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Indeterminacy, EPR and Bell

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Abstract

This article seeks to clarify certain key theoretical, conceptual and philosophical issues in the foundations of microphysics which, to judge from certain recent publications, continue to cause misunderstandings. In particular, we examine the Heisenberg indeterminacy relations, underlining that they are not univocally interpretable and that, at least in the interpretation following directly from the quantum formalism, they are not the target of Einstein, Podolsky and Rosen's criticism. We try to identify the essential goal and premises of this famous argument, with the help of a simple example. Finally, we examine briefly the Bell inequalities, emphasizing that, given their generality, the net consequence of their experimental violation cannot be circumvented neither by the abandonment of determinism nor by any local realistic reinterpretation of measurement results, as attempted in an article recently published in this journal.

1. Introduction

Despite its unprecedented practical success, quantum mechanics (QM) has, since its inception, led to deep disagreement concerning its conceptual basis and philosophical implications. No short article can appropriately deal with such a vast and complex subject. Our purpose here is to clarify a few key topics which, to judge from certain recent publications, continue to cause serious misunderstandings. An effort is made to keep the discussion accessible to a general audience of physicists and physics students unfamiliar with the specialized literature on the foundations of microphysics. The exposition is partly designed to contrast with certain unwarranted claims made in an article recently published in this journal (Tartaglia 1998). In section 2 we examine the chief interpretative stands that may be taken concerning the Heisenberg indeterminacy principle. Sections 3 and 4 deal with the Einstein–Podolsky–Rosen (EPR) argument and the Bell inequalities, respectively. References to some major works in this literature are provided.

2. The Heisenberg relations

A fundamental difference between quantum and classical mechanics arises, as is well known, from the way the *states* of physical objects are characterized. Whereas in classical mechanics the state of a particle is represented by a set of six numbers—the three components of its position and of its momentum—in quantum mechanics the pure states of an object are complex-valued functions—usually referred to as *wavefunctions*—or, more generally, vectors in a Hilbert space. In both classical and quantum mechanics the purpose of defining states is to allow the prediction of the physical properties belonging to the object. In the former theory, the specification of the state allows, in principle, the prediction of *all* the dynamical quantities of the object, such as its kinetic energy, angular momentum, etc. Quantum mechanical states, however, do *not* afford a complete value assignment to all the quantities which can legitimately be measured—and therefore, apparently—attributed to the object. This fact leads immediately to the suspicion that the theory is *incomplete* as a description of physical reality. The apparent incompleteness of QM is at the root of most, if not all, the intriguing features of this theory, and separated the founding fathers into two opposite camps: most of them *denied* that there is anything missing in the quantum mechanical theoretical description, whereas Einstein, Schrödinger and de Broglie insisted that the theory is, indeed, incomplete. The two most powerful arguments to sustain this view appeared in 1935: Einstein, Podolsky and Rosen's argument concerning pairs of correlated quantum objects (EPR 1935) and Schrödinger's argument concerning the measurement process, known as the 'cat paradox' (Schrödinger 1980).

Before examining briefly the former argument in the following section, let us notice that in order to maintain their case, the defenders of the completeness of QM had to resort to rather peculiar, non classical views concerning the relation of theory with experience and with reality. Given the undeniable fact that no quantum mechanical state affords the values of all the physical attributes of a quantum object, they claimed that the attributes lacking theoretical values are somewhat 'undefined', 'inapplicable', 'fuzzy', 'unknowable *in principle*' or something of the sort. If the concept of a physically describable reality is not altogether abandoned (a stand commonly taken by proponents of the so-called 'Copenhagen' interpretation), a justification for such claims can be sought along two main directions:

(1) reality is conceived as an (almost) literal counterpart of the wavefunction: quantum objects are wave-like; or

(2) reality is formed of more or less classical particles, but the act of observation introduces an unavoidable and uncontrollable disturbance in the states of the particles, so that some of their properties are always beyond our knowledge. The positivist doctrine (fashionable in the 1930s) is then called upon to discharge QM from the task of describing these properties. The theory can, thus, be considered complete, at least with respect to what can be known about reality.

