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2015 J. Phys.: Conf. Ser. 630 012056

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Journal of Physics: Conference Series **630** (2015) 012056 doi:10.1088/1742-6596/630/1/012056

Running of the contact interactions in chiral N3LO potentials from subtractive renormalization *

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Abstract.
In this work a subtracted kernel renormalization procedure (SKM) is applied to the chiral NN potential up to next-to-next-to-next-to-leading-order (N^3LO) to obtain the running of the renormalized contact strengths with the subtraction scale μ and the phase shifts for all uncoupled waves with contact interaction (S, P, D). We use two potentials constructed within the framework of Weinberg's approach to ChEFT, which provide a very accurate description of NN scattering data below laboratory energies $E \sim 350\ MeV$, namely Epelbaum, Glöckle and Meissner (N3LO-EGM) and Entem and Machleidt (N3LO-EM). For both potentials, we consider a large cutoff (30 fm^{-1}) and analyze the phases and the running of the contact strengths with the subtraction point μ by making a fit of the K-matrix with five subtractions to the K-matrix from the Nijmegen II potential at low energies ($E \leq 20$ MeV).

1. Introduction

Quantum Chromodynamics (QCD) is widely accepted as the fundamental theory of strong interactions. The degrees of freedom described in the QCD Lagrangian are quarks and gluons. The strength of the interaction is strongly energy dependent and decreases as the energy increases. As a consequence, the QCD coupling constant α_s decreases for large momentum and the quarks inside the hadrons behave as free particles. Thus, in the high-energy regime α_s is very small and QCD can be treated perturbatively. On the other hand, in the low-energy regime the QCD coupling constant increases and perturbation theory can no longer be applied. Therefore, QCD is not effective to describe the strong interactions at low energies. To avoid this problem alternative approaches to treat strong interactions in the non-perturbative regime have been developed over the last two decades.

At low-energies the relevant degrees of freedom are baryons and mesons rather than quarks and gluons. Hence, it is more convenient to use Effective Fields Theories (EFT) which are

*Presented by E.F.B. at XXXVII RTFNB, 37th Brazilian Workshop on Nuclear Physics, 8-12 September 2014, Maresias, SP, Brazil

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constructed with nucleons and pions as the relevant degrees of freedom. Weinberg proposed an EFT formalism for the nucleon-nucleon (NN) interaction based on a chiral expansion of an effective Lagrangian known as Chiral Perturbation Theory (ChPT) [1, 2, 3]. This approach, first applied by Ordóñez, Ray and van Kolck [4] allows one to work with the NN interaction perturbatively where the classes of the Feynman diagrams required to complete each order are organised by a power counting scheme called Weinberg's Power Counting (WPC). At leading order (LO) the NN potential consists of one-pion-exchange (OPE) plus a contact term. At next-to-leading order (NLO) two-pion-exchange (TPE) and $\mathcal{O}(p^2)$ contact interactions are added, at next-to-next-to-leading order (N2LO) there is an additional set of TPE diagrams and finally, at next-to-next-to-next-to-leading order (N3LO) corrections to both OPE and TPE are included along with $\mathcal{O}(p^4)$ contact interactions.

At any order in the chiral expansion the NN interaction contributions have ultraviolet (UV) divergencies and therefore a renormalization procedure is required. Usually a momentum cutoff (sharp or smooth) is applied to the potential so that the integral in the Lippmann-Schwinger (LS) equation becomes finite. An alternative procedure based on a kernel subtraction scheme has been proposed to modify the scattering equation and keep the potential uncut. This method was first introduced in Ref.[5] where the authors applied a single subtraction to renormalize the NN interaction at leading order.

In this formalism only one scale parameter is introduced, the subtraction point μ, which is the scale where a physical input for the two-body scattering amplitude has to be supplied in order to fix the renormalized counter-terms. The number of subtractions necessary to render the amplitude finite depends on the degree of divergence of the interaction. Each subtraction introduces a factor of p^{-2} in the kernel of the LS equation. So, if the interaction has terms with $\mathcal{O}(p^d)$, at least $d+1$ subtractions have to be performed in the LS equation kernel to generate finite results.

