino experiment, we find that the values above $G_X/G_F = 92.4$ are excluded by more than 2σ C.L., where G_F is the Fermi constant and G_X is the field strength of the ν_s HI model. Using this model, we can also probe other new physics scenarios. We find that the region allowed by the $(g-2)_\mu$ discrepancy is entirely ruled out for $M_X \lesssim 100$ MeV. Finally, the secret interaction of sterile neutrinos has been to solve a conflict between the sterile neutrinos and cosmology. It is shown here that such an interaction is excluded by MINOS for $g'_s > 1.6 \times 10^{-2}$. This exclusion, however, does depend on the value of g'_l .

DOI: [10.1103/PhysRevD.93.053003](https://doi.org/10.1103/PhysRevD.93.053003)**I. INTRODUCTION**

Most of the data collected from the neutrino oscillation experiments are in agreement with the three-neutrino hypothesis [1]. However, the observation of the reactor anomaly, which is a deficit of electron antineutrinos produced in the reactors [2], together with the results of the MiniBooNE experiment [3] which shows evidence for $\nu_\mu \rightarrow \nu_e$ conversion, cannot be explained by the usual three-neutrino scenario [4]. The most popular way to clarify these anomalies is to assume there exists 1 (or more) neutrino state(s) which does not have any weak interaction (therefore is sterile) but can mix with the active neutrinos in the Standard Model (SM) and change their oscillation behavior pattern.

Although most of the anomalies seen in the neutrino sector are in favor of the sterile models with mass ~ 1 eV, there are conflicts between the sterile hypothesis and cosmology. Such light additional sterile states thermalize in the early Universe through their mixing with the active neutrinos; therefore, we effectively have additional relativistic number of neutrinos which can be parametrized by ΔN_{eff} . In the standard model of cosmology we have $\Delta N_{\text{eff}} = 0$. Massive sterile neutrinos with mass ~ 1 eV and large enough mixing angles to solve the reactor anomalies imply full thermalization at the early Universe. This means that for any additional species of sterile neutrinos, we should have $\Delta N_{\text{eff}} = 1$. However, this is not consistent with the big bang nucleosynthesis and the Planck results, which state $\Delta N_{\text{eff}} < 0.7$ with 90% C.L. [5].

It was recently proposed in [6,7] that this problem could be solved if the sterile neutrino state interacted with a new gauge boson X with mass \sim a few MeV. This can easily produce a large field strength for the sterile neutrinos. In this way the sterile state experiences a large thermal potential which suppresses the mixing between the active and sterile states in the early Universe. Therefore, the abundance of the sterile neutrinos remains small, and its impact on the big bang nucleosynthesis (BBN), cosmic microwave background (CMB), and the large scale structure formation would be negligible; hence, the sterile state can be consistent with the cosmological model.

In this work, we investigate the possibility of the sterile neutrino states interacting with a new gauge boson X , with mass \sim MeV, which has couplings with the sterile neutrinos and the charged leptons in the SM. This new interaction of the sterile neutrinos was first mentioned in [8]. The “ ν_s hidden interaction” (ν_s HI) model produces a neutral current (NC) matter potential for the sterile states proportional to G_X , where G_X is the field strength of the new interaction. The NC matter potential in the ν_s HI model changes the oscillation probability of neutrinos and antineutrinos drastically. Therefore, using the data of a neutrino oscillation experiment such as the MINOS experiment [9], we can test the ν_s HI model.

An advantage of the ν_s HI model is that through it we can use the data of neutrino oscillation experiments to test other new physics scenarios which imply having couplings with a light gauge boson, such as the explanation of the $(g-2)_\mu$ discrepancy with a light gauge boson [10] and the secret interaction of sterile neutrinos proposed in [6,7] which solves the tension between the sterile hypothesis and cosmology.

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II. THE FORMALISM

We enlarge the SM with one extra species of the sterile neutrinos which do not couple with the SM gauge bosons, but have interactions with a new $U_X(1)$ gauge symmetry (the ν_s HI model). The new gauge boson couples to the sterile neutrinos and charged leptons with coupling constants g'_s and g'_l , respectively, [11] where for simplicity, we have assumed equal coupling constants for the charged leptons. The strength of this new interaction is given by

$$\frac{G_X}{\sqrt{2}} = \frac{g'_s g'_l}{4M_X^2}, \quad (1)$$

where M_X is the mass of the new gauge boson.

