Novel FEM Approach for the Analysis of Cylindrically Symmetric Photonic Devices

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***Abstract*—A novel scheme based on a 2-D finite element method (2-D-FEM) for the frequency domain, in cylindrical coordinates in conjunction with the perfectly matched layers (PML), is proposed and validated here. This scheme permits the analysis and simulation of photonic devices, including discontinuities along the propagation direction. Also, the present approach takes into account the dispersive nature of metals at optical wavelengths.**

***Index Terms*—Discontinuities, finite element, focusing, lens, metamaterials, near-field, optical fibers, optics of metals, photonic devices.**

I. Introduction

WAVEGUIDE discontinuities junctions are basic and important structures for designing photonic devices and circuits. The finite element method (FEM) [1]–[4] is one of the powerful analysis methods for such discontinuity problems. In [1] and [2], powerful algorithms based on a 2-D finite element method (2-D-FEM) in rectangular coordinates without the need of a mode expansion technique for the analysis of waveguiding structures with open boundaries have been proposed. The scheme proposed in [1] is based on the use of a sequence of Padé approximation for modelling open boundaries at the input or output ports. Though essentially analogous to that 2-D-FEM scheme of [2], however, does not use PML in the open boundaries of the considered domain.

Here, the approach presented in [1] has been modified in conjunction with the use of port truncation by PML boundary condition [2] and extended to analyze discontinuity junctions with cylindrical symmetry by solving for the Helmholtz equation in cylindrical coordinates. Anisotropic perfectly matched layers (PMLs) are used to avoid reflections from the computational boundaries and numerical integration is used to compute the finite element matrices. All the above considerations reduce the computational effort and computation time.

To show the validity and usefulness of the present approach, numerical analysis for a fiber bragg gratings, fiber coupling losses due to an air gap and a metallo-dielectric lens that focus electromagnetic radiation in a spot smaller than the wavelength size are presented.

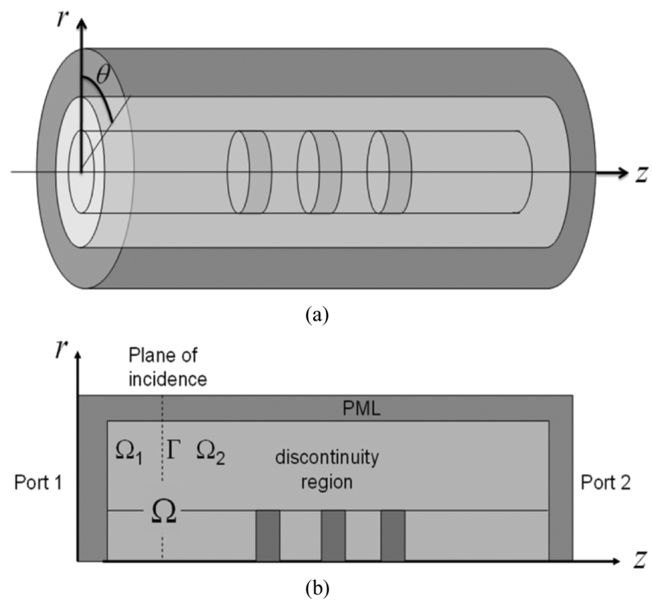

Fig. 1. (a) Schematic of the discontinuities in cylindrical coordinates with cylindrical symmetries and (b) framework for the 2-FEM analysis, where the junction or discontinuities is inside the region Ω_2.

II. Finite Element Formulation

We considered a general structure with cylindrical symmetry as shown in Fig. 1(a), where the computational domain can be considered on the plane $r - z$ due to the symmetry of the problem, see Fig. 1(b). There is no variation of the field along the θ coordinate $(d/d\theta = 0)$.