Failure to distinguish clearly these two positions has often led to confusions in the historical debate concerning QM. In his classic 1927 article on the indeterminacy relations, for instance, Heisenberg first deduces the relations from the mathematical properties of (a particular form of) wavefunctions, and then tries to confirm them physically by the famous gamma-ray microscope thought experiment, in which the disturbance assumption plays a central role. We shall not give details here about that paper, or about the hot discussions it engendered even before publication, since the subject has been thoroughly examined by other authors (see, e.g., Jammer (1974), ch 3; Heisenberg's views are further developed in Heisenberg (1949)). What is important for us now is to make clear the physical content of the relations. Unfortunately, however, several interpretations are possible.

It can be shown in a rigorous way (which is not done in Heisenberg's paper) that the position–momentum indeterminacy relation follows from a classical analysis of wavepackets, together with the de Broglie formula relating wavelength and momentum, $p = h/\lambda$. (The energy–time relation follows from the application of the Planck–Einstein formula, $E = h\nu$, to wavepackets, but the derivation is more controversial.) If it is further assumed—according to position 1, above—that the quantum states, i.e. wavepackets, are (almost) literal counterparts of reality, it is evident that position and momentum are generally *undefined* for a quantum object. In this interpretation the Heisenberg relations are better called '*indeterminateness* relations'.

A different interpretation results from the adoption of position 2. In the microscope thought experiment, such as analysed by Heisenberg in *The Physical Principles of the Quantum Theory* (1949), an electron with '*precisely known*' velocity (p 20) and unknown position interacts with a photon. Some (imprecise) knowledge of the electron's position is thereby gained, but '*after* the experiment' (p 21), the electron is left with an unknown velocity, since it gets some unknown momentum from the photon. *Our* uncertainties about the final values of the electron's position and momentum are shown to obey the inequality $\delta x \delta p \geqslant h/4\pi$. The relation is, thus, *epistemic* (i.e. concerning *knowledge*), and its application to reality requires, as suggested by a previous comment, a positivistic *tour de force*: what is unknowable is not real, or at least is of no concern to science.

Both interpretative stands of the Heisenberg relations and, more generally, of the relationship between quantum states and reality, are open to methodological and philosophical criticism. Concerning position (1) it may be objected, for instance, that tailoring reality in literal conformity with wavefunctions is an unwarranted metaphysical assumption, which begs the very question of the completeness of the theory. Also, this position would require that objects be conceived as existing in more than three dimensions. In addition, this position renders acute the measurement problem. Position (2), on the other hand, can be accused of being philosophically incongruous, since it at once assumes and, in a sense, denies classically well defined properties to quantum objects. Furthermore, the disturbance doctrine has been subjected to strong criticism in the literature (see, e.g., Brown and Redhead (1981)). Also, it can be argued that physical theories or principles can be criticised, but not defended, by thought experiments (cf Popper (1968), appendix *xi). Finally, the microscope thought experiment is not directly relevant to questions relating to the simultaneous measurability of position and momentum. This last point deserves some amplification, since it leads to an altogether different approach to the Heisenberg relations, compatible with the incompleteness thesis.

Supporters of this thesis tend to favour the so-called *statistical interpretation* of QM (Ballentine 1970), which conceives the theory as formally analogous to classical statistical mechanics. The quantum states are not taken to describe individual objects, but statistical ensembles of objects. Lacking space to present the many arguments for and against this interpretation, we notice here only that it provides a natural background for understanding the Heisenberg relations. The quantities involved in them (Δx, Δp, etc) are interpreted as standard deviations in the quantum ensembles, the relations being thus simply *statistical scatter relations* (Popper 1968, ch 9). In contrast with the two previous approaches, this fact can be rigorously *proved* from the quantum formalism *without any extraneous assumptions*, philosophical or otherwise. (See Robertson (1929) for a pioneering proof; it was generalized by Schrödinger one year later; cf Jammer (1974), pp 72–73.)

Heisenberg's microscope and other thought experiments with similar purposes can also be easily accommodated by this third approach. Here it is necessary to make reference to the important distinction between *measurement* and *state preparation* (Margenau 1958). In the light of this distinction, what the experiments show is that the particular devices involved are not suitable for *preparing* dispersion-free ensembles of quantum objects, i.e. ensembles all members of which have the same sharply defined values of position and momentum (or other pairs of conjugate variables). With hindsight, we can see that Heisenberg himself has effectively acknowledged this point in the above-mentioned book, since he affirms (p 20) that his relation 'does not refer to the past' (measurement), but to the future (state preparation). Notice, indeed, that his microscope is *not* meant to *measure* the velocity of the electron. Like any ordinary microscope, it is a device for measuring position only. Thus, although we cannot use this apparatus to prepare an ensemble of electrons (or whatever) with the same, precisely definite positions and momenta, the experiment is of no direct concern to the issue of simultaneous *measurability* of these quantities, contrary to a still widespread belief. Furthermore, there are in the literature theoretical analyses purporting to show that the simultaneous measurability of conjugate variables does not necessarily conflict with the quantum formalism (Park and Margenau 1968, Prugovecki 1967; cf however Uffink 1994). In addition, some thought experiments have been suggested to illustrate this possibility (Robinson 1969, Ballentine 1970).