The active neutrinos of the SM have charged and neutral current interactions with the W^\pm and Z bosons. Their matter potential is therefore given by $V_\alpha(r) = \delta_{ae} V_{CC}(r) + V_{NC}(r) = \sqrt{2} G_F N_e(r) (\delta_{ae} - 1/2)$, where $\alpha = e, \mu, \tau$ and $V_{CC(NC)}$ is the charged (neutral) current potential of the active neutrinos. The factor G_F is the Fermi constant, while $N_e(r)$ is electron number density of the earth given by the PREM model [12]. We have assumed that the electron and neutron number densities are equal for our practical purposes.

The sterile neutrinos which couple to the X boson will also have neutral current matter potential which is proportional to the strength field of the new interaction,

$$V_s(r) = -\frac{\sqrt{2}}{2} G_X N_e(r) \equiv \alpha V_{NC}(r), \quad (2)$$

where the dimensionless parameter α is defined as

$$\alpha = \frac{G_X}{G_F}. \quad (3)$$

For $\alpha \rightarrow 0$, we recover the minimal $3 + 1$ sterile neutrino model. In the minimal “ $3 + 1$ ” model [13] the flavor and mass eigenstates of neutrinos are related through the unitary $(3 + 1) \times (3 + 1)$ PMNS matrix U : $\nu_\alpha = \sum_{i=1}^{3+1} U_{\alpha i}^* \nu_i$. The oscillation probability of neutrinos is described using the active-active and active-sterile mixing angles, as well as the mass squared differences Δm_{21}^2 , Δm_{31}^2 , and Δm_{41}^2 , where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$.

The evolution of neutrinos in the ν_s HI model can be found by solving the following Schrödinger-like equation,

$$i \frac{d}{dr} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \left[\frac{1}{2E_\nu} U M^2 U^\dagger + V^{\nu_s \text{HI}}(r) \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix}, \quad (4)$$

where U is the 4×4 PMNS matrix [4], which is parametrized by the active-active mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$

as well as three active-sterile mixing angles $(\theta_{14}, \theta_{24}, \theta_{34})$. The matrix

$$M^2 = \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2, \Delta m_{41}^2)$$

is the matrix of the mass squared differences. Using Eq. (2), the matter potential matrix in the ν_s HI model will be (after subtracting the constant $V_{NC}(r) \times \mathbb{1}$)

$$V^{\nu_s \text{HI}}(r) = \text{diag}(V_{CC}(r), 0, 0, V_s(r) - V_{NC}(r)) \\ = \sqrt{2} G_F N_e(r) \text{diag}\left(1, 0, 0, \frac{(1-\alpha)}{2}\right). \quad (5)$$

The same evolution equation applies to antineutrinos with the replacement $V^{\nu_s \text{HI}}(r) \rightarrow -V^{\nu_s \text{HI}}(r)$. We consider the ν_s HI model with $\alpha > 0$. In an effective two-neutrino scheme the so called MSW resonance [14] happens when $\frac{\Delta m^2}{2E_\nu} \cos \theta = V$. Since in the ν_s HI model the sterile states have nonzero matter potential, the potential would be positive in a $\nu_\mu - \nu_s$ system (for $\alpha > 1$), which means that at energies where the resonance condition is carried out, ν_μ converts to ν_s .

An interesting place to test the ν_s HI model is the MINOS long-baseline neutrino experiment [9]. The MINOS experiment which has a baseline of 735 km detects both muon and antimuon neutrinos, and it is one of few experiments that is both sensitive to neutrino and antineutrino oscillation probabilities. For the baseline and energy range of the MINOS experiment, the oscillation probabilities of the neutrinos and antineutrinos are very similar in the usual three-neutrino scenario. However, this does not hold in the ν_s HI model anymore.

