Spontaneous decay rates in active waveguides

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We present a new method of measuring the guided, radiated, and total decay rates in uniform waveguides. It is also shown theoretically that large modifications of the total decay rate can be achieved in realistic erbium-doped fiber amplifiers and erbium-doped waveguide amplifiers with effective mode area radii smaller than ≈1 μm. © 2005 Optical Society of America

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It is well known that the spontaneous decay rates (SDRs) of emitting sources such as atoms or quantum wells can be greatly modified in optical microcavities.[1,2] Controlled spontaneous emission plays a key role in a new generation of micro-optic and nano-optic devices. High-performance microcavity lasers, for instance, have been experimentally demonstrated.[3,4] SDR modifications are also expected to affect the performance of optical waveguide amplifiers as the guided mode area radii of these devices become smaller than ≈1 μm.[5,6] To characterize one-dimensional nano- and micro-optic devices, a key decomposition of the total decay rate (τ_0^{-1}) into two components was introduced in Refs. 7 and 8. The decay rate was divided into guided modes (τ_g^{-1}) and radiated modes (τ_r^{-1}), where the total decay rate is $\tau_0^{-1}=\tau_g^{-1}+\tau_r^{-1}$. Unlike with large devices, where just the total decay rate must be considered for their characterization, the modeling of nano- and micro-optic devices requires the measurement of both components of the decay rate. The usual way to determine the SDR of an emitting source embedded in a uniform waveguide is to measure the exponential decay rate of the amplified spontaneous emission output power when the pump source is switched off.[9] The SDR is given by the exponential decay coefficient of the ASE output power.

Two natural questions arise here: (1) How can τ_g and τ_r be measured? (2) What is actually measured when one is using the classical method to determine the SDR? In this Letter we answer these two questions. We show that the classical method of measuring τ_0 in uniform waveguides gives τ_r if it is used with long waveguides and actually gives τ_0 in short waveguides (assuming no reflections at the waveguide ends). We also show how these measures are modified in lossy media, i.e., when a background loss coefficient is incorporated into the rate and propagation equations. Then we show how these ideas are useful for devices of practical interest. Three cases are considered: erbium-doped waveguide amplifiers (EDWAs), erbium-doped fiber amplifiers (EDFAs), and semiconductor optical amplifiers (SOAs).

We employ an analytical solution for the longitudinal z dependence of the rate equations presented in Ref. 6 to investigate measurement of the SDR in uniform waveguides. The analytical solution presented there is valid only for waveguides in which the excited-state population of the emitting source, $N_2(z)$, is constant along the fiber. Since the decay rate is measured when $N_2 \to 0$ along z, this approximation is valid when one is measuring τ_0. The rate equation is[5]

$$\frac{\partial \mathcal{N}_2(z,t)}{\partial t} = -\frac{\mathcal{N}_2(z,t)}{\tau_0} - \frac{1}{\rho S}\sum_{n=1}^{M}\{[(\alpha_n+\gamma_n)\times\mathcal{N}_2(z,t)-\alpha_n]P_n(z,t)\}, \quad (1)$$

where $\mathcal{N}_1+\mathcal{N}_2=1$ are the normalized population of the upper and lower levels of the emitting source, τ_0 is the spontaneous lifetime of the upper level, ρ is the number density of active ions, S is the doped region area, and α_n and γ_n are the absorption and gain constants. The propagation equation is

$$\frac{\partial P_n(z,t)}{\partial z} = u_n\{[(\alpha_n+\gamma_n)\mathcal{N}_2(z,t)-\alpha_n-\alpha_{\text{loss}}]\times P_n(z,t)+2\gamma_n\Delta\nu\mathcal{N}_2(z,t)\}, \quad (2)$$

where $P_n(z,t)$ is the optical power (in photons per unit time) at location z of the nth beam with the wavelength centered at λ_n ($n \leq M$), $u_n=1$ for forward-traveling beams and $u_n=-1$ for backward-traveling beams, α_{loss} is the attenuation coefficient given by the background loss of the fiber glass host, $\Delta\nu$ is the frequency interval between two successive wavelengths considered in the model, and the factor 2 in the last term stands for two possible polarizations. Solving Eqs. (1) and (2) for $\mathcal{N}_2(z,t)=\mathcal{N}_2(t)$, i.e., $\mathcal{N}_2$ constant along z, the output power is[6]