The 2-D scalar wave equation in frequency domain governing the linearly polarized (LP) modes on the plane $r - z$ is written as [3]

$$\frac{s_r}{r}\frac{\partial}{\partial r}\left(pr\frac{s_r}{s}\frac{\partial \phi}{\partial r}\right) + s_z\frac{\partial}{\partial z}\left(p\frac{s_z}{s}\frac{\partial \phi}{\partial z}\right) + k_0^2 qs\phi = 0 \quad (1)$$

where k_0 is the free-space wavenumber, s_r, s, and s_z are parameters related to PMLs [1], [2], adapted here for cylindrical

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coordinates, $p = 1$, $q = n^2$, $\phi = \phi_r$ the scalar field. We consider the incidence plane Γ, which is normal to the z axis, as shown in Fig. 1(b), and divide the domain Ω into two subdomains Ω_1, and Ω_2. To remove the singularity at $r = 0$ in (1), we multiplied (1) by r and applying the Galerkin procedure [4], we obtain

$$\int_\Omega \left[pr\frac{s_r^2}{s}\left(\frac{\partial\phi}{\partial r}\right)\left(\frac{\partial w}{\partial r}\right) + pr\frac{s_z^2}{s}\left(\frac{\partial\phi}{\partial z}\right)\left(\frac{\partial w}{\partial z}\right) - k_0^2 qsr\phi w\right] d\Omega = \int_\Gamma p\frac{s_z^2}{s} rw\frac{\partial\phi}{\partial z} d\Gamma \quad (2)$$

where w represents the weight functions. Following [2], we discretized the analysis region into quadratic triangular elements and the field within each element can be approximated as

$$\phi = [N]\{\phi\}_e \quad (3)$$

where $[N]$ is the shape function vector for the isoparametric triangular element and $\{\phi\}_\mathrm{e}$ is the nodal ϕ vector for each element. Next, considering $\phi = 0$ at the outer boundary of PML, and assembling the complete matrix for Ω, by adding the contributions of all different elements, except for the incidence plane, the boundary integral terms vanish owing to the continuity conditions of electric and magnetic fields, we obtain the following matrix equation:

$$[A]\{\phi\} = \sum_e{}' \int_e p\frac{s_z^2}{s} r\{N\}_\Gamma \left(\frac{\partial\phi_1}{\partial z} - \frac{\partial\phi_2}{\partial z}\right) d\Gamma \quad (4)$$

where [A] is the resulting matrix, assembled as

$$\begin{aligned}[A] = &\sum_e \int_{\Omega e} \left[p\frac{s_r^2}{s} r\frac{\partial\{N\}}{\partial r}\frac{\partial\{N\}^T}{\partial r}\right] drdz \\ &+ \sum_e \int_{\Omega e} \left[p\frac{s_z^2}{s} r\frac{\partial\{N\}}{\partial z}\frac{\partial\{N\}^T}{\partial z}\right] drdz \\ &- \sum_e \int_{\Omega e} \left[k_0^2 qsr\{N\}\{N\}^T\right] drdz \end{aligned} \quad (5)$$

where the components of vector $\{\phi\}$ are the values of ϕ over all nodes corresponding to Ω, the sum $\sum$ is performed over all the elements in Ω, and $\sum'$ is performed over the elements related to Γ. The field on the regions Ω_1, and Ω_2 are expressed as ϕ_1 and ϕ_2, respectively. $\{N\}_\Gamma$ are 1-D ring shape functions [3], $\{N\}$ is the 2-D shape function vector obtained by the discretization of the computational domain Ω. T denotes the transpose matrix.

The field in Ω is expressed as

$$\phi = \phi_{in} + \phi_\mathrm{scat} \quad (6)$$

where ϕ_{in} is the incident field and ϕ_scat is the scattering field.

The continuity condition of electric and magnetic fields on the incidence plane Γ is assumed then (4) is rewritten as

$$[A]\{\phi\} = \sum_e{}' \int_e p\frac{s_r^2}{s} r\{N\}_\Gamma \left(\frac{\partial\phi_{in,1}}{\partial z} - \frac{\partial\phi_{in,2}}{\partial z}\right) d\Gamma. \quad (7)$$

