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Excitons and shallow impurities in GaAs-Ga$_{1-x}$Al$_x$As semiconductor heterostructures within a fractional-dimensional space approach: Magnetic-field effects

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The fractional-dimensional space approach is extended to study exciton and shallow-donor states in symmetric-coupled GaAs-Ga$_{1-x}$Al$_x$As multiple quantum wells. In this scheme, the real anisotropic ‘‘exciton (or shallow donor) plus multiple quantum well’’ semiconductor system is mapped, for each exciton (or donor) state, into an effective fractional-dimensional isotropic environment, and the fractional dimension is essentially related to the anisotropy of the actual semiconductor system. Moreover, the fractional-dimensional space approach was extended to include magnetic-field effects in the study of shallow-impurity states in GaAs-Ga$_{1-x}$Al$_x$As quantum wells and superlattices. In our study, the magnetic field was applied along the growth direction of the semiconductor heterostructure, and introduces an additional degree of confinement and anisotropy besides the one imposed by the heterostructure barrier potential. The fractional dimension is then related to the anisotropy introduced both by the heterostructure barrier potential and magnetic field. Calculations within the fractional-dimensional space scheme were performed for the binding energies of 1s-like heavy-hole direct exciton and shallow-donor states in symmetric-coupled semiconductor quantum wells, and for shallow-impurity states in semiconductor quantum wells and superlattices under growth-direction applied magnetic fields. Fractional-dimensional theoretical results are shown to be in good agreement with previous variational theoretical calculations and available experimental measurements.

I. INTRODUCTION

Low-dimensional semiconductor systems have been of great interest for the past two decades, not only because of the physics underlying various properties of these systems but also due to their importance for potential applications in electronic and optoelectronic devices. For a number of reasons, most work on semiconductor systems has been carried out on III-V semiconductor heterostructures, and GaAs-Ga$_{1-x}$Al$_x$As heterostructures, such as quantum wells (QW’s), symmetric and asymmetric double QW’s, multiple quantum wells (MQW’s), quantum-well wires (QWW’s), quantum dots (QD’s), periodic and quasiperiodic superlattices (SL’s), and so on, have been widely studied. The confinement of carriers in GaAs-Ga$_{1-x}$Al$_x$As MQWs and SLs leads to the formation of subbands with a dispersion that would depend on the strength of the interwell coupling. In particular, impurity and exciton states may be significantly modified by the confinement, and much experimental[1–5] and theoretical[6–13] work have been devoted to the quantitative understanding of the physical properties of shallow impurities and excitons in GaAs-Ga$_{1-x}$Al$_x$As QW’s, MQW’s and SL’s. An external perturbation of a system, such as the application of electric and/or magnetic fields, is a powerful tool for investigating matter properties, and it allows many studies in semiconductor systems. In particular, the application of a magnetic field perpendicular to the semiconductor layers is expected to provide useful band structure data and several magneto-optical studies have been performed that yield experimental[14–22] and theoretical[23–33] valuable information on shallow impurity and exciton states under magnetic fields.

Experimentally, of particular interest to this work, is the investigation by Westgaard *et al.*[4] of exciton states in GaAs-Ga$_{0.7}$Al$_{0.3}$As symmetric coupled double quantum wells (SCDQW’s) by photoluminescence (PL) and photoluminescence excitation (PLE) spectroscopy, the study by Reynolds *et al.*,[16] who measured the exciton diamagnetic shifts in bulk GaAs and in GaAs-Ga$_{1-x}$Al$_x$As MQW’s as functions of applied magnetic fields using high-resolution optical spectroscopy at liquid-helium temperature, and the work by Skromme *et al.*,[15] who studied the cyclotron motion of electrons in coupled-well GaAs-Ga$_{1-x}$Al$_x$As SL’s by PL of conduction-band to acceptor transitions in magnetic fields up to 13 T, applied either parallel or perpendicular to the layers. From the theoretical point of view, the problem of hydrogeniclike systems, in free space or in bulk semiconductors, under a constant external magnetic field was extensively studied by Aldrich and Greene,[23] who used a variational scheme and a cylindrical Gaussian basis. The binding energies of the $1s$-like ground state and of $2p_{\pm}$ excited states of a hydrogenic donor associated with the lowest electron subband and confined in a GaAs-Ga$_{1-x}$Al$_x$As QW were calculated, using a variational procedure, by Chaudhuri and Bajaj[24] and Greene and Bajaj[25] as functions of the GaAs well width and applied magnetic field along the axis of growth of the heterostructure. The magnetic-field dependence of hydrogenic energy levels in GaAs-Ga$_{1-x}$Al$_x$As QW structures was also studied in Ref. 26, whose authors used strong-perturbation theory and a perturbative-variational approach.

The fractional-dimensional space approach[34–42] was successfully used to describe the absorption spectra, and exciton and donor properties in semiconductor QW's, QWW's and SL's, and biexcitons, exciton-phonon interaction, the Stark shift of excitonic complexes, and the refractive index in QW structures. In this approach, the anisotropic problem in a three-dimensional environment is treated as isotropic in an effective fractional-dimensional space, and the value of the fractional dimension D is associated with the degree of anisotropy of the actual three-dimensional system. Recently, we used the fractional-dimensional space scheme to study shallow-impurity and exciton states[40–42] in GaAs-Ga$_{1-x}$Al$_x$As QW's and SL's and proposed a systematic procedure to determine the appropriate fractional dimension of the isotropic space which would model the actual system. As in semiconductor MQW's, variational procedures are very demanding on computer time in comparison with single QW structures, we were motivated to extend our previous fractional-dimensional approach[40–42] to the case of excitons and shallow impurities in doped semiconductor MQW's. Of course, a magnetic field applied to a semiconductor heterostructure introduces an additional degree of confinement and anisotropy besides the one imposed by the heterostructure barrier potential, and interesting changes in impurity- and excitonic-related far-infrared and electronic properties may be achieved. Therefore, the fractional dimension should also depend on the degree of anisotropy introduced by the applied magnetic field, and an extension of the fractional-dimensional space approach to include the effects of applied magnetic field would certainly be of great interest.

This work is concerned with an extension of the fractional-dimensional scheme to the study of direct exciton states and shallow-donor states in GaAs-Ga$_{1-x}$Al$_x$As MQW's. Moreover, the fractional-dimensional approach is extended to include magnetic-field effects in the study of shallow-impurity states in GaAs-Ga$_{1-x}$Al$_x$As semiconductor heterostructures such as QW's and SL's. The study is organized as follows. In Sec. II the theoretical basis of the fractional-dimensional scheme, developed by de Dios-Leyva and co-workers[40–42] for exciton and shallow-donor states in QW's and SL's, is extended to the case of excitons and shallow donors in GaAs-Ga$_{1-x}$Al$_x$As MQS's, and to include the effects of applied magnetic fields perpendicular to the layers in the study of shallow-impurity properties of GaAs-Ga$_{1-x}$Al$_x$As semiconductor heterostructures. Results and discussion are in Sec. III and conclusions in Sec. IV.

II. FRACTIONAL-DIMENSIONAL SPACE APPROACH: MAGNETIC FIELD EFFECTS

We first consider the problem of a shallow donor at the position $\mathbf{r}_i$ in a semiconductor GaAs-Ga$_{1-x}$Al$_x$As heterostructure with growth axis along the z direction (such as a QW, MQW, or a SL), within the effective-mass and nondegenerate parabolic band approximations, under an applied magnetic field. The Hamiltonian for the shallow impurity may be given by

$$H=\frac{1}{2m^*}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2-\frac{e^2}{\varepsilon|\mathbf{r}-\mathbf{r}_i|}+V(z), \tag{2.1}$$

where m^* is the conduction-band effective mass, $V(z)$ is the heterostructure confining potential, and ε is the dielectric constant of the semiconductor GaAs-Ga$_{1-x}$Al$_x$As heterostructure (for simplicity, m^* and ε are taken as the GaAs bulk values throughout the heterostructure). We choose the magnetic field along the z direction and the symmetric gauge for the vector potential, so that $\mathbf{A}=\mathbf{B}x\mathbf{r}/2$, and Hamiltonian (2.1) may be written as

$$H=-\frac{\hbar^2}{2m^*}\nabla^2+\Omega\hat{L}_z+\frac{1}{2}m^*\Omega^2(x^2+y^2)-\frac{e^2}{\varepsilon|\mathbf{r}-\mathbf{r}_i|}+V(z), \tag{2.2}$$

with

$$\Omega=\frac{eB}{2m^*c}=\frac{1}{2}\omega_c \tag{2.3}$$

and $\hat{L}_z$ is the z component of the angular momentum operator. Due to the cylindrical symmetry, we may write as the donor-electron wave function,

$$\Psi(\mathbf{r})=\frac{e^{\mathrm{im}\varphi}}{\sqrt{2\pi}}\psi(\rho,z), \tag{2.4}$$

with the magnetic quantum number $m=0,\pm1,\pm2,\ldots,$ $\rho=(x^2+y^2)^{1/2}$, and using Hamiltonian (2.2), we obtain

$$\begin{aligned}-\frac{\hbar^2}{2m^*}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)-\frac{m^2}{\rho^2}\right]\psi(\rho,z)&\\+\left(\frac{1}{2}m^*\Omega^2\rho^2-\frac{e^2}{\varepsilon|\mathbf{r}-\mathbf{r}_i|}\right)\psi(\rho,z)&\\+\left[-\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial z^2}+V(z)\right]\psi(\rho,z)&\\=\left(E-\frac{m}{2}\hbar\omega_c\right)\psi(\rho,z).&\end{aligned} \tag{2.5}$$