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$$P_n^{\text{out}}(t) = P_n^{\text{in}}(t)G_n(t) + 2\mathcal{N}_n^{\text{sp}}\Delta\nu[G_n(t) - 1], \quad (3)$$

where

$$G_n(t) = \exp[(\alpha_n + \gamma_n)\mathcal{N}_2(t)L - (\alpha_n - \alpha_{\text{loss}})L], \quad (4)$$

$$\mathcal{N}_n^{\text{sp}} = \frac{\gamma_n \mathcal{N}_2(t)}{(\alpha_n + \gamma_n)\mathcal{N}_2(t) - \alpha_n - \alpha_{\text{loss}}}. \quad (5)$$

The rate equation is

$$\frac{\mathrm{d}\mathcal{N}_2(t)}{\mathrm{d}t} = -\frac{\mathcal{N}_2(t)}{\tau_0} - \frac{1}{\rho S L}\sum_{n=1}^{M} [P_n^{\text{out}}(t) - P_n^{\text{in}}(t) - 2\gamma_n \Delta\nu \mathcal{N}_2(t)L + \alpha_{\text{loss}} H_n(t)L], \quad (6)$$

where

$$H_n(t) = \frac{P_n^{\text{in}}(t)}{\ln[G_n(t)]}[G_n(t) - 1] + 2\mathcal{N}_n^{\text{sp}}\Delta\nu \times \left\{\frac{G_n(t) - 1}{\ln[G_n(t)]} - 1\right\}. \quad (7)$$

Here $P_n^{\text{out}}(t) = P_n(L,t)$ and $P_n^{\text{in}}(t) = P_n(0,t)$ are the output and input powers of the nth beam, respectively; $G_n(t)$ is the linear gain; $\mathcal{N}_n^{\text{sp}}$ is the spontaneous-emission factor for the nth mode; and L is the doped fiber length.

In the classical method[7] of determining the SDR, the input power is turned off $[P_n^{\text{in}}(t) = 0]$ and the useful data are collected when the concentration of excited ions is low ($\mathcal{N}_2 \ll 1$). With these two conditions Eqs. (3) and (7) become

$$P_n^{\text{out}}(t) = \frac{2\gamma_n \Delta\nu \mathcal{N}_2(t)}{\alpha_n + \alpha_{\text{loss}}}\{1 - \exp[-(\alpha_n + \alpha_{\text{loss}})L]\}, \quad (8)$$

$$H_n(t) = -\frac{P_n^{\text{out}}(t)}{L(\alpha_n + \alpha_{\text{loss}})} + \frac{2\gamma_n \Delta\nu \mathcal{N}_2(t)}{\alpha_n + \alpha_{\text{loss}}}. \quad (9)$$

Using Eq. (9), we can rewrite Eq. (6) as

$$\frac{\mathrm{d}\mathcal{N}_2(t)}{\mathrm{d}t} = -\frac{\mathcal{N}_2(t)}{\tau_0} - \sum_{n=1}^{M}\left\{\frac{P_n^{\text{out}}(t)}{\rho S L} - \frac{2\gamma_n \Delta\nu \mathcal{N}_2(t)}{\rho S} - \beta_n\left[\frac{P_n^{\text{out}}(t)}{\rho S L} - \frac{2\gamma_n \Delta\nu \mathcal{N}_2(t)}{\rho S}\right]\right\}, \quad (10)$$

where we introduce the effective background loss coefficient for the nth mode $\beta_n = \alpha_{\text{loss}}/(\alpha_n + \alpha_{\text{loss}})$. Looking at Eq. (8), which is linear in $\mathcal{N}_2(t)$, we see that the right-hand side of Eq. (10) is also linear in $\mathcal{N}_2(t)$. Therefore it can be rewritten as $\mathrm{d}\mathcal{N}_2(t)/\mathrm{d}t = -\mathcal{N}_2(t)/\tau_m$, where τ_m is what is actually measured by the classical method and not τ_0. Since τ_m is quite cumbersome, we do not explicitly write it here. But two limiting cases deserve detailed study. Case 1 is short waveguides. In this case $(\alpha_n + \alpha_{\text{loss}})L \ll 1$. With this approximation Eq. (10) reduces to

$$\frac{\mathrm{d}\mathcal{N}_2(t)}{\mathrm{d}t} = -\frac{\mathcal{N}_2(t)}{\tau_0}. \quad (11)$$