To see how the ν_s HI model affects the oscillation probability of neutrinos, we compute the full numerical survival probabilities for muon (anti)neutrino in the case of the standard three-neutrino scenario and in the $3 + 1$ and ν_s HI models. We show in Fig. (1) the survival probability of ν_μ (top) and $\bar{\nu}_\mu$ (bottom) for the standard three-neutrino case (black dashed curve), the $3 + 1$ model with $\alpha = 0$ (red solid curve) and the ν_s HI model with $\alpha = 150$ (the blue dot-dashed curve). To calculate the probabilities, we have fixed the three-neutrino oscillation parameters by the best-fit values of NUFIT [15]: $\Delta m_{21}^2 = 7.5 \times 10^{-5} \text{ eV}^2$, $\Delta m_{31}^2 = 2.4 \times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{12} = 0.3$, $\sin^2 \theta_{23} = 0.6$ and $\sin^2 \theta_{13} = 0.023$. The values of the active-sterile mixing parameters in the $3 + 1$ and ν_s HI models are listed in Fig. 1 [4]. Comparing the three-neutrino case (the black dashed curve) with the $3 + 1$ model (the red solid curve), we see that the effect of the sterile neutrino with mass squared difference $\Delta m_{41}^2 = 1 \text{ eV}^2$ is marginal adding only a very fast oscillation on the top of the oscillation induced by the atmospheric mass squared difference Δm_{31}^2 . However, in the ν_s HI model we have dramatic effects both for neutrino and antineutrino survival probabilities.

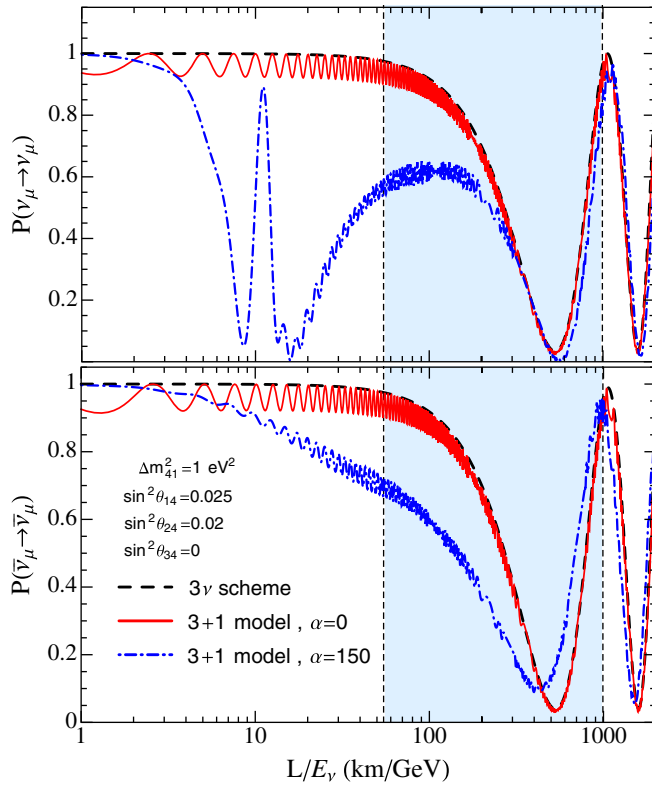


FIG. 1. The muon neutrino and antineutrino survival probabilities as a function of distance over neutrino energy are shown in the top and bottom, respectively. The black-dashed and the red-solid curves correspond to the 3 and 3 + 1 neutrino models, respectively. The blue dot-dashed curves represent the probabilities calculated in the ν_s HI model for $\alpha = 150$. The standard three-neutrino parameters are fixed by the NUFIT best-fit values [15] and the active-sterile mixing parameters are shown in the plot. The blue shaded area is the range of L/E_ν for the MINOS experiment [9].

When the resonance condition is fulfilled, we expect stronger changes for the ν_μ survival probability, while for antineutrinos the changes are milder. This can be seen in the blue dot-dashed curve at the top and bottom of Fig. 1.

III. THE ANALYSIS

We analyze the collected ν_μ and $\bar{\nu}_\mu$ beam data in the MINOS experiment to constrain the α parameter in the ν_s HI model. We calculate the expected number of events in each bin of energy by

$$N_i^{\text{osc}} = N_i^{\text{no-osc}} \times \langle P_{\text{sur}}(s_{23}^2, s_{24}^2, \Delta m_{31}^2, \Delta m_{41}^2; \alpha) \rangle_i, \quad (6)$$

where $s_{ij}^2 \equiv \sin^2 \theta_{ij}$ and $N_i^{\text{no-osc}}$ is the expected number of events for no-oscillation case in the i th bin of energy after subtracting background [9]; while $\langle P_{\text{sur}} \rangle_i$ is the averaged $\nu_\mu \rightarrow \nu_\mu$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$) survival probability in the i th energy bin, calculated using Eq. (4) for the fixed values $\sin^2 \theta_{14} = 0.025$ and $\sin^2 \theta_{34} = 0$ and letting the other parameters to vary.

