THE TWO-COMPONENT VIRIAL THEOREM AND THE PHYSICAL PROPERTIES OF STELLAR SYSTEMS

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ABSTRACT

Motivated by present indirect evidence that galaxies are surrounded by dark matter halos, we investigate whether their physical properties can be described by a formulation of the virial theorem that explicitly takes into account the gravitational potential term representing the interaction of the dark halo with the baryonic or luminous component. Our analysis shows that the application of such a "two-component virial theorem" not only accounts for the scaling relations displayed by, in particular, elliptical galaxies, but also for the observed properties of all virialized stellar systems, ranging from globular clusters to galaxy clusters.

Subject headings: cosmology: theory — dark matter — galaxies: elliptical and lenticular, cD — galaxies: fundamental parameters — galaxies: halos — galaxies: kinematics and dynamics — galaxies: structure

1. INTRODUCTION

It is expected on very fundamental grounds that the state of equilibrium of self-gravitating, time-averaged stationary stellar systems should be well described by the virial theorem. In fact, elliptical galaxies, for instance, show a remarkable homogeneity which is expressed by a very tight kinematical-structural relationship, the so-called "fundamental plane" (FP; Djorgovski & Davis 1987; Dressler et al. 1987). Since it is believed that these galaxies represent equilibrium systems, their interconnected physical properties should reflect their virialized condition. However, the FP is significantly "tilted" relative to the relations expressed by the virial theorem applied to a family of homologous objects. The nature of this discrepancy is controversial and has been extensively debated in the literature (e.g., Graham & Colless 1997; Ciotti, Lanzoni, & Renzini 1996; Pahre, Djorgovski, & de Carvalho 1998, and references therein).

The FP "problem" can be stated as follows. The virial theorem, applied to a stationary self-gravitating system, states that $2K + W = 0$, where K is the kinetic energy and W is the potential energy of the system. This may be rewritten as $\langle v^2 \rangle = GM/r_G$, where r_G is the gravitational radius defined by $r_G = GM^2/|W|$, $\langle v^2 \rangle$ is the mean square velocity of the particles, G is the gravitational constant, and M is the total mass of the system. These physical quantities may be translated to observational ones through the definition of some kinematical-structural coefficients (C_r, C_v) which may or may not be constants among galaxies: $\sigma_0^2 = C_v \langle v^2 \rangle$ and $r_e = C_r r_G$; $I_e = [(M/2)/\pi r_e^2] (M/L)^{-1}$. M/L is the mass-luminosity relation for the system; r_e is its effective radius, that is the radius that contains half of its total luminosity: $L(< r_e) = L_{\rm tot}/2$; σ_0 its central projected velocity dispersion, that is the mean square projected velocity of stars at the galaxy center (measured inside a slit of finite projected width); and $I_e = L(< r_e)/\pi r_e^2$ is the mean surface brightness inside r_e in linear units. Inserting these equations into the virial relation, one finds $r_e = C_{\rm FP} \sigma_0^2 I_e^{-1}$, where $C_{\rm FP}$ depends on the mass-luminosity relation and on the coefficients defined above (C_r, C_v). In contrast, what one observes is that

$r_e \propto \sigma_0^A I_e^B$, with $A \sim 1.53$, $B \sim -0.79$, for elliptical galaxies observed in the near-infrared (Pahre et al. 1998). The reasons for the deviation of the observed relationship as compared to the virial theorem are not well established. One may postulate a systematic variation of the structural coefficients (galaxies would form a nonhomologous family of objects; Capelato, de Carvalho, & Carlberg 1995, 1997; Hjorth & Madsen 1995) or even a systematic trend of the mass-to-light ratio with galaxy mass: $M/L \propto M^\alpha$ (e.g., Dressler et al. 1987).