In the absence of the Coulomb potential, the above equation may be solved, and the solution is

$$\psi_0(\rho,z)=\chi(\rho)f(z), \tag{2.6}$$

with $f(z)$ being the z part of the $\mathbf{k}_\perp=0$ electron envelope wave function for the GaAs-Ga$_{1-x}$Al$_x$As semiconductor heterostructure in the absence of magnetic field and Coulomb potential, and

$$\chi_{n_r,m}(\rho)\sim\xi^{|m|}e^{-(1/2)\xi^2}G_{n_r,m}(\xi), \tag{2.7}$$

with $\xi=\rho/\lambda_B$, and $\lambda_B=\sqrt{2}l_B=\sqrt{2}\sqrt{\hbar c/eB}$ (l_B is the cyclotron radius). In the above, G may be found in terms of the generalized Laguerre polynomials,[43] i.e., $G_{n_r,m}(\xi)\sim L_{n_r}^{|m|}(\xi^2)$, where

$$E_{n_r,m}=\hbar\omega_c\left(n_r+\frac{m+|m|+1}{2}\right)$$

is the energy of the Landau level, and $n_r=0,1,2,\ldots$ is the radial quantum number. The total electron energy E_0 [cf Eq. (2.5)] in the absence of the Coulomb potential is then given by $E_0=E_{n_r,m}+\varepsilon_e$, where ε_e is the confining energy for electrons in the $V(z)$ heterostructure potential [corresponding to the z part of Eq. (2.5), i.e., calculated in the absence of magnetic field and Coulomb potential]. Therefore, for each heterostructure ε_e level, one has an infinite set of Landau levels, and, for a given applied magnetic field, the ground-state level (without the effect of the Coulomb potential) is $\varepsilon_{e,0}+\frac{1}{2}\hbar\omega_c$, with $\varepsilon_{e,0}$ being the heterostructure electron ground state [associated with the wave function $f_0(z)$].

If one includes the effect of the impurity Coulomb potential, and is only concerned with the $1s$-like donor ground state, the eigenfunction corresponding to Eq. (2.2) may be taken as $\psi_E(\mathbf{r})=f_0(z)\chi_{0,0}(\rho)\phi_{E_{1s}}(\mathbf{r})$, where $\chi_{0,0}(\rho)=\exp(-\rho^2/4l_B^2)$ is the $n_r=0$, $m=0$ electron wave function corresponding to the lowest Landau level [cf. Eq. (2.7)], and $\phi_{E_{1s}}(\mathbf{r})$ satisfies (see Appendix A).

$$\begin{aligned}
&-\frac{\hbar^2}{2m^*}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\right]\phi_{E_{1s}}(\mathbf{r})\\
&\quad-\frac{\hbar^2}{2m^*}\frac{1}{h}\left[\frac{\partial h}{\partial r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial h}{\partial\theta}\frac{\partial}{\partial\theta}\right]\phi_{E_{1s}}(\mathbf{r})\\
&\quad-\frac{e^2}{\varepsilon r}\phi_{E_{1s}}(\mathbf{r})=E_{1s}\phi_{E_{1s}}(\mathbf{r}),
\end{aligned} \tag{2.8}$$

with $r=\sqrt{x^2+y^2+(z-z_i)^2}$. The total electron energy is now

$$E=\varepsilon_{e,0}+\tfrac{1}{2}\hbar\omega_c+E_{1s}, \tag{2.9}$$

the impurity binding energy is $-E_{1s}$, spherical coordinates are taken with the origin at the impurity position, and the weight function[40–42] h is given by

$$h=h(r,\theta)=h_0(r\cos\theta+z_i)\exp\left(-\frac{r^2\sin^2\theta}{\lambda_B^2}\right), \tag{2.10}$$

$$h_0(z)=f_0^2(z). \tag{2.11}$$

Moreover, one may write Eq. (2.8) as

$$[H_D+W]\phi_{E_{1s}}(\mathbf{r})=E_{1s}\phi_{E_{1s}}(\mathbf{r}), \tag{2.12}$$

where

$$H_D=-\frac{\hbar^2}{2m^*}\nabla_D^2-\frac{e^2}{\varepsilon r} \tag{2.13}$$

is the D fractional-dimensional space Hamiltonian,[34] and

$$\begin{aligned}
W=&-\frac{\hbar^2}{2m^*}\left(\frac{\beta}{r}+\frac{1}{h_0}\frac{\partial h_0}{\partial r}-\frac{2r\sin^2\theta}{\lambda_B^2}\right)\frac{\partial}{\partial r}-\frac{\hbar^2}{2m^*}\frac{1}{r^2}\\
&\times\left(\beta\cot\theta+\frac{1}{h_0}\frac{\partial h_0}{\partial\theta}-r^2\frac{\sin 2\theta}{\lambda_B^2}\right)\frac{\partial}{\partial\theta},
\end{aligned} \tag{2.14}$$

where $\beta=3-D$, and $h_0=h_0(r\cos\theta+z_i)$. Notice that Eq. (2.14) reduces to Eq. (2.6) of de Dios-Leyva *et al.*[40] in the zero-magnetic field limit. Alternatively, one may write W as

$$W=-\frac{\hbar^2}{2m^*}\left[\left(\frac{\beta}{r}+\frac{1}{h}\frac{\partial h}{\partial r}\right)\frac{\partial}{\partial r}+\frac{1}{r^2}\left(\beta\cot\theta+\frac{1}{h}\frac{\partial h}{\partial\theta}\right)\frac{\partial}{\partial\theta}\right], \tag{2.15}$$

with h given by Eq. (2.10). We now stress that Eq. (2.15) has the same form as Eq. (2.6) of Ref. 40, except that h is defined by Eq. (2.10), and includes the effects of the applied magnetic field. It immediately follows from Eqs. (2.12)–(2.15) that the system ''shallow donor plus heterostructure plus magnetic field'' may be modeled by an effective isotropic hydrogenic system in a fractional D-dimensional space, a problem which may be solved analytically.[34] It is then straightforward to extend the framework developed by de Dios-Leyva *et al.*[40] in the case of shallow donors in semiconductor GaAs-Ga$_{1-x}$Al$_x$As QW's and SL's in order to consider effects of the applied magnetic field within the fractional-dimensional space approach. One then finds that, for a given state of the ''shallow donor plus heterostructure plus magnetic field'' anisotropic system, one may choose the D parameter in order to map the actual system into an effective isotropic D-dimensional space via the condition[40–42]

$$\int\int hr^2\sin\theta\,\phi_E^*W\phi_j\,dr\,d\theta=0, \tag{2.16}$$

where the operator W in Eqs. (2.14) and (2.15) includes the effect of the applied magnetic field. In the above equation, $\phi_E(\mathbf{r})$ is the corresponding impurity eigenfunction, and ϕ_j and E_j are the eigenfunctions and eigenvalues of the D-dimensional Hamiltonian. We would like to stress that the above approach for determining the fractional dimensional parameter D is valid for a general GaAs-Ga$_{1-x}$Al$_x$As semiconductor heterostructure grown along the z axis, such as QW's, double symmetric or asymmetric QW's, multiple QW's, periodic and quasiperiodic (Fibonacci, etc.) SL's, and so on. Notice that Eq. (2.16) depends on $f_0(z)$ [see Eqs. (2.10), (2.11), and (A4)], which would depend on the $V(z)$ heterostructure confining potential (for explicit expressions of $f_0(z)$ in the QW, SCDQW, MQW, and SL cases, for example, see Oliveira,[44] Thoar,[8] Chaudhuri,[7] and Helm *et al.*,[3] respectively).