To analyze the full MINOS data, we define the following χ^2 function,

$$\chi^2 = \sum_i \frac{[(1+a)N_i^{\text{osc}} + (1+b)N_i^b - N_i^{\text{obs}}]^2}{(\sigma_i^{\text{obs}})^2} + \frac{a^2}{\sigma_a^2} + \frac{b^2}{\sigma_b^2}, \quad (7)$$

where i runs over the bins of energy (23 for ν_μ events and 12 for $\bar{\nu}_\mu$ events), N_i^{osc} is the expected number of events defined in Eq. (6), and N_i^b and N_i^{obs} are the background and observed events, respectively. The $\sigma_i^{\text{obs}} = \sqrt{N_i^{\text{obs}}}$ represents the statistical error of the observed events. The parameters a and b take into account the systematic uncertainties of the normalization of the neutrino flux and the background events respectively, with $\sigma_a = 0.016$ and $\sigma_b = 0.2$ [16].

After combining the χ^2 function for the ν_μ and $\bar{\nu}_\mu$ events and marginalizing over all parameters, we find the following best-fit values: $\Delta m_{31}^2 = 2.43 \times 10^{-3} \text{ eV}^2$, $\Delta m_{41}^2 = 4.35 \text{ eV}^2$, $s_{23}^2 = 0.67$, $s_{24}^2 = 0.03$, and $\alpha = 19.95$, while the ratio of the χ^2 value over the number of degrees of freedom is $\chi^2/\text{d.o.f} = 39.7/30$. When we increase α from its best-fit value, we have disagreement between the ν_s HI model and the MINOS data. From this, we can find an upper bound for α at 2σ C.L.:

$$\alpha < 92.4. \quad (8)$$

Using Eq. (1) and Eq. (3), we can write down the coupling g'_i as a function of the gauge boson mass M_X and fixed value of α :

$$g'_i = \sqrt{\frac{2\sqrt{2}\alpha G_F}{\gamma}} M_X = 5.5 \times 10^{-5} \sqrt{\frac{\alpha}{92.4\gamma}} \left(\frac{M_X}{\text{MeV}} \right), \quad (9)$$

where we have assumed the two new coupling constants in our model are related as $g'_s = \gamma g'_l$, in which $\gamma \geq 1$. Therefore, we can use the expression above to find an exclusion region in the $(M_X - g'_l)$ plane. Implementing the relation above for the MINOS experiment, we arrive to the black dashed curve shown in Fig. 2.

A light gauge boson with mass $\sim \text{MeV}$ can be used as a unique explanation for the 3.6σ discrepancy between the experimental measurement and the SM prediction of the muon anomalous magnetic moment, $(g-2)_\mu$ [10]. The purple shaded region in Fig. 2 shows the favored 2σ region from $(g-2)_\mu$ discrepancy. It is shown in Ref. [19] that nearly the entire $(g-2)_\mu$ band is excluded by various experiments if one assumes that the light gauge boson decays to charged leptons with branching ratio (Br) ~ 1 . However, in the ν_s HI model, the primary decay mode of the light gauge boson is into invisibles (such as the light sterile neutrinos) with Br ~ 1 . Therefore, all the $(g-2)_\mu$ band in Fig. 2 will be valid in the ν_s HI model. As Fig. 2