However, it should be noted that elliptical galaxies, as any other collapsed structures, are probably surrounded by massive dark matter halos. The observed FP relations, on the other hand, arise from the observed (i.e., baryonic) component of these systems. It seems thus natural to ask how the equilibrium state of the baryonic component under the influence of its massive halo would modify the simple one-component virial theorem. In fact, attempts to construct two-component models can be found in the recent literature. For instance, Ciotti et al. (1996) indicate that the FP tilt could be explained by massive extended dark matter halos embedding the luminous matter of galaxies with the following caveat: a nonrealistic fine-tuning of the luminous–to–dark matter distributions would be required in order to explain the small scatter of the FP correlations. On the other hand, preliminary results of Kritsuk (1997) suggest that the FP for elliptical galaxies and, also, the observed deviation of dwarf spheroidal galaxies from it may follow from the dynamical equilibrium condition in the framework of a two-component model.

Attempting to visualize the physical properties of virialized stellar systems of various scales into an integrated framework (the κ-space; see Bender, Burstein, & Faber 1992), Burstein et al. (1997, hereafter BBFN97) concluded that globular clusters, galaxies, groups of galaxies, and clusters of galaxies also show systematic trends in their observed properties, populating what they called a "cosmic metaplane" in their parameter space. This metaplane, also tilted with respect to the simple virial expectation, was interpreted as a combination of FP-like tilts associated to the various stellar systems, possibly reflecting their different stellar population and dissipation histories. However, under this interpretation, a fine-tuning mechanism for the variation of M/L with mass, for every stellar system, also had to be invoked in order to preserve the striking appearance of the metaplane (see also Schaeffer et al. 1993). Also, their analysis made evident a "zone of exclusion" in which no stellar system

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could be found. This raises the question of which formation process would generate such a trend and the mechanisms responsible for producing the metaplane itself.

In this Letter we tackle these questions by starting from the hypothesis that self-gravitating stellar systems in the universe are embedded in dark halos. As a consequence, the strict virial theorem must be replaced by a new equilibrium equation that takes explicitly into account the gravitational potential produced by the massive halo in which the luminous component is embedded. With this assumption, we present an alternative model that may naturally explain the issues discussed above. Our Letter is organized as follows: in § 2, we discuss the virial theorem for two-component systems; in § 3, we apply it to observational data; and in § 4, we discuss some of the implications of our results.

2. THE TWO-COMPONENT VIRIAL THEOREM

The scalar virial theorem for the baryonic component of a stellar system (component 2) in steady state equilibrium embedded in its dark matter halo (component 1) may be readily deduced from the Jeans equation by assuming that, in addition to its self-potential, it is also subjected to the external potential produced by the dark matter (see, e.g., Binney & Tremaine 1987; see also Limber 1959; Spitzer 1969; Smith 1980). In this case, a new term is added to the gravitational energy of the system due to the interaction of the two components. Assuming spherical symmetry, we may write the gravitational energy of luminous component W_2 as

$$W_2 = -G \int_0^\infty \frac{\rho_2(r) M_2(r)}{r} dV - G \int_0^\infty \frac{\rho_2(r) M_1(r)}{r} dV, \quad (1)$$

where $M_\mu(r)$ is the total mass of the μ-component within the radius r. If we now further assume that the dark matter halo—component 1—is more extended than the baryonic component, having a not too steep density profile within the interior region containing the luminous component, then we may approximate the second integral, which gives the interaction energy, by

$$W_{21} \equiv -G \int_0^\infty \frac{\rho_2(r) M_1(r)}{r} dV$$

$$\sim -\frac{4\pi}{3} \rho_{0,1} G \int_0^\infty \frac{\rho_2(r) r^3}{r} dV$$

$$= -\frac{4\pi}{3} \rho_{0,1} G M_2 \langle r_2^2 \rangle, \quad (2)$$

where $\rho_{0,1}$ is the mean density of the dark matter halo within the region containing the luminous component and

$$\langle r_2^2 \rangle \equiv \frac{\int r^2 \rho_2(r) dV}{\int \rho_2(r) dV}. \quad (3)$$

Thus, the virial theorem for the collapsed baryonic component,

$2K_2 + W_2 = 0$, may be written as

$$\langle v_2^2 \rangle = \frac{GM_2}{r_{G,2}} + \frac{4\pi}{3} G\rho_{0,1} \langle r_2^2 \rangle, \quad (4)$$

where $r_{G,2}$ is the gravitational radius of the second component.