As we are interested in evaluating the magnetic-field effects on the donor binding energy, which is associated to the $1s$-like ground-state energy E_{1s}, one may approximately choose $\phi_E=\phi_{j=0}$ in Eq. (2.16), with $\phi_{j=0}$ being the $1s$ ground-state exact solution of the D-dimensional Hamiltonian, i.e., $\phi_{j=0}=\phi_{1s}(r)=e^{-\lambda r}$, with $\lambda=2/[a_0^*(D-1)]$, where a_0^* is the donor Bohr radius. For the case of a semiconductor GaAs-Ga$_{1-x}$Al$_x$As QW or SL, one then obtains the following transcendental equation to be solved for the fractional-dimensional parameter D [see Eq. (2.16) and Appendix B]:

$$\left(\beta-2-\frac{1}{2}\alpha^2\lambda_B^2\right)\int_0^\infty F(r,z_i)e^{-ar}dr+\frac{1}{2}\alpha\lambda_B^2\int_0^\infty[f_0^2(r+z_i)+f_0^2(r-z_i)]e^{-ar}dr=0, \tag{2.17}$$

where $\alpha=2\lambda$, $\beta=3-D$, and

$$F(r,z_i)=e^{-r^2/\lambda_B^2}\int_0^r[f_0^2(z+z_i)+f_0^2(z-z_i)]e^{z^2/\lambda_B^2}dz. \tag{2.18}$$

Notice that Eq. (2.17) depends on D and $f_0(z)$ [see Eq. (A4)]. Once the fractional-dimensional parameter D is calculated as a function of the applied magnetic field, the donor binding energies may then be obtained in a straightforward way through $E_B=-E_{1s}=4/(D-1)^2R^*$, where R^* is the donor effective Rydberg. In the particular case of bulk GaAs, f_0 is a constant, the energy $\varepsilon_{e,0}=0$ (the origin of energy at the bottom of the conduction band), and Eq. (2.17) reduces to (see Appendix B)

$$\int_0^\infty\frac{e^{-x}}{4x+k^2}dx=\frac{1}{4-2\beta+k^2}, \tag{2.19}$$

with

$$k=\frac{4\sqrt{2}}{D-1}\frac{l_B}{a_0^*}.$$

Of course, Eq. (2.19) is valid in the case of Wannier excitons in bulk GaAs, with m^* substituted by the exciton reduced mass. Also, as shown in Appendix B [see Eq. (B11)], in the *zero-magnetic-field limit*, Eq. (2.16) leads to[42]

$$(\beta-1)L(\alpha,z_i)-\alpha\frac{\partial}{\partial\alpha}L(\alpha,z_i)=0, \tag{2.20}$$

with

$$L(\alpha,z_i)=\int_0^{+\infty}dr\,\exp(-\alpha r)[h_0(r+z_i)+h_0(r-z_i)], \tag{2.21}$$

which corresponds to Eq. (2.2) of Ref. 42.

Moreover, in the zero-magnetic-field case, it is quite straightforward to extend Eq. (2.20) for donors in GaAs-Ga$_{1-x}$Al$_x$As MQW's in order to include effects due to different masses in the GaAs well and Ga$_{1-x}$Al$_x$As barrier. In what follows, we consider conduction-band effective masses,[4,45] in units of the m_0 free-electron mass, as $m_w=0.0665$ and $m_b=0.0665+0.0835x$, in which w and b are labels for well and barrier, respectively. Following a similar procedure as outlined above and in Appendix A, the operator W may then be written as

$$\begin{aligned}W=&-\frac{\hbar^2}{2m_w}\left[\left(\frac{\beta}{r}\frac{u_0}{h_0}+\frac{1}{h_0}\frac{\partial u_0}{\partial r}\right)\frac{\partial}{\partial r}\right.\\&\left.+\frac{1}{r^2}\left(\beta\frac{u_0}{h_0}\cot\theta+\frac{1}{h_0}\frac{\partial u_0}{\partial\theta}\right)\frac{\partial}{\partial\theta}\right]\\&+\frac{\hbar^2}{2m_w}\left(1-\frac{u_0}{h_0}\right)\left[\frac{1}{r^{D-1}}\frac{\partial}{\partial r}\left(r^{D-1}\frac{\partial}{\partial r}\right)\right.\\&\left.+\frac{1}{r^2\sin^{D-2}(\theta)}\frac{\partial}{\partial\theta}\left(\sin^{D-2}(\theta)\frac{\partial}{\partial\theta}\right)\right],\end{aligned} \tag{2.22}$$

which corresponds to Eq. (2.8) in Ref. 41 for excitons (with the exciton reduced mass μ_w substituted by the donor effective mass m_w in the GaAs well). In Eq. (2.22), $h_0=h_0(r\cos\theta+z_i)$ and $u_0=u_0(r\cos\theta+z_i)$, and are given by $h_0(z)=f_0^2(z)$ and $u_0(z)=m_wh_0(z)/m(z)$ [where $f_0(z)$ is the z part of the electron ground-state envelope wave function—see Eq. (A4)—for the MQW], with $m(z)$ given by

$$\begin{aligned}m^{-1}(z)=&\frac{1}{m_w}+\left(\frac{1}{m_b}-\frac{1}{m_w}\right)(\Theta[L_b/2-|z|]\\&+\Theta[|z|-(L_w+L_b/2)]),\end{aligned} \tag{2.23}$$

for a SCDQW, with the origin $z=0$ taken at the center of the central barrier, and

$$\begin{aligned}m^{-1}(z)=&\frac{1}{m_w}+\left(\frac{1}{m_b}-\frac{1}{m_w}\right)(\Theta[|z-(L_w+L_b/2)|-L_b/2]\\&+\Theta[(3L_w/2+L_b-|z|)]\\&+\Theta[|z+(L_w+L_b/2)|-L_b/2])\end{aligned} \tag{2.24}$$

for a symmetric coupled triple QW (SCTQW), with the origin $z=0$ taken at the center of the central well. In the above, L_b and L_w are the widths of the barrier and well regions of the MQW, respectively, and Θ is the Heaviside function. One then obtains [Eq. (2.16), with $h=h_0$] the following transcendental equation to be solved for the fractional-dimensional parameter D,

$$(2\beta-3)L(\alpha,z_i)+G(\alpha,z_i)-\alpha\frac{\partial[L(\alpha,z_i)+G(\alpha,z_i)]}{\partial\alpha}=0, \tag{2.25}$$

where

$$L(\alpha,z_i)=\int_0^\infty e^{-az}[h_0(z+z_i)+h_0(z-z_i)]dz \tag{2.26}$$

and

$$G(\alpha,z_i)=\int_0^\infty e^{-az}[u_0(z+z_i)+u_0(z-z_i)]dz. \tag{2.27}$$

Notice that if the effective mass is taken as constant and equal to the GaAs value throughout the heterostructure, then $m(z)=m_w$, $u_0=h_0$, and $G=L$, with the result that one recovers Eq. (2.20), as expected. Once Eq. (2.25) is solved for the fractional dimension, the $1s$-like MQW shallow-donor binding energies may then be obtained through $E_B=4/(D-1)^2R^*$, where R^* is the shallow donor effective Rydberg.

In the zero-magnetic-field limit, one may also deal quite straightforwardly with the problem of a direct heavy-hole exciton in a semiconductor GaAs-$\mathrm{Ga}_{1-x}\mathrm{Al}_x\mathrm{As}$ SCDQW (growth axis along the z direction). We work within the effective-mass and nondegenerate parabolic band approximations, and for simplicity we assume the relative motion of the carriers and that of the center of mass are independent, although one may only make this separation in the plane of the well.[46,47] We take the effective masses, in units of the free-electron mass, as[4,45] $m_{ew}=0.0665$, $m_{eb}=0.0665+0.0835x$, $m_{hw}=0.34$, and $m_{hb}=0.34+0.42x$, in which w and b are labels for well and barrier, respectively, and e and h denote an electron and a heavy hole, respectively. One may then show (this is a straightforward extension of the formalism of Ref. 41) that the fractional-dimensional parameter of the effective ''exciton plus semiconductor SCDQW'' is obtained by

$$(2\beta-3)L(\alpha)+G(\alpha)-\alpha\frac{d[L(\alpha)+G(\alpha)]}{d\alpha}=0, \tag{2.28}$$

with

$$L(\alpha)=\int_0^{\infty}e^{-az}h_0(z)dz \tag{2.29}$$

and

$$G(\alpha)=\int_0^{\infty}e^{-az}u_0(z)dz, \tag{2.30}$$

which is very similar to Eq. (2.25) for shallow donors, except that the functions h_0 and u_0 depend now on f_e^0 and f_h^0 (the z part of the electron and hole ground-state envelope wave functions for the SCDQW), and are given by[41]