A generalisation of the SKM method for any number of subtractions was developed in Ref. [6] and it was shown that a non-relativistic Callan-Symanzik equation arises from the condition that the two-body amplitude has to be invariant under the change of the subtraction point. The renormalization group flow equation tell us how the driving terms have to be modified with the renormalization scale in order to keep the scattering amplitude invariant.

In previous works this formalism was applied with three [7] and four [8, 9] subtractions in the study of higher orders of NN interaction. In the latter, the Callan-Symanzik flow equation with four subtractions was numerically integrated for the ${}^1S_0$ channel and renormlization group invariance of the method was shown explicitely.

2. Chiral forces at N3LO: renormalization with five subtractions

The NN potential at N3LO can be separated in three main contributions:

$$V_{\rm N3LO} = V_{\rm OPE} + V_{\rm TPE} + V_{\rm Contact} \; . \tag{1}$$

The first term is the one-pion-exchange potential which describes the long range part of the interaction. The second is the two-pion-exchange potential that is associated with the mid-range part and the last one is the contact term which parametrizes the short range part of the interaction.

At N3LO, there are contact interactions only in the partial waves with $J \leq 2$ and for uncoupled channels we have twelve Low-Energy Constants (LEC):

$$\begin{aligned} V^{\rm 1S0}_{\rm Contact} &= C_0 + C_2\;(p^2+p'^2) + C_4\;(p^2\times p'^2) + C'_4\;(p^4+p'^4)\;, \\ V^{\rm 3P0,\;1P1,\;3P1}_{\rm Contact} &= C_1\;(p\times p') + C_3\;(p\times p')\;(p^2+p'^2)\;, \\ V^{\rm 1D2,\;3D2}_{\rm Contact} &= D_4\;(p^2\times p'^2)\;. \end{aligned} \tag{2}$$

The OPE and TPE terms are given in terms of well known physical quantities like the axial coupling constant, g_a, the pion weak-decay constant, f_π, the nucleon mass, m, and the pion mass, m_π. The LECs C_i and D_4 stand for the strengths of the contact interactions which have to be adjusted to reproduce the two-nucleon observables.

Both pion-exchange and contact interaction terms can lead to UV divergences when the effective NN potential at a given order in the chiral expansion is iterated in the LS equation, requiring a regularization and renormalization procedure in order to obtain well-defined finite solutions. Both a sharp cutoff renormalization scheme [10] or a smooth cutoff regularization function [11] can applied with very successful results.

In this work we renormalize the N3LO interactions by applying five subtractions to the kernel of the LS equation for the K-matrix with an iterative procedure at a given momentum scale μ. The LS equation for the K-matrix with five subtraction is given by

$$K_5(p,p';k,\mu) = V^{(5)}(p,p';k,\mu) + \frac{2}{\pi}\mathcal{P}\int dq\ q^2\ V^{(5)}(p,q;k,\mu)\ G^{(5)}(q,k;\mu)\ K_5(q,p';k,\mu)\ , \quad (3)$$

where $G^{(5)}$ is the five-fold subtracted Green's function

$$G^{(5)}(q,k;\mu) = \left(\frac{\mu^2+k^2}{\mu^2+q^2}\right)^5 \frac{1}{k^2-q^2}\ , \quad (4)$$

and $V^{(5)}$ is the so called fifth-order driving term

$$V^{(5)}(p,p';k,\mu) = V^{(4)}(p,p';k,\mu) + \frac{2}{\pi}\mathcal{P}\int dp\ p^2\ V^{(4)}(p,q;k,\mu)\ \frac{(\mu^2+k^2)^4}{(\mu^2+q^2)^5}\ V^{(5)}(q,p';k,\mu)\ . \quad (5)$$

Thus, for a given subtraction scale μ we determine the recursive driving terms by applying the recursive procedure described in detail in our previous works. Then we insert the driving term $V^{(5)}$ in Eq.3 and a finite solution for the K-matrix is obtained which can be used to calculate the NN phase-shift at a given energy. In the next section we will show some numerical results for all uncoupled channels up to $J = 2$.