This result shows that only for short waveguides does the classical method[7] furnish the total SDR of the ion. It is interesting to note that Eq. (11) is valid whether or not we have background loss ($\alpha_{\text{loss}} \neq 0$). Case 2 is long waveguides. Here $(\alpha_n + \alpha_{\text{loss}})L \gg 1$. Now Eq. (10) becomes

$$\frac{\mathrm{d}\mathcal{N}_2(t)}{\mathrm{d}t} = -\frac{\mathcal{N}_2(t)}{\tau_r} - \sum_{n=1}^{M}\beta_n \frac{\mathcal{N}_2(t)}{\tau_{g_n}}, \quad (12)$$

where we use the decomposition[7,8] of τ_0^{-1} in guided and radiated modes ($\tau_0^{-1} = \tau_g^{-1} + \tau_r^{-1}$) and the fact that $\tau_g^{-1} = \sum_{i=1}^{M}\tau_{g_n}^{-1}$, where $\tau_{g_n}^{-1} = 2\gamma_n \Delta\nu/\rho S$ is the guided decay rate into the nth mode. For sufficiently low background loss $\beta_n \approx 0$, which implies that the classical method now furnishes τ_r^{-1}.

The fact that τ_g^{-1} in a given mode can be written as $\tau_{g_n}^{-1} = 2\gamma_n \Delta\nu/\rho S$ is pointed out here for what is to our knowledge the first time. It arises naturally from the interpretation given in Ref. 6 for this term as being the photons captured by the guided modes per unit time. Unlike in Refs. 7 and 8, in which τ_g^{-1} was given as a function of the dipole moment matrix element between the emitting source's excited and ground states, here we use the easily measurable gain constant γ_n.

Therefore the total (τ_0^{-1}) and the radiated (τ_r^{-1}) decay rates in a given uniform waveguide can, in principle, be measured separately by employing two different waveguide lengths. τ_g^{-1} can also be determined by simply subtracting the latter from the former ($\tau_g^{-1} = \tau_0^{-1} - \tau_r^{-1}$). In any case, τ_g^{-1} is also easily obtained from the waveguide's intrinsic parameters, as discussed in the previous paragraph. We observe that in Subsection 2.B.3 of Ref. 5 the results presented above were outlined for $\alpha_{\text{loss}} = 0$. But here we explicitly perform the calculations and reinterpret these results in light of the decomposition of the decay rate into guided and radiated modes.

We end by studying the range of waveguide lengths at which τ_0^{-1} and τ_r^{-1} can be measured. We use three sets of parameters of practical interest. They represent typical EDFAs, EDWAs, and SOAs. It is worth mentioning that the term proportional to $\mathcal{N}_2^2$, which is usually included in modeling EDWAs and SOAs, can be neglected since $\mathcal{N}_2 \ll 1$.

In usual EDFAs and EDWAs, with optical mode areas larger than $\approx 1\ \mu\text{m}^2$, τ_g^{-1} is negligible and the total lifetime is equal to the radiated lifetime ($\tau_0 \approx \tau_r \approx 10$ ms). However, when the optical mode area becomes smaller, τ_g^{-1} starts to have a nonnegligible and measurable value. For instance, using the parameters shown in Table 1, a 1-μm optical mode radius, and Eq. (10), we determine the measured lifetime as a function of the waveguide length for typical EDFAs and EDWAs (assuming perfect detection). Without loss of generality, we use an effective gain and absorption constant along the total transition bandwidth of ≈ 15 THz centered at 1.55 μm. We can observe from Fig. 1 that a 1-μm optical mode radius is small enough to cause a variation of $\approx 5\%$ between the total and the radiated lifetimes, measured at small and large waveguide lengths, respectively. Although small, an optical mode radius of ≈ 1 μm is already commercially available in photonic crystal fibers. Moreover, it has been shown that waveguides with a high index contrast (and, consequently, small mode areas) have several advantages,[12] which envisages future construction of very small mode area devices. To study how the total and radiated decay rates can be measured in such devices, we perform simulations assuming the values shown in Table 1 for an EDWA but with an optical mode area of 0.02 μm^2, which was obtained, for instance, in Ref. 3. The results are shown in Fig. 2. Of course, such highly confined EDWAs would use materials that would not necessarily have the parameters shown in Table 1. But Fig. 2 illustrates the effects that would always occur at smaller mode areas: the difference between the total and the radiated lifetimes increases (as a consequence of larger τ_g^{-1}), the difference between the decay rates in lossy and nonlossy waveguides increases, and, at last, the waveguide lengths that are