$$h_0(z)=\int_{-\infty}^{+\infty}[f_e^0(z)]^2[f_h^0(\xi-z)]^2d\xi, \tag{2.31}$$

$$u_0(z)=\mu_w\int_{-\infty}^{+\infty}\frac{[f_e^0(z)]^2[f_h^0(\xi-z)]^2}{\mu(\xi,z)}d\xi, \tag{2.32}$$

and

$$\begin{aligned}\mu^{-1}(\xi,z)=&\frac{1}{\mu_w}+\left(\frac{1}{m_{eb}}-\frac{1}{m_{ew}}\right)\{\Theta[L_b/2-|\xi|]\\&+\Theta[|\xi|-(L_w+L_b/2)]\}+\left(\frac{1}{m_{hb}}-\frac{1}{m_{hw}}\right)\\&\times\{\Theta[L_b/2-|\xi-z|]+\Theta[|\xi-z|\\&-(L_w+L_b/2)]\},\end{aligned} \tag{2.33}$$

where μ_w is the reduced mass of the heavy-hole exciton in the GaAs QW. Notice that Eq. (2.33) reduces to Eq. (2.5) of Ref. 41, if $L_b=0$, i.e., if one has a QW instead of a SCDQW. Finally, we would like to stress that, in the above fractional-dimensional space approach, the fractional dimension is chosen via an analytical procedure [cf. Eq. (2.16)], and involves no ansatz, and no fitting with experiment or previous variational calculations.

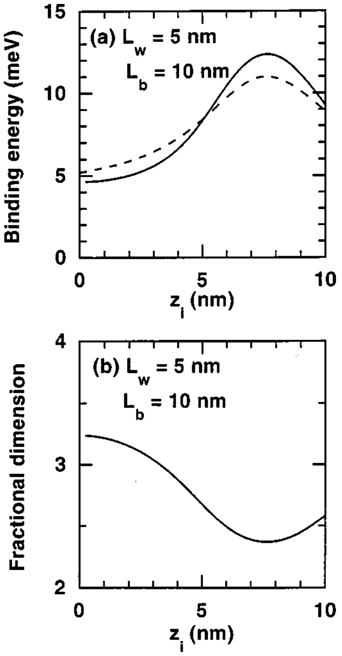

FIG. 1. Shallow-donor $1s$-like binding energies (a) and corresponding fractional dimensions (b) as functions of the z_i impurity positions in a 10-nm-barrier, 5-nm-well GaAs-$\mathrm{Ga}_{0.6}\mathrm{Al}_{0.4}\mathrm{As}$ SCDQW. Solid curves correspond to the present fractional-dimensional calculation and dashed lines are variational results from Thoai (Ref. 8). The impurity position is referred to the center of the central barrier.

III. RESULTS AND DISCUSSION

In what follows, we will first present our theoretical results in the fractional-dimensional space approach, in the zero-magnetic-field limit, in the case of shallow donors in GaAs-$\mathrm{Ga}_{1-x}\mathrm{Al}_x\mathrm{As}$ SCDQW's and SCTQW's, and including effects due to different masses in the GaAs well and $\mathrm{Ga}_{1-x}\mathrm{Al}_x\mathrm{As}$ barrier. Second, we will deal with the zero-magnetic-field limit of a direct heavy-hole exciton in a semiconductor GaAs-$\mathrm{Ga}_{1-x}\mathrm{Al}_x\mathrm{As}$ SCDQW, and finally we will present results for shallow-impurity states in semiconductor quantum wells and superlattices under applied magnetic fields along the heterostructure growth direction.

The fractional-dimensional $1s$-like shallow donor ground-state theoretical results [see Eq. (2.25)], for a 10-nm-barrier, 5-nm-well GaAs-$\mathrm{Ga}_{0.6}\mathrm{Al}_{0.4}\mathrm{As}$ SCDQW, are compared in Fig. 1 with the corresponding variational calculations by Thoai,[8] as functions of the z_i impurity positions referred to the center of the central barrier. From Fig. 1(a), one may notice the overall agreement between the present fractional-dimensional results and Thoai's variational[8] donor binding energies. In Fig. 1(b) we also display the corresponding fractional dimension used in the calculation of the $1s$-like donor binding energies. Notice that the fractional dimension may even be larger than three [cf. Fig. 1(b)]: for the donor at the center of the central barrier ($z_i=0$), and as the donor GaAs effective Bohr radius is ≈ 10 nm, one finds that the donor envelope wave function is essentially taken away from the impurity center and is concentrated at the two neighboring wells (due to the anisotropy caused by the SCDQW potential), with a strong distortion with respect to the isotropic three-dimensional (3D) situation, leading to a smaller con-

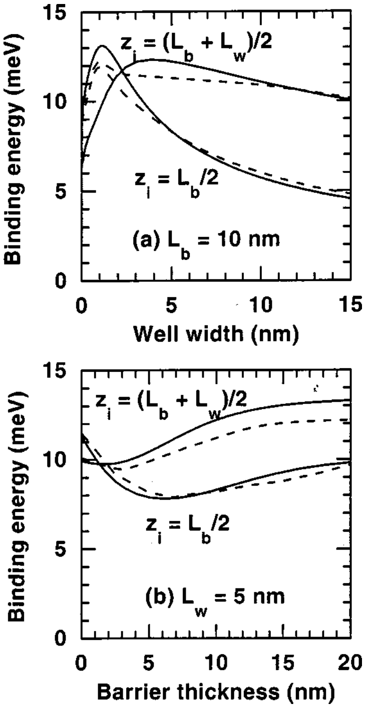

FIG. 2. Well-width dependence in a 10-nm-barrier GaAs-$Ga_{0.6}Al_{0.4}As$ SCDQW (a) and barrier-width dependence in a 5-nm well GaAs-$Ga_{0.7}Al_{0.3}As$ SCDQW (b) of the 1*s*-like ground-state binding energies of donors for impurity positions at the edge of the barrier ($z_i = L_b/2$) and at the center of the GaAs well [$z_i = (L_b + L_w)/2$]. The solid curves correspond to the present fractional-dimensional results, whereas dashed lines are variational calculations by Thoai (Ref. 8). The impurity position refers to the center of the central barrier.

finement (and binding energy) than in the 3D situation, and to an ''effective'' isotropic medium with a fractional dimension larger than three. In other words, anisotropic situations with a ''smaller-than-isotropic'' 3D confinement lead to an effective isotropic medium with a fractional dimension $D > 3$. Notice that stronger confinement (and larger donor 1*s*-like binding energy) occurs for the donor at the center of one of the GaAs wells ($z_i = 7.5$ nm), as one would expect. In that case, the donor envelope wave function is more concentrated around the impurity site than in the isotropic hydrogenic 3D case, and the fractional-dimensional parameter D is then < 3, i.e., a ''larger-than-isotropic'' 3D confinement leads to fractional dimensions $D < 3$.

Figure 2 compares the fractional-dimensional 1*s*-like donor ground-state binding energies for GaAs-$Ga_{1-x}Al_xAs$ SCDQW heterostructures, for varying barrier or well thicknesses, with the corresponding variational calculations by Thoai,[8] for impurity positions at the edge of the barrier ($z_i = L_b/2$) and at the center of the GaAs well [$z_i = (L_b + L_w)/2$]. From Fig. 2, one again notes the good agreement between the present fractional-dimensional results and Thoai's variational[8] donor binding energies. The fractional-dimensional 1*s*-like donor binding energies (for the impurity at the center of the central GaAs well) and corresponding fractional dimensions are shown in Fig. 3 as functions of the barrier (well) thicknesses in GaAs-$Ga_{0.6}Al_{0.4}$ As SCTQW's for different well (barrier) thicknesses. One notices that the present fractional-dimensional results are in quite good agreement with the variational calculations by Chaudhuri.[7] We note that in Figs. 1, 2, and 3 we chose the same parameters and coupled-well barrier potential as in works by Thoai[8] and Chaudhuri,[7] respectively.