3. Numerical results

In the following we present the numerical procedure used to fix the strengths of the contact interactions and some results for the NN phase shifts and the running of the contact strengths with the renormalization scale are shown.

3.1. Fitting Procedure

We compute the mean square function

$$\mathcal{F}(\mu) = \frac{1}{N}\sum_{j=1}^{N}\left[K_5(k_j;\ \mu) - K_{\rm ref}(k_j)\right]^2\ , \quad (6)$$

and obtain $C(\mu)$ by minimizing $\mathcal{F}(\mu)$ for each value of the renormalization scale μ. We use $N = 21$ points in the interval $0 \leq k_j \leq 0.5$ fm^{-1} ($0 \leq E_j \leq 20$ MeV). Once the strengths of the contact interactions have been determined, the phase shifts can be obtained as

$$\delta_\mu(k_j) = \text{atan}\left[-k_j \times K_5(k_j;\ \mu)\right]\ . \quad (7)$$

Note that we fit only the low-energy part of the K-matrix so that the results for higher energy will be just predictions.

Table 1. Strengths of the renormalized contact interactions, all given in fm, for the reference scale μ_0, in fm^{-1}, for the EGM potential.

	μ_0	C_0	$\sqrt[3]{C_1}$	$\sqrt[3]{C_2}$	$\sqrt[3]{C_3}$	$\sqrt[5]{C_4}$	$\sqrt[5]{C_4'}$	$\sqrt[5]{D_4}$
$^1S_0$	0.97	-1.248	-	-2.451	-	3.176	-0.646	-
$^3P_0$	3.00	-	-1.707	-	-0.288	-	-	-
$^1P_1$	1.90	-	0.429	-	0.416	-	-	-
$^3P_1$	1.50	-	-0.499	-	0.722	-	-	-
$^1D_2$	5.25	-	-	-	-	-	-	-0.346
$^3D_2$	0.47	-	-	-	-	-	-	-0.614

Table 2. Strengths of the renormalized contact interactions, all given in fm, for the reference scale μ_0, in fm^{-1}, for the EM potential.

	μ_0	C_0	$\sqrt[3]{C_1}$	$\sqrt[3]{C_2}$	$\sqrt[3]{C_3}$	$\sqrt[5]{C_4}$	$\sqrt[5]{C_4'}$	$\sqrt[5]{D_4}$
$^1S_0$	1.00	-0.838	-	-2.798	-	3.388	-0.817	-
$^3P_0$	3.05	-	3.672	-	-0.3	-	-	-
$^1P_1$	1.70	-	0.574	-	0.541	-	-	-
$^3P_1$	1.50	-	-0.584	-	0.649	-	-	-
$^1D_2$	4.00	-	-	-	-	-	-	3.094
$^3D_2$	0.47	-	-	-	-	-	-	-0.423

In Tables 1 and 2 we show the strengths of the renormalized contact interactions for the best fit in each partial wave for the N3LO-EGM and N3LO-EM respectively. We define the scale which provides the best fit as the reference scale μ_0.

In Figs. 1 and 2 we show the phase-shifts for the $^1D_2$ and $^3D_2$ waves for both N3LO-EM and N3LO-EGM chiral potentials compared to the Nijmegen partial wave analysis [12]. The magenta and yellow bands correspond to a range for the renormalization scale μ. With the exception of the $^1D_2$ with the N3LO-EGM potential which doesn't show a good fit, there is a narrow range of the renormalization scale where the fits are very good up to $E_{\text{lab}} = 100$ MeV.

The running of the contact strength D_4 for the two potentials are displayed in Fig. 3 for the uncoupled D-waves. In the case of the $^3D_2$ channel (blue line), the running is nearly identical for the two potentials and is similar to what was obtained by Epelbaum and Meissner with cutoff regularization [13]. The strengths for the $^1D_2$ channel (red line) is almost scale invariant in the range $0 < \mu < 2\ \text{fm}^{-1}$.