We see that in the presence of an extended dark matter halo, the virial theorem gets an extra term on its right-hand side, which accounts for the interaction with the extended dark matter halo (this is also known as the "Limber effect"). As we will see in the next section, this term is essential for our understanding of the systematic trends of the observed properties of the stellar systems we discussed before. In terms of the observational quantities, the modified virial theorem writes as

$$\sigma_0^2 = C^*(I_e r_e + b r_e^2), \quad (5)$$

where

$$C^* = 2\pi G C_r C_v \left(\frac{M}{L}\right)_2$$

and

$$b = \frac{2}{3} \frac{R}{C_r} \left(\frac{M}{L}\right)_2^{-1} \rho_{0,1}, \quad (6)$$

with $R = \langle r_2^2 \rangle / r_e^2$.

Notice that in these equations all the structural coefficients as well as M/L refer to the baryonic component. Parameter b has dimension of a luminosity density, whereas C^* has dimension of a less intuitive quantity (i.e., GM/L).

Equation (6) is specially interesting, since it relates the parameter b to the central density of the dark matter halo. We numerically analyzed various equilibrium models (specifically, Jaffe, King, and Sersic models; see Binney & Tremaine 1987; Ciotti 1991; Ciotti & Lanzoni 1997) and found that $C_r C_v \sim 0.2$, whereas R/C_r varies significantly, depending on the models: $R/C_r \sim 10$–25 for King or Jaffe models and ~ 10–60 for the Sersic models. We adopted $R/C_r \sim 20$ as a typical value. It is important to stress that for galaxies this approximation can introduce a factor of 2 difference in the parameter b.

3. APPLYING THE TWO-COMPONENT VIRIAL THEOREM

We will apply the two-component virial theorem (2-VT) in the context of the κ-space parameter framework. This will allow us to directly compare the 2-VT predictions with the extensive data provided by BBFN97. In this coordinate system the FP is seen edge-on, projected on the (κ_1, κ_3)-plane, and the 2-VT (eq. [5]) can be expressed as

$$\kappa_3 = \frac{\log C^*}{\sqrt{3}} + \frac{1}{\sqrt{3}} \log (1 + b10^\omega), \quad (7)$$