If incident fields $\phi_{in,1}$ and $\phi_{in,2}$ can be expressed as a linear combination of the eigenmodes of the input/output waveguides, then (7) becomes

$$[A]\{\phi\} = [Q]\{\psi\}_\Gamma \quad (8)$$

where

$$[Q] = \sum_e{}' \int_e pr\frac{s_r^2}{s}\{N\}\{N\}^T dy \quad (9)$$

$$\{\psi\}_\Gamma = 2\sum_m j\beta_m A_m\{f_m\}_\Gamma \quad (10)$$

A_m are the modal expansion's coefficients and $f_m(\mathrm{r})$ and β_m are the transverse field distribution and the propagation constant of the m-th mode, respectively, and $\{f_m\}_\Gamma$ is the transverse field vector on the incidence plane. 2-D numerical integrations [5] have been used to calculate the fundamental matrixes in order to open the possibility to treat problems with curved shaped geometries.

III. Numerical Results

In order to validate the proposed approach, a modulated refractive index fiber bragg grating (FBG), see Fig. 2(a), has been analyzed [6]. The grating is composed by 20 periods of a sinusoidal index change of the refractive index of the fiber core between 1.50 and 1.55 as shown in Fig. 2(b). The grating period is 0.5 μm and it starts at $z = 6$ μm, the number of periods was fixed to be 20, the cladding refractive index is fixed to $\mathrm{n_{cl}} = 1.45$ and the core radius, $a = 1$ μm. The incident field is the fundamental linearly polarized mode LP_{01}, computed at each wavelength by solving the transcendental equation for LP modes [6] and placed at the plane of incidence at $z = 1$ μm. The computational domain was -2 μm $< z <$ 20 μm and $0 < r < 6$ μm divided in 15705 elements (31850 nodes), where the PMLs are the outer 1 μm and 2 μm in r and z coordinates, respectively. The reflection coefficient is shown in Fig. 2(c) and it has been obtained by computing the ratio between the intensity of the field at the output and input ports placed at $z = 1$ μm and $z = 17$ μm, respectively. An excellent agreement with previously published results has been found [6].

Next, we have analyzed the effects of the air gap length between two identical single mode fibers in order to simulate a horizontal displacement, d, in an optical fiber junction. The fiber parameters are: $n_\mathrm{core} = 1.45$, $n_\mathrm{cladding} = 1.446$ and core radius $=$ 4 μm and the material between the fibers is air ($n = 1.0$). The computational domain was -2 μm $< z <$ 15 μm and $0 < r <$ 14 μm divided in 17667 elements (35680 points), where the PMLs are the outer 2 μm. The plane of incidence is placed at z = 1 μm and the input and output planes to calculate the reflection and transmission were placed at $z = 1$ μm and $z = 12$ μm, respectively. The air gap start at $z = 6$ μm. The incident field is the fundamental linearly polarized mode LP_{01}.

We compute the transmission and reflection coefficients for a range of wavelengths of interest in optical communications (1.30 μm $< \lambda <$ 1.65 μm) and we considered several gap lengths (0 μm $< d <$ 1.0 μm). The results can be seen in