Figs. 4 shows the phase-shifts for the $^1P_1$ and $^3P_1$ channels. In these channels a proper choice for the renormalization scale range (see figure legends) allows a good prediction for the phases up to $E_{\text{lab}} = 100$ MeV like in the D-waves. The associated running of C_1 and C_3 are displayed in Fig. 5, where we can observe that the running for C_1 is rather stable (upper panels) but for C_3 there are regions with stability followed by sharp variations (lower panels). Note that C_3 in the $^3P_1$ channel changes sign for N3LO-EM in the range $1 < \mu < 2\ \text{fm}^{-1}$ and for N3LO-EGM there is a maximum for C_3 at $\mu = 1.5\ \text{fm}^{-1}$ in the $^3P_1$ channel and at $\mu = 2.0\ \text{fm}^{-1}$ in the $^1P_1$ channel. In the range $2 < \mu < 4\ \text{fm}^{-1}$ the strengths of C_3 are nearly the same for the $^1P_1$ and $^3P_1$ channels for both N3LO-EM and N3LO-EGM.

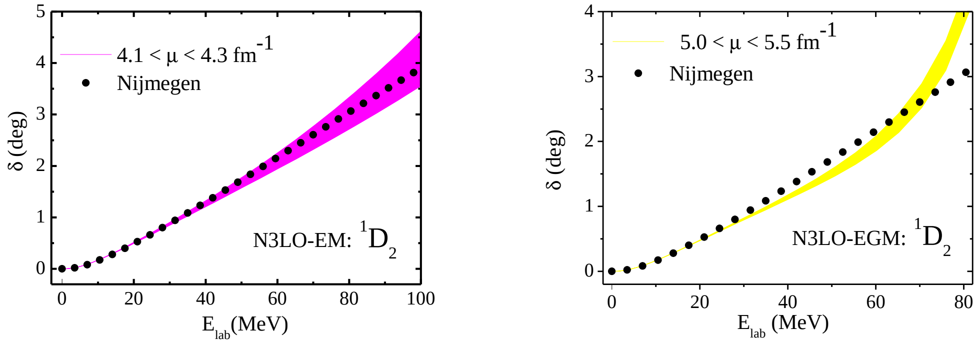

Figure 1. Phase-shifts for the $^1D_2$ channel from both N3LO-EM (left) and N3LO-EGM (right) chiral potentials compared to the Nijmegen partial wave analysis.

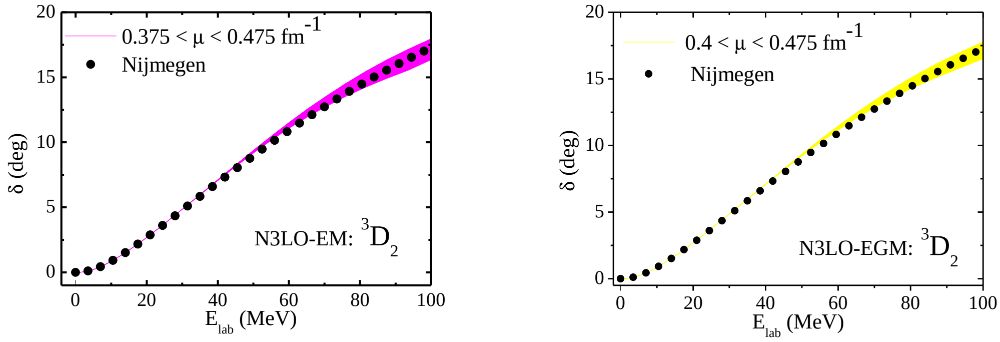

Figure 2. Phase-shifts for the $^3D_2$ channel from both N3LO-EM (left) and N3LO-EGM (right) chiral potentials compared to the Nijmegen partial wave analysis.