where

$$\omega \equiv \frac{\kappa_1 - \sqrt{3}\kappa_2}{\sqrt{2}} = -\log \frac{I_e}{r_e}, \quad (8)$$
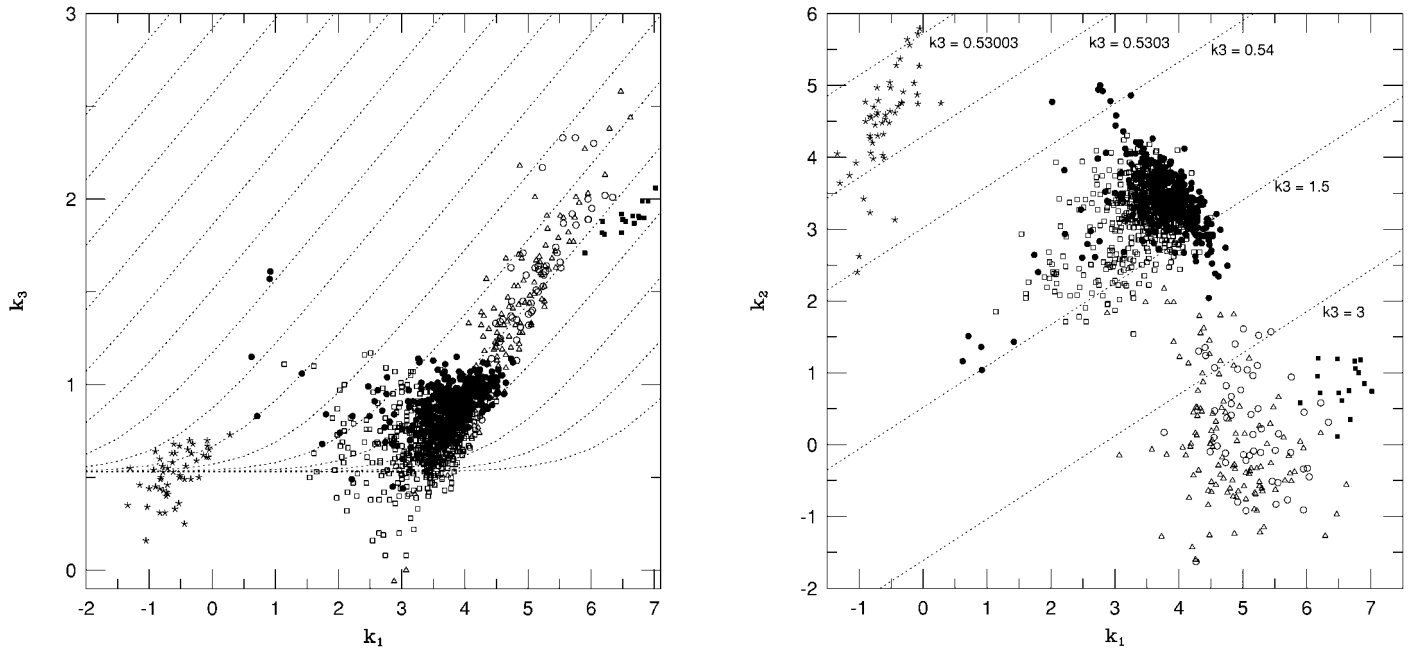
FIG. 1.—Projection in the κ-space of the data presented by BBFN97. The symbols are for groups dominated by elliptical galaxies (*open circles*), elliptical galaxies (*filled circles*), spiral galaxies (*open squares*), clusters of galaxies (*filled squares*), globular clusters (*stars*), and groups dominated by spiral galaxies (*triangles*). (*a*) $\kappa_1 \times \kappa_3$, where the dotted lines indicate the variation of κ_2 from -2.5 to 5.0. For both projections, the 2-VT model is constrained by $C^* = 8.28$ and $b = 200$. (*b*) The projection $\kappa_1 \times \kappa_2$, where the dotted lines represent different values of κ_3 as indicated.

that is, ω is measuring the central luminosity density of the stellar systems.

From equations (7) and (8) we see that the 2-VT defines a surface in the κ-space whose main characteristics may be best viewed through the curve defined by its intersection with the (κ_3, ω)-plane, perpendicular to the (κ_1, κ_2)-plane. A brief analysis of equation (7) shows that it intercepts the κ_3-axis at $\log C^*/\sqrt{3}$. If there were no dark halo, $b = 0$, recovering the usual one-component virial theorem, $\kappa_3 = cte$. Departure from this horizontal line at a given ω depends on the term $b10^\omega$ and thus on the density of the dark halo. For $b10^\omega \gg 1$ it tends to a straight line with a fixed slope of $1/\sqrt{6}$ intercepting the κ_3-axis at $\log (C^*b)/\sqrt{3}$. That is, the 2-VT predicts an *asymptotic, characteristic, fixed tilt* relative to the one-component virial theorem. Notice that, within a factor depending on the structural coefficients of the baryonic component, the value of the mean central density of the dark matter halo is given by this intercept.