Fractional-dimensional theoretical results for the heavy-hole 1*s*-like direct exciton transition energies (exciton peak

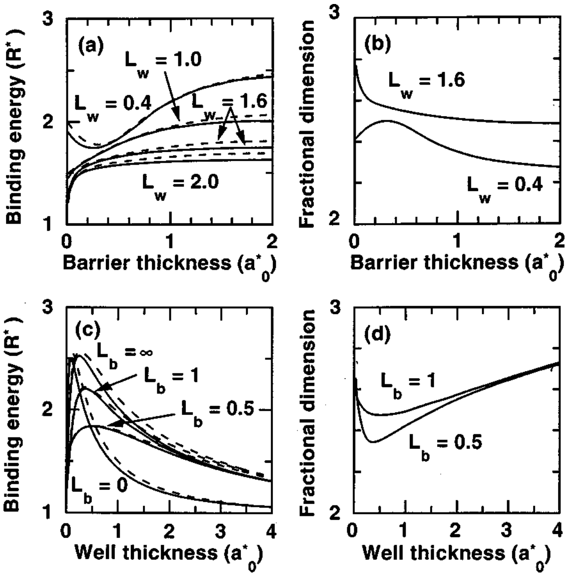

FIG. 3. Donor 1*s*-like binding energies and fractional dimensions in GaAs-$Ga_{0.6}Al_{0.4}As$ SCTQW's for different GaAs layer [(a) and (b)] or barrier [(c) and (d)] thicknesses. Solid lines correspond to the present fractional-dimensional results, and dashed lines are the variational calculations by Chaudhuri (Ref. 7). Energies and distances are expressed in terms of the impurity effective Rydberg (R^*) and the effective Bohr radius (a_0^*), respectively, and the impurity is at the center of the central GaAs well.

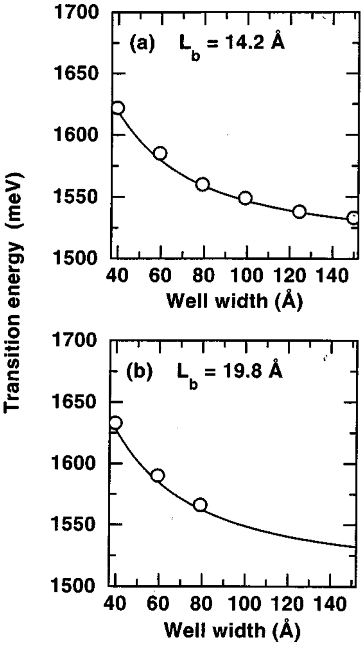

FIG. 4. Heavy-hole 1*s*-like direct exciton theoretical transition energies (solid lines) as functions of the well width in 14.2-Å-barrier (a) and 19.8-Å-barrier (b) GaAs-$Ga_{0.7}Al_{0.3}As$ SCDQW's. Open dots are experimental values by Westgaard *et al.* (Ref. 4).

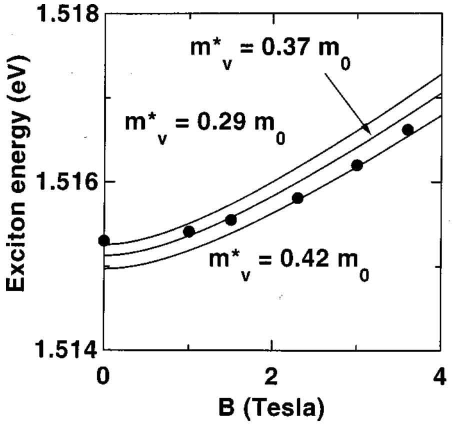

FIG. 5. Heavy-hole 1*s*-like exciton transition energies in bulk GaAs for three values of the heavy-hole effective mass, in units of the m_0 free-electron mass, as functions of the magnetic field. Experimental data of Reynolds *et al.* (Ref. 16) are shown as solid circles.

position) are shown in Fig. 4 as functions of the well width in 14.2-Å-barrier (a) and 19.8-Å-barrier (b) GaAs-Ga$_{0.7}$Al$_{0.3}$As SCDQW's, and compared with the experimental values by Westgaard *et al.*[4] Calculations were performed by using Eq. (2.28), and once the D fractional dimension is obtained (we use the same parameters and coupled-well barrier potential as Westgaard *et al.*[4]), the 1*s*-like heavy-hole exciton binding energies may then be calculated in a straightforward way through[34,35] $E_B = 4/(D-1)^2$ Ry*, where Ry* is the exciton effective Rydberg. Notice that the excellent agreement between our results and the PLE measurements by Westgaard *et al.*[9] (see Fig. 4) suggests that the exciton peak positions are not much modified by the coupling between symmetric and antisymmetric exciton states, which is not taken into account in the present calculation.

The fractional-dimensional space approach for applied magnetic fields along the heterostructure growth direction was first applied to the case of bulk GaAs under magnetic fields [cf. Eq. (2.19)]. In Fig. 5, the magnetic-field-dependent heavy-hole 1*s*-like exciton transition energies (exciton peak positions) in bulk GaAs are shown, within the fractional-dimensional approach, and compared with experimental measurements by Reynolds *et al.*[16] We have assumed different values for the GaAs heavy-hole effective mass, as the literature[4,28,32,45] quotes values ranging from 0.3 up to 0.6, in units of the free-electron mass. Within this uncertainty, agreement with experiment[16] is quite good. Of course, results for shallow donors or excitons in bulk GaAs are identical within a hydrogeniclike treatment, provided they are written in the corresponding reduced units. We note that in the limit of vanishing magnetic fields we find the effective fractional-dimension space as three dimensional ($D=3$), and its dimension decreasing with increasing strength of the magnetic field. Moreover, we note that as the strength of the field increases, the system becomes more and more anisotropic and the simple approximation $\phi_E = \phi_{j=0}$ in Eq. (2.16) should break down. One would then expect the present fractional-dimensional results to be quantitatively reliable only for low and moderate strengths of the applied magnetic field.

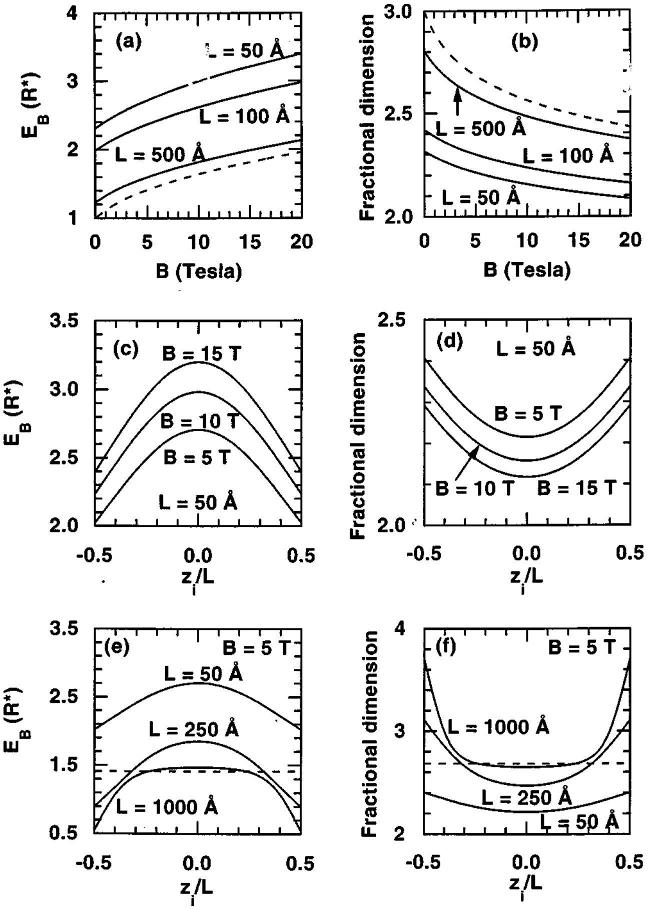

FIG. 6. (a) Magnetic-field dependence of the on-center shallow-donor binding energies, for various values of the GaAs-Ga$_{0.7}$Al$_{0.3}$As QW widths. (c) donor binding energies as functions of the impurity position, for various values of the magnetic field, in a 50-Å-width GaAs-Ga$_{0.7}$Al$_{0.3}$As QW. (e) Donor binding energies for various values of the GaAs-Ga$_{0.7}$Al$_{0.3}$As QW width, under a fixed magnetic field of 5 T. Fractional-dimension parameters in (b), (d), and (f) correspond to results in (a), (c), and (e), respectively. The magnetic field is applied along the growth direction of the QW, and results in the bulk limit are shown as dashed lines. The impurity position is measured from the center of the GaAs QW.