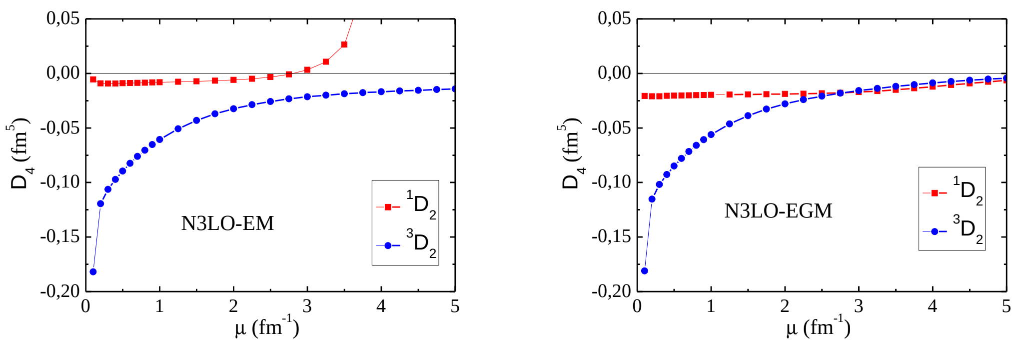

Figure 3. Running of the contact strength D_4 with the renormalization scale μ for the uncoupled D-waves with both N3LO-EM (left) and N3LO-EGM (right) chiral potentials.

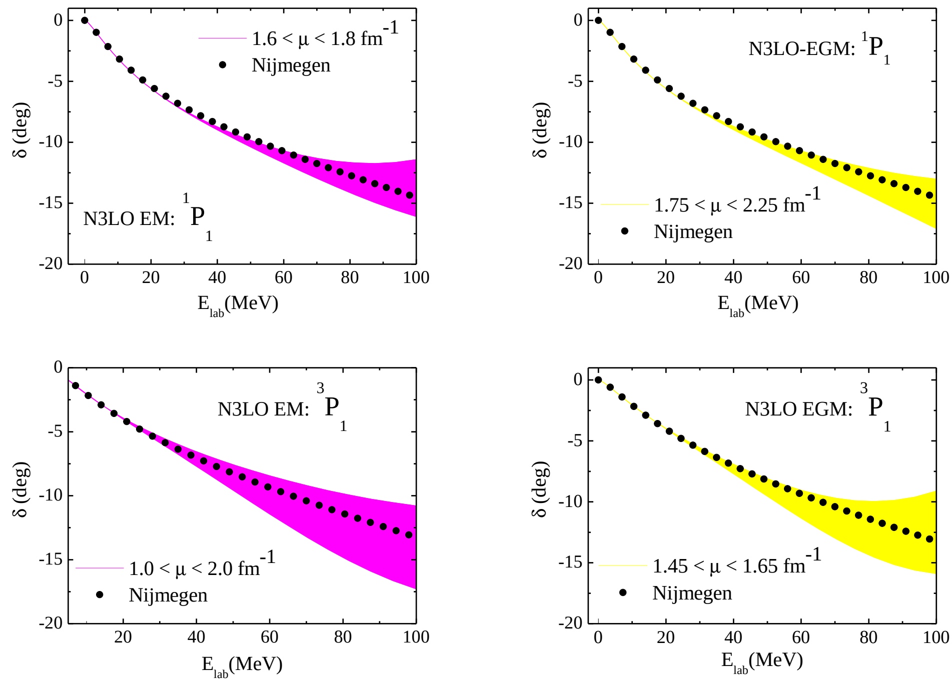

Figure 4. Phase-shifts for the $^1P_1$ (top) and $^3P_1$ (bottom) channels from both N3LO-EM and N3LO-EGM chiral potentials compared to the Nijmegen partial wave analysis.