Table 1. Parameters Used in the Simulations

Parameter	γ_{eff} (m^{-1})	α_{eff} (m^{-1})	τ_r (ms)	ρ (m^{-3})
EDFA[6]	0.2	0.2	10	1.0×10^{24}
EDWA[10]	20	49	22	1.4×10^{26}

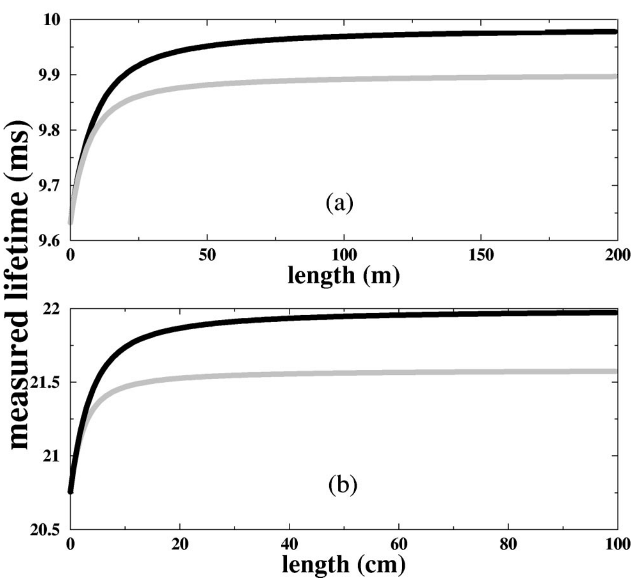

Fig. 1. Simulations of the measured lifetimes as a function of the waveguide length for typical (a) EDFAs and (b) EDWAs. Black curves, nonlossy waveguides; gray curves, $\alpha_{\mathrm{loss}}=0.3$ dB/m (EDFAs) (Ref. 11) and 1 dB/cm (EDWAs),[10] values typical of fluorozirconate EDFAs and silica-based EDWAs, respectively.

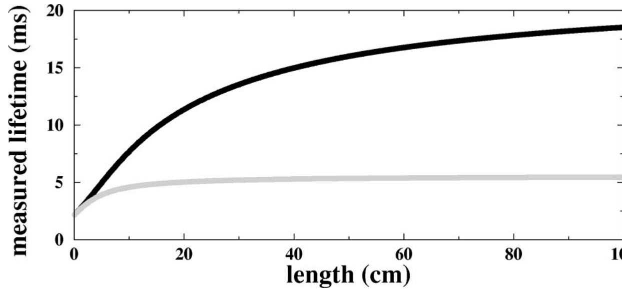

Fig. 2. Theoretical lifetimes as a function of the waveguide length for an EDWA with a 0.02-μm^2 optical mode area. Other parameters are given in Table 1.

necessary to obtain the radiated lifetime with a given accuracy also increase.

In SOAs the condition $\mathcal{N}_2+\mathcal{N}_1=1$ does not hold, but it is easy to show that the method of measuring the radiated and the total lifetime presented here also works. We observe that γ_n can be written as $\sigma_n^e\Gamma\rho/S$, where σ_n^e is the emission cross section at wavelength λ_n and Γ is the overlap factor between the optical mode and the doped region area.[13] Then, τ_g^{-1} is given by $\tau_g^{-1}=2\sigma_n^e\Gamma/S$. Using $\sigma_n^e\Gamma=2.0\times 10^{-16}\,\mathrm{cm}^2$ for a typical SOA,[14] we found that τ_g^{-1} becomes comparable with τ_r^{-1} of $\approx(200\ \mathrm{ps})^{-1}$ only at an optical mode radius smaller than ≈ 1 nm, which is far from possible with present technologies.

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