Summing up, at least three interpretations are possible for the Heisenberg relations. They express: (1) indeterminateness, (2) uncertainty, or (3) dispersion in statistical ensembles.

Failure to identify these rather distinct stands is responsible for much obscurity in the literature, past and present.

In Tartaglia (1998), for instance, the Heisenberg principle is first formulated in terms of quantities having ‘undefined’ values, indicating thus commitment to position (1). But later the author appears to resort to the disturbance doctrine, akin to position (2), since he talks of measurements ‘inducing’ measured objects into, or ‘leaving’ them in, such and such states. Whatever the case may be, he is firmly on the completeness side of the interpretative divide (up to now, at least; cf the following section). In this condition, it would be most unnatural for him to downplay—as he effectively does in the whole article—the discrepancies between the quantum and the classical worldviews.

Furthermore, Tartaglia’s metaphorical remark that the Heisenberg principle ‘simply states, in a refined and mathematical way, the fact that anyone may be standing or seated, but no-one may simultaneously be standing or seated’ (p 308) is clearly incorrect. To be standing and to be seated are different ‘values’ of the *same* ‘property’ of a human body (its ‘position’). No critic of QM has ever absurdly blamed the theory for failing to attribute simultaneously two different values to the same property of an object (e.g. two different values of momentum to an electron). Contrary to Tartaglia’s assertion on p 308, this theory *is* ‘mysterious, exotic or peculiar’, to the extent in which it fails to assign values to *pairs* (or, more generally, sets) of quantities which are perfectly compatible according to classical physics, and that can, furthermore, be ordinarily measured at any moment (at least one at a time). As we have already pointed out, it is this theoretical trait of QM that lies at the root of most of the perplexities involving its interpretation.

3. The EPR argument

In 1935, the issue of the completeness of QM was in the forefront of the conceptual discussions, and the completeness thesis was already prevalent. Although its proponents generally mixed up positions (1) and (2), as well as a variety of anti-realist stands, the disturbance assumption was central in the defence of completeness. The EPR argument was designed exactly to show the limitations of this assumption. It cleverly exploits the properties of certain correlated, quantum mechanically ‘entangled’ physical systems. In such systems, knowledge of certain magnitudes can in principle be obtained without any physical interaction of the object with the measuring agent, if a certain *locality* condition is satisfied. Roughly put, this is the plausible hypothesis—well backed by relativity theory—that all physical influences take finite time to propagate in space (there is no ‘action-at-a-distance’).

The trouble is, EPR effectively argue, that among the quantities whose values can be predicted with certainty by way of the EPR correlations there are quantities to which QM does *not* attribute definite values whatsoever. The theory seems, therefore, to be incomplete. (It must be stressed that the EPR correlations follow from the very formalism of QM and, as we know today, are verified experimentally.)

Historical research has revealed that the EPR paper was written down by Podolsky alone, and that Einstein was very unsatisfied with the way it came out. Einstein’s own incompleteness argument, which he expressed in private correspondence and in some of his published writings, has different premises and structure, thereby avoiding several fatal obscurities of the EPR text (see Fine 1986, Howard 1985, Deltete and Guy 1991). Furthermore, it can be shown that, following certain hints made by Einstein, yet another variant of the argument can be formulated, which is much simpler and, therefore, stronger than the former two.

Before presenting an informal version of this simple variant, we would like to remark that Tartaglia’s rendering of the EPR argument at the opening of section 3 of his 1998 is *very far* from corresponding to any of these three versions. Also, the explicit conclusion of the EPR-type arguments is *not* that the uncertainty principle is violated, as Tartaglia implies. Nowadays historians of science generally agree with Pais (1982) that by 1935 Einstein had already abandoned his former hope of refuting that principle (see, e.g., Paty 1995). Quite independently of this historical point, inspection of the EPR article does not indicate that that was the authors' goal, especially if the Heisenberg relations are taken as dispersion relations following directly from the quantum formalism. In this interpretation they are fully compatible with the incompleteness thesis, as indicated in the previous section.