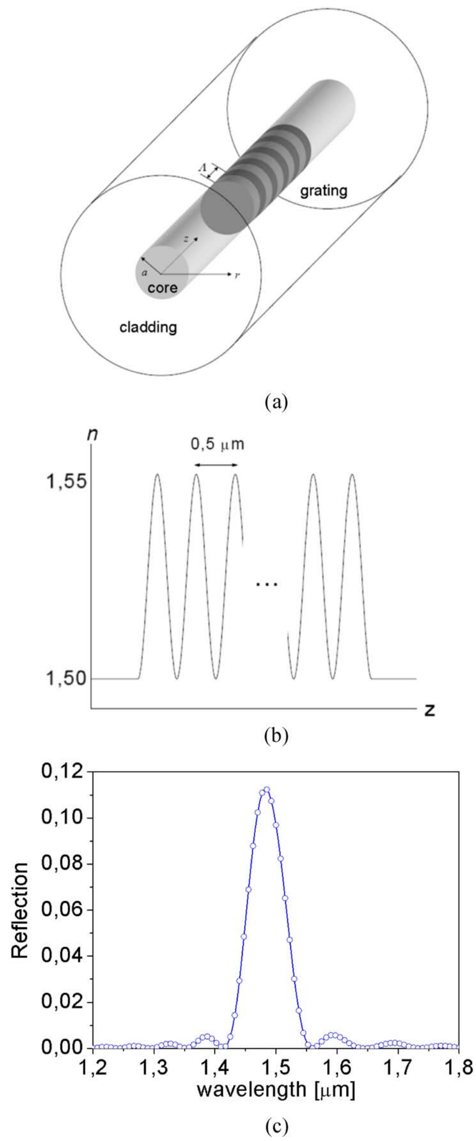

Fig. 2. (a) Schematic illustrating the configuration of the fiber Bragg grating (FBG). (b) Refractive index change in the core of the fiber and (c) reflected power as a function of the wavelength.

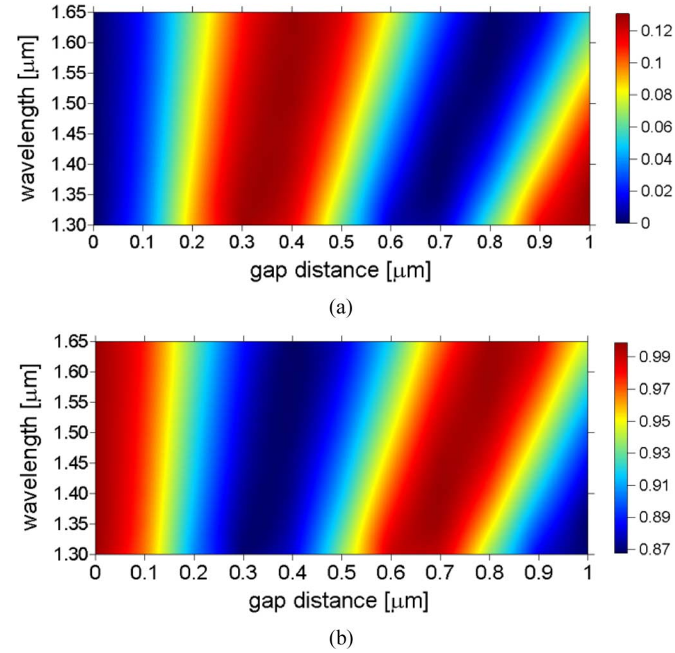

Fig. 3. (a) Reflected and (b) transmitted coefficients as a function of the wavelength and the gap distance between two fibers.

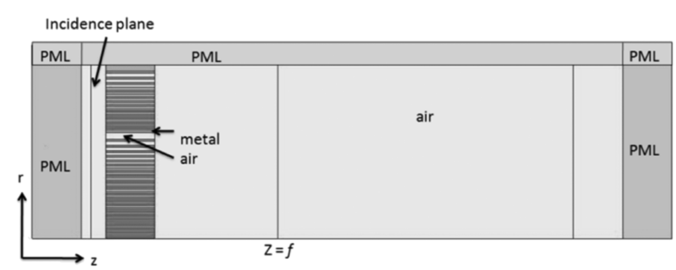

Fig. 4. Schematic of the near-field structure consisting of nano-slit array with different nanocapacitors and nanoinductors films using dielectric and plasmonic structures.

Fig. 3. Although analytical models [8], [9] predict a very smooth decaying of the transmission coefficient as a function of the gap length, they do not show the Fabry-Perot effect due to the cavity formed by the interface fiber-air-fiber as obtained by the present approach. By fixing the operation wavelength, and varying the gap length, we can observe that the reflection (or transmission) has a periodical variation with a period equal to $\lambda/2$, then we can have maximums and minimums for both coefficients depending on the gap length. They can be explained by the fabry perot effect due to cavity formed by the two ends of the fiber.

As a third example, we analyze here the focusing of light by using a nano-slit lens or also called Near-Field Focusing Plate [12], [13]. At optical frequencies, this structure can be implemented by alternating nanocapacitors and nanoinductors which can be obtained by using dielectric and plasmonic structures, respectively [11]. This structure, in its rectangular counterpart case, has been analyzed in [10] and it focuses the light in the transversal direction because of its 2-D nature. In here, we consider an analog geometry with cylindrical symmetry obtained by considering the revolution of the upper half of the original device.