We now use the fractional-dimensional space approach to investigate shallow-donor properties in GaAs-Ga$_{1-x}$Al$_x$As QW's under magnetic fields applied in the growth direction [see Eq. (2.17)]. Figure 6 presents the theoretical fractional-dimensional results for the 1*s*-like shallow-donor binding energies (we use the same parameters and barrier potential as in Ref. 26), and corresponding fractional dimensions for donors in GaAs-Ga$_{0.7}$Al$_{0.3}$As QW's. Results are presented as functions of applied magnetic field (and various widths of the GaAs QW) or donor position in the QW (and varying magnetic field). The present calculations show an increase in the donor binding energy as the donor confinement increases, i.e., with the decreasing width of the QW [Figs. 6(a) and 6(e)] or the increasing strength of the magnetic field [Figs. 6(a) and 6(c)]. In the limit of vanishing applied magnetic field and large well widths, the system should exhibit a 3D behavior, and the fractional dimension is 3 [see Figs. 6(a) and 6(b)]. Of course, large values of the field and/or small values of the well width lead to a

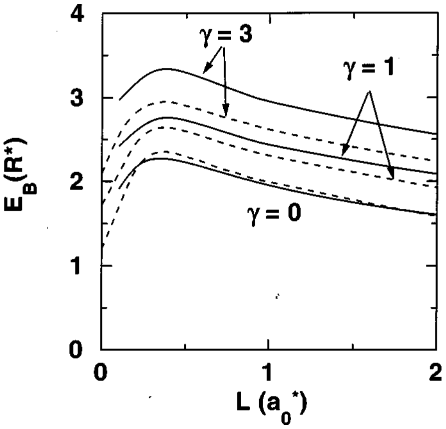

FIG. 7. GaAs QW thickness dependence of the on-center donor binding energies in GaAs-$Ga_{0.7}Al_{0.3}$As QW's. Dashed curves are from Ref. 26 whereas full curves are theoretical results within the fractional dimensional space approach.

decrease of the corresponding fractional dimension of the effective isotropic medium [Figs. 6(b), 6(d), and 6(f)], as the anisotropy increases. Notice in Fig. 6 that the impurity binding energy is larger for donors at the well center than at the on-edge position, as expected due to the larger confinement of the donor-electron wave function. In Fig. 6(f), one finds that, for GaAs-(Ga, Al)As QW's of width $L=250$ and 1000 Å, the fractional dimension may even be larger than 3. For donors at the GaAs QW on-edge position, the donor-electron wave function may be strongly distorted and taken away from the impurity center, leading to a smaller confinement (and binding energy) than in the 3D situation, and to a fractional-dimensional parameter $D>3$. Moreover, the application of a magnetic field increases the donor confinement, and for strong enough magnetic fields, one may expect that, for on-edge donors under magnetic fields, a competing situation would occur with respect to the effective fractional-dimension parameter, with an interplay between an ''expelling'' barrier potential and a ''confining'' magnetic field.

The GaAs QW thickness dependence of the on-center (i.e., the donor is at the GaAs QW center) donor binding energies in GaAs-$Ga_{0.7}Al_{0.3}$As QW's are shown in Fig. 7, which displays the present theoretical fractional-dimensional results in comparison with the perturbative-variational results of Ref. 26. The parameter $\gamma=\hbar\omega_c/2R^*=e\hbar B/2m^*cR^*$ is a measure of the applied magnetic field, with the effective mass $m^*=0.067m_0$ and reduced Rydberg $R^*=5.83$ meV refering to shallow donors. Note that, for low and moderate strengths of the applied magnetic field (essentially up to $\gamma\approx 1$, i.e., up to $B\approx 7$ T), the present fractional-dimensional results are in overall agreement with the calculations in Ref. 26.

Finally, in Fig. 8 we present the fractional-dimensional theoretical results of the transitions involving the lowest Landau-level conduction electrons and acceptor states ($e-A^0$ transitions) of a GaAs-$Ga_{1-x}Al_x$As SL under magnetic fields applied along the growth-axis direction, as compared with the experimental peak positions by Skromme *et al.*[15] In SL calculations, we have used a GaAs dielectric constant ε of 12.35, and a 65% (35%) rule for the conduction (valence)-barrier potential with respect to the total band-gap offset. The band-gap discontinuity was taken[45] as E_g (eV) $=1.247x$, where x is the Al concentration, and the m^* hole effective mass was taken[28] as 0.29 in units of the free-electron mass. For simplicity, m^* and ϵ are taken as the GaAs bulk values throughout the heterostructure. Note that theoretical transitions are to acceptors at the edge and center of the well whereas the measured energy peak which shows up in a photoluminescence experiment should depend on the acceptor distribution in the SL as well as on the temperature, which would affect the hole population at the acceptor states. The excellent agreement between experiment and the theoretical predictions for transitions involving on-center donors indicates that, as a consequence of the quite large width of the GaAs well (width of 540 Å) and a quite small barrier (of 10 Å), most of the acceptors behave essentially as on-center acceptors in the bulk limit, as one would expect.

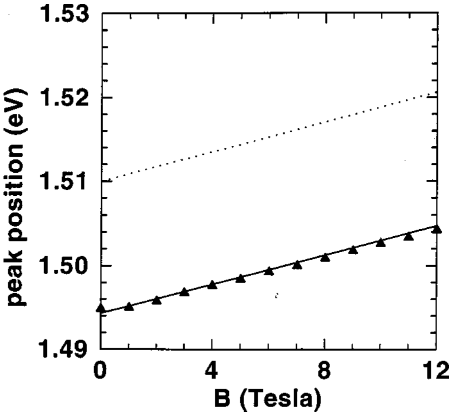

FIG. 8. Magnetic-field dependence of the photoluminescence peak positions corresponding to the e-A^0 transitions in a GaAs-$Ga_{1-x}Al_x$As superlattice with well width $a=540$ Å, barrier width $b=10$ Å, $x=0.27$ (Al content), and $T=13.8$ K (temperature). The full (dotted) curve corresponds to theoretical transitions to acceptors at the center (edge) of the well. Experimental results of Skromme *et al.* (Ref. 15) are shown as solid triangles, and the magnetic field is applied along the growth direction of the superlattice.

IV. CONCLUSIONS

Summing up, we have extended the fractional-dimensional space approach, in which a real anisotropic semiconductor heterostructure in a 3D environment is treated as isotropic in an effective fractional-dimensional space, to include magnetic-field effects in the study of shallow-impurity states in GaAs-(Ga,Al)As QW's and SL's. In the fractional-dimensional scheme, the value of the fractional dimension is associated to the degree of anisotropy of the actual 3D semiconductor system. In the present study, the magnetic field was applied along the growth direction of the semiconductor heterostructure, and introduces an additional degree of confinement and anisotropy besides the one imposed by the heterostructure barrier potential. The fractional dimension is then related to the anisotropy introduced both by the heterostructure barrier potential and magnetic field. Fractional-dimensional calculations for 1s-like bulk GaAs exciton transition energies were shown to be in good overall agreement with experimental measurements by Reynolds *et al.*[16] Also, our calculated results for shallow-impurity states in GaAs-(Ga,Al)As semiconductor QW's under applied magnetic fields were shown to be in overall agreement, under low and moderate strengths of the applied magnetic fields, with previous calculations in Ref. 26. Moreover, our theoretical results for transitions from the lowest magnetic Landau conduction level to on-edge and on-center acceptor states were in quite good agreement with experimental data from Skromme *et al.*[15] The present work on the fractional-dimensional space approach to include effects of applied magnetic fields upon shallow-impurity properties in semiconductor heterostructures may be extended to study exciton states in theses systems, and would be of importance in the quantitative understanding of future experimental work in the area.

Finally, one should mention that the fractional-dimensional approach yields results in agreement with the exact results in two and three dimensions, and its applications to excitons and shallow impurities in semiconductor heterostructures such as QW's, MQW's, and SL's give overall agreement with previous variational calculations, as this work and previous works have shown. The applicability of the fractional-dimensional approach to semiconductor systems of lower dimensionality such as quasi-one-dimensional QWW's and quasi-zero-dimensional QD's, however, is still an open problem and it is not clear how meaningful the fractional-dimensional results would be in the limit[48] of $D\to 1$ or $D\to 0$.

ACKNOWLEDGMENTS

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APPENDIX A

By substituting

$$\psi_E(\mathbf{r}) = f_0(z)\,\chi_{0,0}(\rho)\,\phi_{E_{1s}}(\mathbf{r}) \tag{A1}$$

into

$$H\psi_E(\mathbf{r}) = E\,\psi_E(\mathbf{r}), \tag{A2}$$

with H given by Eq. (2.2), and using

$$\nabla^2\psi_E = \chi_{0,0} f_0 \nabla^2\phi_{E_{1s}} + f_0\phi_{E_{1s}}\nabla^2_\rho\chi_{0,0} + \chi_{0,0}\phi_{E_{1s}}\frac{d^2 f_0}{dz^2} + 2\,f_0\nabla_\rho\chi_{0,0}\cdot\nabla_\rho\phi_{E_{1s}} + 2\chi_{0,0}\frac{df_0}{dz}\,\frac{\partial\phi_{E_{1s}}}{\partial z}, \tag{A3}$$

$$-\frac{\hbar^2}{2m^*}\,\frac{d^2 f_0}{dz^2} + V(z) f_0 = \varepsilon_{e,0} f_0, \tag{A4}$$

$$\left[-\frac{\hbar^2}{2m^*}\nabla^2_\rho + \frac{1}{2}m^*\Omega^2\rho^2\right]\chi_{0,0} = \hbar\Omega\,\chi_{0,0}, \tag{A5}$$