Tartaglia's analysis of EPR suffers from other serious defects. He claims that the EPR paradox 'is not a real paradox', and 'has no particular subtlety in it' (pp 307, 309). This is because quantum objects could be understood as carrying 'instruction leaflets' telling them what to do 'when encountering a measuring apparatus or in general undergoing an interaction with something else' (p 308). Let us investigate this point more closely.

The kernel of the EPR-type arguments lies in the existence of certain quantum correlations between spatially separated, apparently non-interacting objects. Such correlations can easily be explained if it is assumed that the objects possess *classical* properties subjected to conservation laws, as the following comparison illustrates.

Suppose that two men, A and B, lying in separate, incommunicable rooms, perform the task of opening a large number of closed boxes, numbered severally, containing just one glove each, and taking note of the number of each box together with the 'value' of its glove, left- (L) or right- handed (R). Suppose that the tables of results are now presented to a third, independent person, who notices at once that whenever glove *n* in the table of A is L, glove *n* in B's table is R, or *vice versa*. The two tables are strictly correlated. If the person is asked to explain this phenomenon, she has conceivably only two options (if coincidence is ruled out): (i) the boxes contained 'classical' gloves which are intrinsically R or L all the time, independently of being observed, and these properties became correlated in the past by some process involving the two gloves[1]; or (ii) the gloves are strange objects lacking any R or L property until they are observed by the men; the act of observation somewhat 'creates' the properties at random. But by some non-local mechanism whenever a man observes his glove, the corresponding glove in the other room also acquire a definite R or L property, with the opposite 'value'. Now, it is obvious that our ordinary knowledge about gloves rules out explanation (ii)[2].

In the situation envisaged by EPR, formally *identical* correlation tables are obtained when, for instance, position or momentum properties of each member of the pair of quantum objects are measured. The analogy is perfect in Bohm's version of the argument, involving spin components of pairs of spin-$\frac{1}{2}$ particles. We are, therefore, strongly tempted to adopt a type-(i) explanation for these cases too: the objects of the pairs have well defined positions, momenta, spin components, etc, and these properties become correlated when the pairs are created.

However, it is rash to assume without further ado that quantum objects are like gloves. After all we have independent evidence that they are weird creatures. Maybe they display some properties that are not intrinsic to them, and arise only when they come under the eyes, or consciousness of an 'observer', as some defenders of the completeness thesis have proposed. In the EPR experiment, however, this 'creative' role of observation would have to extend to distant and, for all we know, physically isolated objects! In other words, Einstein effectively realized that type-(ii) explanations for the EPR phenomenon are ruled out by the well grounded principle of locality. We are, thus, left with type-(i) explanations: the objects of the pairs have definite, correlated properties all the time, independently of measurement. But QM does *not* prescribe values to the positions, momenta, spin components, etc of the objects in the EPR states, being therefore incomplete.

This version of the argument does not require that the controversial issue of the simultaneous definability or measurability of conjugate magnitudes be considered at all. It also makes clear that the only substantial premise of the argument open to discussion is locality.

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[1] For instance, at the end of the production line in an ordinary glove factory the gloves of each pair are separated at random and put into two boxes labelled with the same number, the boxes being, then, sent to A and B.

[2] This knowledge also rules out a metaphysical variant of explanation (i) which must, however, be mentioned, in view of our later analysis of Tartaglia's position. The world could in principle be such that although the gloves lack actual L or R properties until observed, the results obtained upon observation are not random (as in (ii)), but are deterministically encoded in their states or, to use Tartaglia's notion, in certain 'instruction leaflets'.

To deny its conclusion, i.e. to assert that QM is complete, requires the violation of this most plausible condition. The argument is indeed paradoxical, in the original sense of the word: something that is not expected, contrary to common belief.