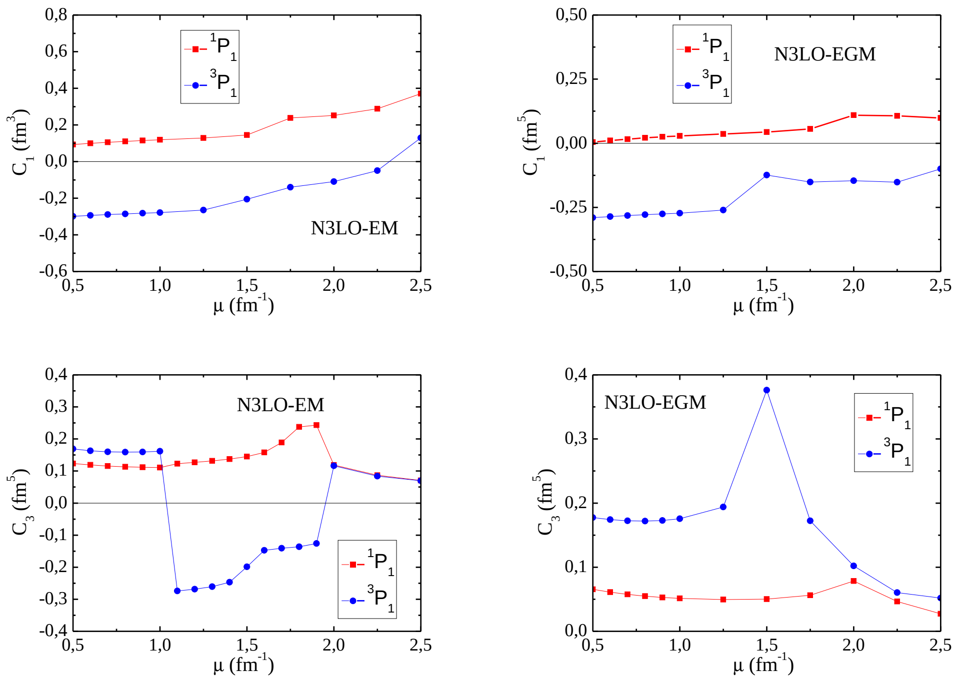

Figure 5. Running of the contact strengths C_1 and C_3 with the renormalization scale μ for the uncoupled P-waves with both N3LO-EM (left) and N3LO-EGM (right) chiral potentials.

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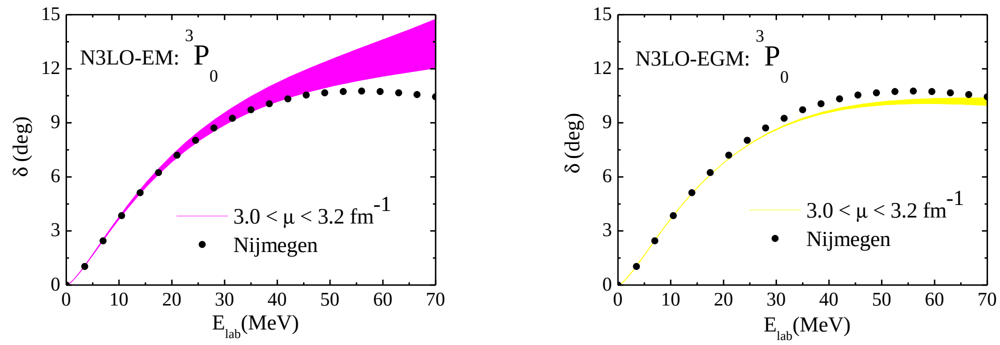

Figure 6. Phase-shifts for the $^3P_0$ channel from both N3LO-EM and N3LO-EGM chiral potentials compared to the Nijmegen partial wave analysis.