one obtains

$$\left[-\frac{\hbar^2}{2m^*}\nabla^2 - \frac{e^2}{\varepsilon|\mathbf{r}-\mathbf{r}_i|}\right]\phi_{E_{1s}}(\mathbf{r}) - \frac{\hbar^2}{m^*}\,\frac{\nabla[f_0(z)\chi_{00}(\rho)]}{f_0(z)\chi_{00}(\rho)}\cdot\nabla\phi_{E_{1s}}(\mathbf{r}) = E_{1s}\phi_{E_{1s}}(\mathbf{r}), \tag{A6}$$

with

$$E = \varepsilon_{e,o} + \hbar\Omega + E_{1s}\,. \tag{A7}$$

By defining

$$h(\vec{\mathbf{r}}) = f_0^2(z)\exp\left(-\frac{\rho^2}{\lambda_B^2}\right), \tag{A8}$$

one may write Eq. (A6) as

$$\left[-\frac{\hbar^2}{2m^*}\nabla^2 - \frac{e^2}{\varepsilon|\mathbf{r}-\mathbf{r}_i|}\right]\phi_{E_{1s}}(\mathbf{w}) - \frac{\hbar^2}{2m^*}\,\frac{\nabla h}{h}\cdot\nabla\phi_{E_{1s}}(\mathbf{r}) = E_{1s}\phi_{E_{1s}}(\mathbf{r}). \tag{A9}$$

If one considers spherical coordinates with the origin at the impurity position, one finds

$$-\frac{\hbar^2}{2m^*}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\right]\phi_{E_{1s}}(\mathbf{r}) - \frac{\hbar^2}{2m^*}\,\frac{1}{h}\left[\frac{\partial h}{\partial r}\,\frac{\partial}{\partial r} + \frac{1}{r^2}\,\frac{\partial h}{\partial\theta}\,\frac{\partial}{\partial\theta}\right]\phi_{E_{1s}}(\mathbf{r}) - \frac{e^2}{\varepsilon r}\,\phi_{E_{1s}}(\mathbf{r}) = E_{1s}\phi_{E_{1s}}(\mathbf{r}), \tag{A10}$$

which corresponds to Eq. (2.8), with $r = \sqrt{x^2+y^2+(z-z_t)^2}$, and h now given by

$$h(\vec{\mathbf{r}}) = h(r,\theta) = f_0^2(r\cos\theta + z_i)\exp\left(-\frac{r^2\sin^2\theta}{\lambda_B^2}\right). \tag{A11}$$

APPENDIX B

For the $1s$-like shallow-impurity ground state one may approximately choose the wave function $\phi_{E_{1s}} = \phi_{j=0}$ in Eq. (2.16), with the fractional-dimensional wave-function $\phi_{j=0} = \phi_{1s}(\mathbf{r}) = e^{-\lambda r}$, with $\lambda = 2/a_0^*(D-1)$. By using Eq. (2.14), one may write Eq. (2.16) as

$$\int\!\!\int dr\,d\theta\,hr^2\sin\theta\left(\frac{\beta}{r} + \frac{1}{h_0}\,\frac{\partial h_0}{\partial r} - \frac{2r\sin^2\theta}{\lambda_B^2}\right)e^{-2\lambda r} = 0. \tag{B1}$$

By defining

$$\sigma(r) = \int_0^\pi d\theta\sin\theta h_0(r\cos\theta + z_i)\exp\left(-\frac{r^2\sin^2\theta}{\lambda_B^2}\right), \tag{B2}$$

and $\alpha = 2\lambda$, Eq. (B1) transforms into

$$\beta\int_0^{+\infty} dr\, r\exp(-\alpha r)\sigma(r) + \int_0^{+\infty} dr\, r^2\exp(-\alpha r)\frac{d}{dr}\sigma(r)=0. \tag{B3}$$

Using $z=r\cos\theta$ in Eq. (B2), one obtains

$$\sigma(r)=\frac{1}{r}F(r,z_i)=\frac{1}{r}\exp\left(-\frac{r^2}{\lambda_B^2}\right) \times\int_0^r dz[f_0^2(z+z_i)+f_0^2(z-z_i)]\exp\left(\frac{z^2}{\lambda_B^2}\right), \tag{B4}$$

and Eq. (B3) reduces to

$$(\beta-1)\int_0^{+\infty} dr\exp(-\alpha r)F(r,z_i) + \int_0^{+\infty} dr\, r\exp(-\alpha r)\frac{\partial F(r,z_i)}{\partial r}=0. \tag{B5}$$

According to Eq. (B4), one may write

$$\frac{\partial F(r,z_i)}{\partial r}=-\frac{2r}{\lambda_B^2}F(r,z_i)+[f_0^2(r+z_i)+f_0^2(r-z_i)], \tag{B6}$$

and substituting Eq. (B6) into Eq. (B5) one finds

$$(\beta-1)\int_0^{\infty} dr\exp(-\alpha r)F(r,z_i) -\frac{2}{\lambda_B^2}\int_0^{+\infty} dr\, r^2\exp(-\alpha r)F(r,z_i) +\int_0^{+\infty} dr\, r\exp(-\alpha r)[f_0^2(r+z_i)+f_0^2(r-z_i)]=0. \tag{B7}$$

In the zero-magnetic-field limit, the function F in Eq. (B4) may be written as

$$F_0(r,z_i)=\int_0^r dz[f_0^2(z+z_i)+f_0^2(z-z_i)], \tag{B8}$$

and Eq. (B7) reduces to

$$(\beta-1)\int_0^{+\infty} dr\exp(-\alpha r)F_0(r,z_i)+\int_0^{+\infty} dr\, r\exp(-\alpha r) \times[f_0^2(r+z_i)+f_0^2(r-z_i)]=0. \tag{B9}$$

Integrating by parts the first integral of Eq. (B9) and defining

$$L(\alpha,z_i)=\int_0^{+\infty} dr\exp(-\alpha r)[f_0^2(r+z_i)+f_0^2(r-z_i)], \tag{B10}$$

one may find the transcendental equation to be solved for the fractional-dimensional parameter in the limit of zero magnetic field, i.e.,

$$(\beta-1)L(\alpha,z_i)-\alpha\frac{\partial}{\partial\alpha}L(\alpha,z)=0, \tag{B11}$$

which was previously obtained in Ref. 42. On the other hand, integrating by parts the second integral in Eq. (B5), one obtains

$$(\beta-2)\int_0^{+\infty} dr\, F(r,z_i)\exp(-\alpha r) +\alpha\int_0^{+\infty} dr\, rF(r,z_i)\exp(-\alpha r)=0, \tag{B12}$$

and, taking into account Eq. (B4), one may write

$$\left(\beta-2-\frac{1}{2}\alpha^2\lambda_B^2\right)\int_0^{+\infty} drF(r,z_i)\exp(-\alpha r) +\frac{1}{2}\alpha\lambda_B^2\int_0^{+\infty} dr[f_0^2(r+z_i)+f_0^2(r-z_i)]\exp(-\alpha r)=0, \tag{B13}$$

which is Eq. (2.17), i.e., the transcendental equation to be solved for the fractional-dimensional parameter D taking into account the effects of the applied magnetic field.

One may obtain the transcendental equation to be solved for the fractional-dimensional parameter D for the three-dimensional limit case, in which the confinement potential $V(z)$ is taken equal to zero. In that case, the ground-state eigenfunction of Eq. (2.1) may be taken as $\psi_E(\vec{\mathbf{r}})=\chi_{0,0}(\rho)\phi_{E_{1s}}(\vec{\mathbf{r}})$, which is equivalent to taking f_0 in Eq. (2.11) as a constant. In that case [see Eq. (2.14)],

$$W^{3D}=-\frac{\hbar^2}{2m^*}\left[\left(\frac{\beta}{r}-\frac{2r\sin^2\theta}{\lambda_B^2}\right)\frac{\partial}{\partial r}+\left(\frac{\beta}{r^2}\cot\theta-\frac{\sin 2\theta}{\lambda_B^2}\right)\frac{\partial}{\partial\theta}\right], \tag{B14}$$

and, following a similar procedure to that outlined above for the case of a semiconductor heterostructure under applied magnetic fields, from Eq. (2.16) one obtains, after some tedious although straightforward manipulations, that

$$\int_0^{\infty}\frac{e^{-x}}{4x+k^2}dx=\frac{1}{4-2\beta+k^2}, \tag{B15}$$

with

$$k=\frac{4\sqrt{2}}{D-1}\,\frac{l_B}{\alpha_0^*},$$

which is Eq. (2.19), valid for the bulk case. Alternatively, one may obtain Eq. (B15) [or Eq. (2.19)] as the appropriate limit of Eq. (B13).