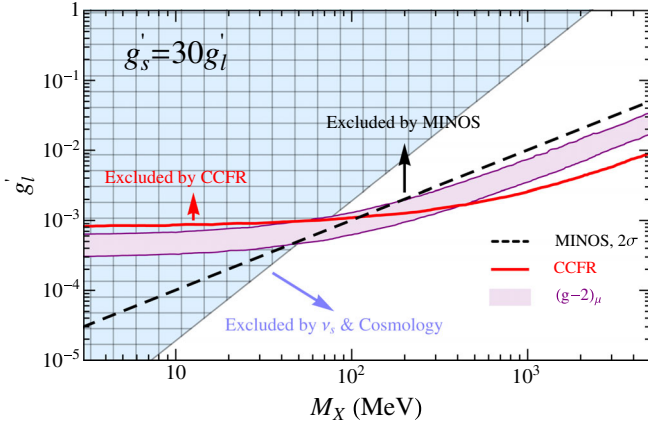


FIG. 2. We have shown the region of interest for the ν_s HI model with a light gauge boson with mass M_X and couplings g'_l and $g'_s = \gamma g'_l$. The result of the analysis of the ν_s HI model with the MINOS data is shown by the black dashed curve with 2σ C.L. (for $\gamma = 30$). The purple shaded region is the region favored by the $(g-2)_\mu$ discrepancy, while the red curve is the CCFR [17] measurement of the neutrino trident cross-section [18]. The blue shaded region is where the tension between the sterile neutrino and cosmology is relieved for $f(g'_s, M_X) = 100$ and $\gamma = 30$ (See Eq. (10) and the discussion after that).

shows, by comparing our results on the MINOS analysis of the ν_s HI model with the $(g-2)_\mu$ band we can exclude all the masses below $M_X \sim 100\sqrt{\gamma/30}$ MeV with 2σ C.L.

Another piece of information comes from neutrino trident production: the process in which the $\mu^+\mu^-$ pair is produced from the scattering of ν_μ off the Coulomb field of a nucleus. The red solid curve in Fig. 2 represents the results of the constraints from CCFR experiment on measurement of the neutrino trident cross-section [17]. As it can be seen from Fig. 2, by combining the result of the CCFR experiment with our result from the MINOS analysis, there is only a tiny region in the $(g-2)_\mu$ band which is allowed by all experiments.

The sterile neutrino states with 1 eV mass have dramatic effects in cosmology due to their thermalization in the early Universe and are disfavored by the Planck data. This tension could be removed if the sterile states have interactions with a light gauge boson in the so called secret interaction model [7,20]. This will produce a temperature dependent matter potential for the sterile states which is $V_{\text{eff}} = -\frac{7\pi^2}{45} \frac{g_s^2}{M_X^4} E_\nu T_s^4$ [7], where T_s is the temperature of the sterile sector and $E_\nu \ll M_X$. Therefore, the oscillation of the active to sterile neutrinos would be suppressed if $|V_{\text{eff}}| \gg |\frac{\Delta m_{41}^2}{2E_\nu}|$ [7]. We define the following function:

$$\frac{V_{\text{eff}}}{\Delta m_{41}^2/2E_\nu} \equiv f(g'_s, M_X) = \frac{14\pi^2 g_s^2 E_\nu^2}{45\Delta m_{41}^2} \left(\frac{T_s}{M_X}\right)^4. \quad (10)$$

Hence, the cosmology condition in the secret interaction model would be satisfied if $f(g'_s, M_X) \gg 1$. Similar to

Eq. (9) we can find the values of the coupling constant which satisfy the cosmology condition:

$$g'_s \gg \sqrt{\frac{45}{14\pi^2}} \frac{\sqrt{\Delta m_{41}^2}}{E_\nu} \left(\frac{M_X}{T_s}\right)^2. \quad (11)$$

Assuming that the cosmology condition is satisfied for $f(g'_s, M_X) = 100$, then using Eq. (9) and the relation between the 2 coupling constants $g'_s = \gamma g'_l$, the values of g'_s above

$$g'_s = 1.6 \times 10^{-2} \left(\frac{T_s}{\text{MeV}}\right)^2 \left(\frac{E_\nu/\text{MeV}}{\sqrt{\Delta m_{41}^2/\text{eV}^2}}\right) \frac{\alpha}{92.4} \frac{\gamma}{30} \quad (12)$$

is excluded by the MINOS analysis. Therefore, using the MINOS data, we find that at the time of BBN ($E_\nu \approx T_s \approx 1$ MeV) and for $\Delta m_{41}^2 = 1$ eV², the values of the coupling constant above $g'_s = 1.6 \times 10^{-2}$ is excluded with more than 2σ C.L. (for $\gamma = 30$). The blue shaded region in Fig. 2 shows the cosmology condition for the values mentioned above.