In Figure 1*a*, we plot the data on the κ-space, projected on the (κ_1, κ_3)-plane for self-gravitating stellar systems spanning all scales—from globular clusters to rich galaxy clusters—using data presented in BBFN97. The 2-VT curves, given by $C^* = 8.28$ and $b = 200$, are shown as dotted lines for the various ranges of the κ parameters. This figure shows the striking compatibility of the cosmic metaplane with the theoretical predictions of the two-component virial theorem—specifically, *the fixed asymptotic tilt relative to the strict virial theorem.*

We establish the 2-VT relation (eq. [7]) by assuming two different hypotheses about the mass-luminosity relation of the baryonic component: (1) that its value is about the same as found for the globular clusters, which seems reasonable since these systems are very well described by the one-component virial theorem, that is $b10^{\omega\text{glob clust}} \ll 1$ (see Bellazzini 1998); and (2) by adjusting the value of the κ_3 intercept [that is, $(M/L)_2$]

to a maximum value still giving a reasonable fit to the groups and clusters of galaxies. In doing that, we were attempting to take into account the presence of nonstellar baryonic mass and the remaining galactic dark halos in these systems.

We find for case 1 $(M/L)_2 \sim 1.6$ $(C^* = 8.28)$, a value that agrees fairly well with those for globular clusters (e.g., Pryor & Meylan 1993). For case 2, $C^* = 39.2$ gives $(M/L)_2 \sim 7.4$. The central densities of the dark matter halos were estimated after adjusting the b parameter. For the (elliptical) galaxies we found $\rho_{0,1} \sim 2.3 \times 10^{-2}$ $M_\odot$ pc^{-3} $(b = 200)$, whereas for the elliptical-dominated groups and clusters of galaxies, $\rho_{0,1} \sim 5.8 \times 10^{-6}$ $M_\odot$ pc^{-3} $(b = 0.20$ for case 1 and $b = 0.004$ for case 2). The corresponding values for the spiral galaxies and for the spiral-dominated groups are about a factor 2–3 smaller due to the fact that these systems appear slightly displaced toward larger values of ω.

In Figure 1*b* we clearly see that the points do not fill the space continuously. On the contrary, they are arranged in some bands defined by specific values of κ_3, which are related to specific values of w through equation (7). The parameter w rules the luminosity density in the systems, and hence it is associated with their dissipation histories and the epoch when the collapse happened (i.e., the density fluctuation spectrum). Thus, in the context of a hierarchical clustering scenario, smaller systems collapse before and are more concentrated, presenting higher luminosity densities (w more negative); while larger objects, collapsed later, present lower luminosity densities (w more positive). The scatter in the perpendicular direction to w probably reflects a change in mass which produces the bands seen in Figure 1*b*. The gaps between different objects on the (κ_1, κ_2)-plane were first noted by BBFN97, but now we have quantified this feature by the parameter w. A full account of the role of the parameter w is beyond the scope of the present Letter.

4. DISCUSSION

This work is based on the hypothesis that self-gravitating, equilibrium stellar systems in general possess an extended dark matter halo. In order to describe their equilibrium state, a modified, two-component virial theorem, which predicts the existence of a fundamental surface, must be taken into account. We found remarkable compatibility of this hypothesis with the observed properties of a great range of stellar systems. Particularly, the cosmic metaplane, first discussed by BBFN97 as an ensemble of interrelated fundamental planes, is shown to reasonably follow the fundamental surface here derived.

Furthermore, our analysis reinforces the view that the FP relations should arise as a correction to the observed (luminous) parameter relations for the presence of the dark (unseen) matter surrounding these systems. However, as pointed out by Ciotti et al. (1996), this does not completely solve the FP problem, since a fine-tuning of the dark–to–luminous matter distributions is required in order to explain the small scatter of the FP cor-

relations. Although no such mechanism has been proposed or known up to now, there is at least one piece of evidence that it may exist, as evidenced by the extremely small scatter of the FP solutions displayed by the end products of hierarchical merger simulations discussed by Capelato et al. (1995, 1997). This would suggest that indeed the fine-tuning mechanism is related to the hierarchical scenario of formation of galaxies. Alternatively, an explanation for the FP, avoiding fine-tuning of any type, is given by our model, which includes a small curvature in the FP correlation. However, giving the clustering scale represented by elliptical galaxies, the scatter of the FP should be known with much higher accuracy.

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