[1] R. C. Miller, A. C. Gossard, W. T. Tsang, and O. Munteanu, Phys. Rev. B **25**, 3871 (1982).

[2] B. V. Shanabrook, J. Comas, T. A. Perry, and R. Merlin, Phys. Rev. B **29**, 7096 (1984).

[3] M. Helm, F. M. Peeters, F. DeRosa, E. Colas, J. P. Harbison, and L. T. Florez, Phys. Rev. B **43**, 13 983 (1991).

[4] T. Westgaard, Q. X. Zhao, B. O. Fimland, K. Johannessen, and L. Johnsen, Phys. Rev. B **45**, 1784 (1992).

[5] G. Oelgart *et al.*, Phys. Rev. B **49**, 10 456 (1994).

[6] G. Bastard, *ibid.* **24**, 4714 (1981); R. L. Greene and K. K. Bajaj, Solid State Commun. **45**, 825 (1985); C. Mailhiot, Y-C. Chang, and T. C. McGill, Phys. Rev. B **26**, 4449 (1982); W. T. Masselink, Y-C. Chang, and H. Morkoç, Phys. Rev. B **28**, 7373 (1983).

[7] S. Chaudhuri, Phys. Rev. B **28**, 4480 (1983).

[8] D. B. T. Thoai, Physica B **175**, 373 (1991).

[9] E. L. Ivchenko and A. V. Kavokin, Fiz. Tekh. Poluprovodn. **25**, 1780 (1991) [Sov. Phys. Semicond. **25**, 1070 (1991)].

[10] L. E. Oliveira and L. M. Falicov, Phys. Rev. B **34**, 8676 (1986); L. E. Oliveira and J. López-Gondar, *ibid.* **41**, 3719 (1990); L. E. Oliveira and G. D. Mahan, *ibid.* **47**, 2406 (1993).

[11] G. Bastard, E. E. Mendez, L. L. Chang, and L. Esaki, Phys. Rev. B **26**, 1974 (1982).

[12] L. C. Andreani and A. Pasquarello, Phys. Rev. B **42**, 8928 (1990).

[13] R. P. Leavitt and J. W. Little, Phys. Rev. B **42**, 11 774 (1990).

[14] A. A. Reeder, J.-M. Mercy, and B. D. McCombe, IEEE J. Quantum Electron. **24**, 1690 (1988).

[15] B. J. Skromme, R. Bhat, M. A. Koza, S. A. Schwarz, T. S. Ravi, and D. M. Hwang, Phys. Rev. Lett. **65**, 2050 (1990).

[16] D. C. Reynolds, K. K. Bajaj, C. W. Litton, R. L. Greene, P. W. Yu, C. K. Peng, and H. Morkoc, Phys. Rev. B **35**, 4515 (1987).

[17] D. C. Rogers, J. Singleton, R. J. Nicholas, C. T. Foxon, and K. Woodbridge, Phys. Rev. B **34**, 4002 (1986).

[18] I. Aksenov, J. Kusano, Y. Aoyagi, T. Sugano, T. Yasuda, and Y. Segawa, Phys. Rev. B **51**, 4278 (1995).

[19] A. I. Tartakovskii, V. B. Timofeev, V. G. Lysenko, D. Birkedal, and J. Hvam, Zh. Eksp. Teor. Fiz. **112**, 1106 (1997) [JETP **85**, 601 (1997)].

[20] O. Jaschinski, M. Vergöhl, J. Schoenes, A. Schlachetzki, and P. Bönsch, Phys. Rev. B **57**, 13 086 (1998).

[21] M. S. Salib, H. A. Nickel, G. S. Herold, A. Petrou, B. D. McCombe, R. Chen, K. K. Bajaj, and W. Schaff, Phys. Rev. Lett. **77**, 1135 (1996); H. A. Nickel, G. S. Herold, M. S. Salib, G. Kioseoglou, A. Petrou, B. D. McCombe, and D. Broido, Physica B **249-251**, 598 (1998).

[22] J. Černe, J. Kono, M. S. Sherwin, M. Sundaram, A. C. Gossard, and G. E. W. Bauer, Phys. Rev. Lett. **77**, 1131 (1996); J. Kono, M. Y. Su, J. Černe, M. S. Sherwin, S. J. Allen, Jr., T. Inoshita, T. Noda, and H. Sakaki, Physica B **249-251**, 527 (1998).

[23] C. Aldrich and R. L. Greene, Phys. Status Solidi B **93**, 343 (1979).

[24] S. Chaudhuri and K. K. Bajaj, Solid State Commun. **52**, 967 (1984).

[25] R. L. Greene and K. K. Bajaj, Phys. Rev. B **31**, 913 (1985).

[26] Zhong-Jun Shen, Shu-Au Zhong, and Shhi-Hei Gu, Phys. Status Solidi B **164**, 221 (1991).

[27] A. Latgé, N. Porras-Montenegro, M. de Dios-Leyva, and L. E. Oliveira, Phys. Rev. B **53**, 10 160 (1996); L. H. M. Barbosa, A. Latgé, M. de Dios Leyva, and L. E. Oliveira, Solid State Commun. **98**, 215 (1996); J. Phys.: Condens. Matter **9**, 3138 (1997).

[28] A. Latgé, N. Porras-Montenegro, M. de Dios-Leyva, and L. E. Oliveira, J. Appl. Phys. **81**, 6234 (1997); Phys. Status Solidi B **210**, 655 (1998).

[29] G. E. W. Bauer and T. Ando, Phys. Rev. B **37**, 3130 (1988).

[30] G. Duggan, Phys. Rev. B **37**, 2759 (1988).

[31] P. Peyla, R. Romestain, Y. Merle d' Aubigné, G. Fishman, A. Wasiela, and H. Mariette, Phys. Rev. B **52**, 12 026 (1995).

[32] S. N. Walck and T. L. Reinecke, Phys. Rev. B **57**, 9088 (1998).

[33] T. Inoshita and H. Sakaki, Physica B **249-251**, 534 (1998).

[34] D. R. Herrick and F. H. Stillinger, Phys. Rev. A **11**, 42 (1975); F. H. Stillinger, J. Math. Phys. **18**, 1224 (1977), and references therein.

[35] X-F. He, Solid State Commun. **61**, 53 (1987); Phys. Rev. B **42**, 11 751 (1990); **43**, 2063 (1991).

[36] H. Mathieu, P. Lefebvre, and P. Christol, J. Appl. Phys. **72**, 300 (1992); Phys. Rev. B **46**, 4092 (1992).

[37] P. Lefebvre, P. Christol, and H. Mathieu, Phys. Rev. B **46**, 13 603 (1992); **48**, 17 308 (1993); P. Christol, P. Lefebvre, and H. Mathieu, J. Appl. Phys. **74**, 5626 (1993); IEEE J. Quantum Electron. **30**, 2287 (1994); P. Lefebvre, P. Christol, H. Mathieu, and S. Glutsch, Phys. Rev. B **52**, 5756 (1995).

[38] Q. X. Zhao, B. Monemar, P. O. Holtz, M. Willander, B. O. Fimland, and K. Jojannessen, Phys. Rev. B **50**, 4476 (1994).

[39] A. Thilagam, Phys. Rev. B **55**, 7804 (1997); **56**, 4665 (1997); **56**, 9798 (1997); J. Appl. Phys. **82**, 5753 (1997); Phys. Rev. B **59**, 3027 (1999); C. Tanguy, P. Lefebvre, H. Mathieu, and R. J. Elliot, J. Appl. Phys. **82**, 798 (1997).

[40] M. de Dios-Leyva, A. Bruno-Alfonso, A. Matos-Abiague, and L. E. Oliveira, J. Phys.: Condens. Matter **9**, 8477 (1997).

[41] A. Matos-Abiague, L. E. Oliveira, and M. de Dios-Leyva, Phys. Rev. B **58**, 4072 (1998).

[42] E. Reyes-Gómez, L. E. Oliveira, and M. de Dios-Leyva, J. Appl. Phys. **85**, 4045 (1999).

[43] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1964).

[44] L. E. Oliveira, Phys. Rev. B **38**, 10 641 (1988).

[45] S. Adachi, J. Appl. Phys. **58**, R1 (1985); H. C. Casey and M. B. Panish, *Heterostructure Lasers* (Academic, New York, 1978).

[46] G. Bastard, E. E. Mendez, L. L. Chang, and L. Esaki, Phys. Rev. B **26**, 1974 (1982).

[47] J. W. Brown and H. N. Spector, Phys. Rev. B **35**, 3009 (1987).

[48] R. E. Moss, Am. J. Phys. **55**, 397 (1987).