IV. CONCLUSIONS

We have investigated the possibility that the light sterile neutrinos as suggested by the reactor anomaly have hidden interaction with an MeV-scale gauge boson. In the hidden interaction (ν_s HI) model, the sterile neutrinos have neutral current matter potential. Therefore, we can use the data of the neutrino experiments to constrain this model and probe other new physics scenarios. The field strength of this model is described by G_X . In this work we studied the ν_s HI model using the MINOS experiment and showed that the values above $G_X/G_F = 92.4$ are excluded.

One consequence of the ν_s HI model is constraining other new physics scenarios such as explaining the $(g-2)_\mu$ discrepancy with a light gauge boson. We showed that, using the ν_s HI model, the $(g-2)_\mu$ region is entirely ruled out for $M_X \lesssim 100\sqrt{\gamma/30}$ MeV by the MINOS data. Also, the secret interaction of sterile neutrinos which is introduced in the literature to solve the tension between the sterile neutrinos and cosmology is excluded by MINOS for $g'_s > 1.6 \times 10^{-2} \frac{\gamma}{30}$ for any value of M_X , where g'_s is the coupling between the sterile states and the light gauge boson. We can use the data of the future neutrino oscillation experiments such as DUNE [21] to further test the ν_s HI model and get a definite answer on the presence of the light gauge boson.

ACKNOWLEDGMENTS

Z. T. thanks the useful discussions with Joachim Kopp and Pedro Machado. O. L. G. P. thanks the hospitality of IPM and the support of FAPESP Funding Grant No. 2012/16389-1.

- [1] M. C. Gonzalez-Garcia and Y. Nir, *Rev. Mod. Phys.* **75**, 345 (2003).
- [2] G. Mention, M. Fechner, Th. Lasserre, Th. A. Mueller, D. Lhuillier, M. Cribier, and A. Letourneau, *Phys. Rev. D* **83**, 073006 (2011); P. Huber, *Phys. Rev. C* **84**, 024617 (2011).
- [3] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), *Phys. Rev. Lett.* **105**, 181801 (2010).
- [4] K. N. Abazajian *et al.*, [arXiv:1204.5379](https://arxiv.org/abs/1204.5379).
- [5] P. A. R. Ade *et al.* (Planck Collaboration), [arXiv:1502.01589](https://arxiv.org/abs/1502.01589).
- [6] S. Hannestad, R. S. Hansen, and T. Tram, *Phys. Rev. Lett.* **112**, 031802 (2014).
- [7] B. Dasgupta and J. Kopp, *Phys. Rev. Lett.* **112**, 031803 (2014).
- [8] M. Pospelov, *Phys. Rev. D* **84**, 085008 (2011).
- [9] P. Adamson *et al.* (MINOS Collaboration), *Phys. Rev. Lett.* **110**, 251801 (2013).
- [10] M. Pospelov, *Phys. Rev. D* **80**, 095002 (2009).
- [11] All the $SU(2)_L$ doublets are assumed to be singlets of $U(1)_X$. However, the charged leptons could be connected to the X boson through, e.g., a Higgs loop.
- [12] A. M. Dziewonski and D. L. Anderson, *Phys. Earth Planet. Inter.* **25**, 297 (1981).
- [13] O. L. G. Peres and A. Y. Smirnov, *Nucl. Phys.* **B599**, 3 (2001).
- [14] S. P. Mikheyev and A. Y. Smirnov, *Sov. J. Nucl. Phys.* **42**, 913 (1985).
- [15] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, *J. High Energy Phys.* **12** (2012) 123.
- [16] By marginalizing the χ^2 function of Eq. (7) for the three-neutrino case, we find that our best-fit values for $\sin^2 2\theta_{23}$ and Δm_{31}^2 are fairly close to the values reported by the Collaboration.
- [17] S. R. Mishra *et al.* (CCFR Collaboration), *Phys. Rev. Lett.* **66**, 3117 (1991).
- [18] W. Altmannshofer, S. Gori, M. Pospelov, and I. Yavin, *Phys. Rev. Lett.* **113**, 091801 (2014).
- [19] H. Davoudiasl, H.-S. Lee, and W. J. Marciano, *Phys. Rev. D* **89**, 095006 (2014).
- [20] J. Kopp and J. Welter, *J. High Energy Phys.* **12** (2014) 104.
- [21] J. M. Berryman, A. de Gouvêa, K. J. Kelly, and A. Kobach, *Phys. Rev. D* **92**, 073012 (2015).