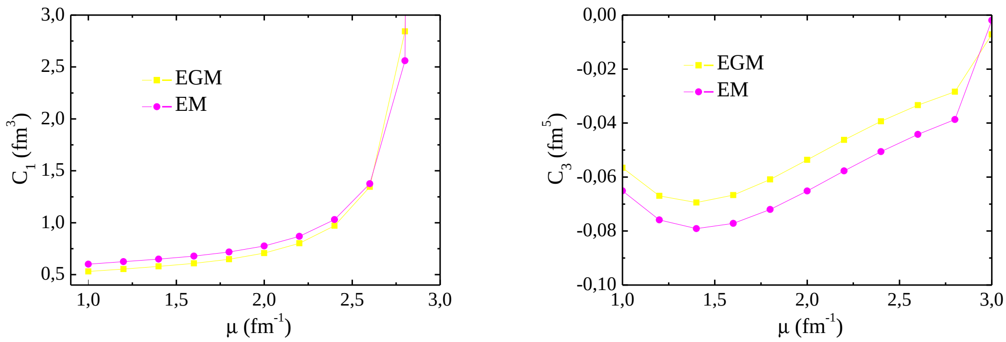

Figure 7. Running of the contact strengths C_1 and C_3 with the renormalization scale μ for the $^3P_0$ channel with both N3LO-EM and N3LO-EGM chiral potentials.

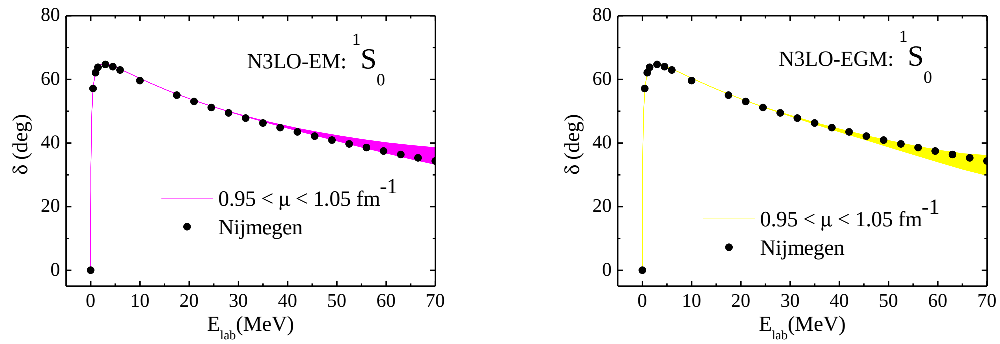

Figure 8. Phase-shifts for the $^1S_0$ channel from both N3LO-EM and N3LO-EGM chiral potentials compared to the Nijmegen partial wave analysis.