The lens is composed by concentrically disposed rings of silver and air with the same dimensions as in [10]. Here, we exchanged the vertical coordinate (x) for the radial coordinate (r) as can be seen in the structure's scheme of Fig. 4. The wavelength of the incident light is 650 nm and the relative permittivity used for silver at this wavelength is $\varepsilon_{\mathrm{r}} = -17.36 - j0.715$. This geometry will focus the light in the three dimensions.

The computational domain was $-0.5\ \mu\mathrm{m} < z < 6\ \mu\mathrm{m}$ and $0 < r < 2\ \mu\mathrm{m}$ divided in 22558 elements (45591 points), where the PMLs are the outer 0.2 μm and 0.2 μm in r and z coordinates, respectively. The incident field is a plane wave and the plane of incidence is placed at $z = 0.1\ \mu$m. The field intensity has been monitored at $z = 1.67\ \mu$m. The near-field focusing structure starts at $z = 0.25\ \mu$m and it has a length of 0.5 μm in the propagation direction.

From Fig. 5, we can observe the lens behaviour of the analyzed geometry where the input light focuses at $z = 1.67\ \mu$m.

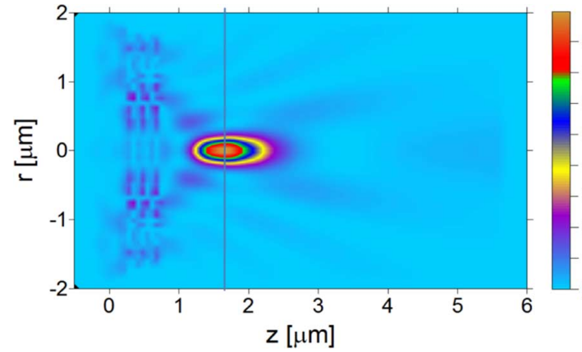

Fig. 5. Field Intensity for the Near-Field Focusing structure of the metallic nano-slits array. The focus is formed at $z = 1.67\ \mu$m or 2.57λ.

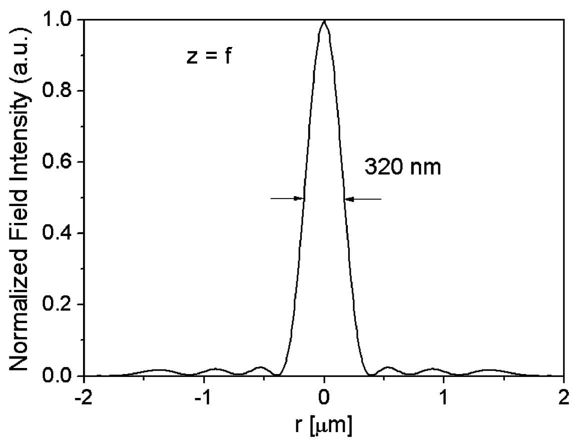

Fig. 6. Normalized field intensity at $z = 1.67\ \mu$m.

The normalized field intensity at the focus distance, shown in Fig. 6, exhibits a full-width at half-maximum (FWHM) of 320 nm or $\lambda/2$. By designing judiciously the lens' geometry a further FWHM's reduction is possible, due to the presence of surface plasmon polaritons. Therefore, the present system is capable to exhibit sub-wavelength resolution.

IV. Conclusion

In conclusion, three numerical examples with cylindrical symmetry have been presented to show the validity and usefulness of the proposed approach. We have analyzed the reflection and transmission characteristics of two discontinuities in optical fibers: one for a fiber bragg grating (FBG) were the refractive index contrast is small and another for the air gap between two fibers where the refractive index contrast is relatively high in the discontinuity region. Additionally we analyzed the near-field focusing of a cylindrical lens composed by nano-slits in a metallo-dielectric device. The obtained results are in good agreement with the previously published ones.

The nonlinear formulation of the present scheme and other specific design of Near-field focusing structures applied for coupling light to optical fibers are under analysis and results will be report in a near future.

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