Tartaglia claims that his ‘resolution’ of the EPR ‘paradox’ avoids the violation of locality. This is possible. He proposes that no action-at-a-distance is involved because the objects respond locally to the instructions contained in their ‘leaflets’. This is possible too. But contrary to what Tartaglia claims, this effectively amounts to reverting to an ordinary, type-(i) explanation for the correlations. Notice, indeed, that his distinction between the objects’ properties having values (as in type-(i) explanations) and the objects being described by ‘leaflets’—i.e. states—telling *precisely* what values will be obtained for these properties upon measurement is a metaphysical distinction with no experimental consequences. Physically, Tartaglia’s ‘instruction leaflets’ must, therefore, play exactly the role of classical, objective, complete states, and there is no point in distinguishing such ‘leaflets’ from the ordinary assignment of states and properties to the objects. Contradicting thus the stand on the completeness issue he took when he presented the Heisenberg principle, Tartaglia now effectively sides with Einstein and others who defend that QM is incomplete.

4. The Bell inequalities

The idea that it is necessary somehow to complete QM is strongly suggested by the EPR argument. It was proposed that the quantum mechanical state description should be supplemented by certain parameters, which misleadingly became known as ‘hidden variables’. However, the programme of developing hidden-variable theories (HVTs) did not take off as soon as it could be expected, due to factors that will not be examined here. The first major advance was made by David Bohm, who in 1952 proposed a HVT capable of reproducing all the empirical predictions of QM (Bohm 1952a, b).

Among other peculiar traits, Bohm’s theory involved explicitly *non-local* mechanisms: some physical objects—including the entangled EPR systems—would be connected by interactions violating Einstein’s locality principle. Intrigued by this curious aspect, J S Bell set out to discover whether non-locality is an inevitable trait of *any* theory purporting to complete QM while preserving its empirical predictions. In a brilliant work (Bell 1964), he proved that the answer is positive—a historical irony, since locality is the basic premise of the main argument indicating that QM should be completed!

Bell’s result has the form of an inequality, setting an upper bound to an expression measuring the ‘degree’ of correlation in EPR-type systems for which the correlations are not absolute. This limit is obeyed by any local HVT and violated by QM. To be more precise, the original proof concerned exclusively *deterministic* HVTs, i.e. theories assigning precise values to all dynamical quantities of the objects. The result was later generalized by Bell and others (Bell 1971, Clauser and Horne 1974) to cover *stochastic* theories, which afford only probabilities for measurement results. The issue of checking experimentally these inequalities became, thus, of paramount importance. A series of experiments undertaken in subsequent years has strikingly shown that they are violated. Local HVTs are, therefore, ruled out both by QM and by experience. (The most important test is reported in Aspect *et al* (1982).)

Tartaglia’s account of this issue is defective. He explicitly declares it to be possible to satisfy at once the three premises he identifies in the inequality: realism, determinism and locality (p 310). But this can only be done by holding that the experimental results are wrong or that Bell’s proof is logically unsound. Tartaglia attempts a third way: Bell’s reasoning would in fact involve a fourth, ‘hidden’ hypothesis, ‘the assumption that the property being measured is something possessed as such by the quantum object and being there at any moment’ (p 310). It can, however, be shown that this way out is untenable.

First, the assumption is *explicitly avoided* in the derivation of the generalized inequalities, constraining stochastic local HVTs. All that such proofs assume is that, given the complete state specification, the experimental results in one branch of the apparatus are independent from the choice made by the experimenter in the other branch as to which quantity will be measured, and from the results he obtains (see Jarrett (1984)). Unfortunately, Tartaglia does not even mention this important extension of Bell's original result.

Secondly, even if we restrict ourselves to the deterministic case, Tartaglia's trick does not work. The quantities entering into the derivation of the deterministic inequalities (such as the quantities in Tartaglia's equation (1)) can be taken either as actually possessed, local values or as *measurement results*, provided they are predictable by local states. Now what Tartaglia's 'leaflets' do is, according to him, exactly to 'tell', *locally and deterministically*, the objects 'what to do when encountering a measuring apparatus'[3] But this is just what the local states do in the ordinary derivation of the inequality. Tartaglia's claim that his 'leaflet' approach renders the Bell inequality 'deprived of meaning', because in this approach the physical magnitudes are not assumed to possess actual values, is thus unwarranted.

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[3] The 'chameleon' metaphor Tartaglia uses to explain the meaning of his strange 'leaflets' reinforces this interpretation. Notice, incidentally, that the metaphor is *completely inappropriate* to explain the EPR and the Bell phenomena, since each chameleon responds locally to its 'measuring device', but the chameleons in the pairs are definitely *not* correlated, neither in their 'leaflets' nor in their final response to the environment.