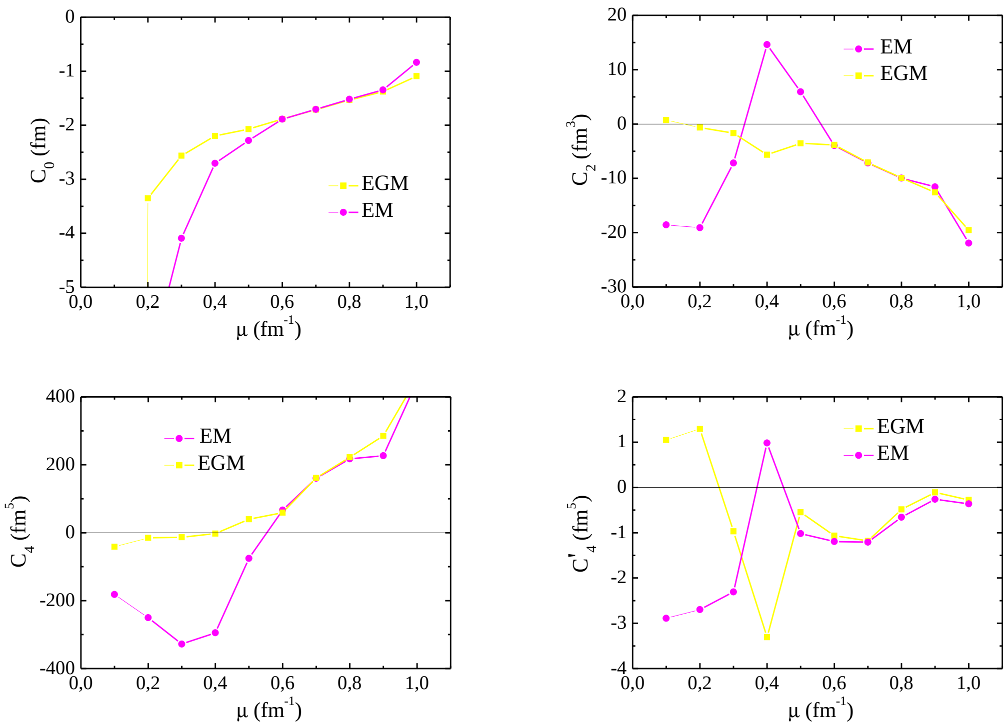

Figure 9. Running of the contact strengths C_0, C_2, C_4 and C_4' with the renormalization scale μ for the $^1S_0$ channel with both N3LO-EM and N3LO-EGM chiral potentials.

Now we turn to the channels with $J = 0$. The phase-shifts for the $^3P_0$ wave are shown in Fig. 6 where we can observe that the experimental data are described only for very low energies ($E_{\rm lab} \sim 50$ MeV) and for the same range of the renormalization scale, $3.0 < \mu < 3.2$ fm^{-1}, the N3LO-EM provides a wider band containing the at low energies.

The running for C_1 and C_3 in the $^3P_0$ channel, displayed in Fig. 7, are very similar for both N3LO-EM and N3LO-EGM. In fact, the difference between the strengths of the contact interactions for the two potentials is nearly constant.

The phase-shifts for the $^1S_0$ channel are very well described up to $E_{\rm lab} \sim 70$ MeV with renormalization scales in the range $0.95 < \mu < 1.05$ fm^{-1} for both N3LO-EM and N3LO-EGM, as can be seen in Fig. 8. This is consistent with what has been obtained with four subtractions at N2LO in Ref. [9].

Fig. 9 shows the running of the four contact terms present in the $^1S_0$ wave and we can observe that the strengths of the interactions for N3LO-EM and N3LO-EGM match for renormalization scales in the range $0.6 < \mu < 1.0$ fm^{-1}.

4. Summary and outlook

In this paper we have used the subtracted kernel renormalization method (SKM) with multiple subtractions to obtain the phase shifts and the running of the contact strengths for all uncoupled channels up to $J = 2$. We have considered two state of the art chiral potentials at N3LO and performed five subtractions while keeping a large instrumental cutoff $\Lambda = 30$ fm^{-1}.

In future works we will study the renormalization group invariance with N3LO potentials by integrating the non-relativistic Callan-Symanzik flow equation in order to obtain an exact solution (non-perturbative) for the driving terms in several channels of the NN interaction.

Acknowledgements
S.S. is partially supported by FAPESP and V.S.T. would like to thank FAEPEX, FAPESP and CNPq for financial support.

References
[1] Weinberg S 1990 *Phys. Lett.* B **251** 288.
[2] Weinberg S 1991 *Nucl. Phys.* B **363** 3.
[3] Weinberg S 1992 *Phys. Lett.* B **295** 114.
[4] Ordóñez C, Ray L and van Kolck U 1994 *Phys. Rev. Lett.* **72** 1982.
[5] Frederico T, Timóteo V S and Tomio L 1999 *Nucl. Phys.* A **653** 209.
[6] Frederico T, Delfino A and Tomio L 2000 *Phys. Lett.* B **481** 143.
[7] Timóteo V S, Frederico T, Tomio L and Delfino A 2005 *Phys. Lett.* B **621** 109.
[8] Timóteo V S, Frederico T, Delfino A and Tomio L 2011 *Phys. Rev.* C **83** 064005.
[9] S. Szpigel and V. S. Timóteo 2012 *J. Phys.* G **39** 105102.
[10] Entem D R and Machleidt R 2003 *Phys. Rev.* C **68** 041001
[11] Epelbaum E, Glöckle W, and Meißner U G 2005 *Nucl. Phys.* A **747** 362.
[12] Stoks V G J, Klomp R A M, Terheggen C P F and de Swart J J 1994 *Phys. Rev.* C **49** 2950.
[13] Epelbaum E and Meissner U -G 2013 *Few Body Systems* **54** 2175.