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Chaos in the parallel sheared plasma flow driven electromagnetic turbulence in nonuniform magnetoplasmas

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By employing the two-fluid model, a system of nonlinear equations for low-frequency electromagnetic waves in nonuniform collisional magnetoplasmas has been derived. The plasma contains both the equilibrium density gradient and sheared flows. In the linear limit, a local dispersion relation has been obtained and analyzed in several interesting limiting cases. It is found that equilibrium sheared plasma flows cause instabilities of Alfvén-type waves even in the absence of the density gradient. The numerical results also show a large growth rate of electromagnetic parallel velocity shear (PVS) mode compared to the electrostatic mode for some ionospheric parameters. For this case, the temporal nonlinear behavior of the relevant governing mode coupling equations is governed by six coupled equations, which are a generalization of the Lorenz–Stenflo equations and which admit chaotic trajectories. The results of this investigation should be useful for understanding the linear and nonlinear properties of electromagnetic waves that are generated by sheared plasma flows in magnetized plasmas. © *1999 American Institute of Physics.*
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I. INTRODUCTION

There has been a renewed interest in the study of the parallel ion velocity shear (PIVS) instability, which has been observed in some recent laboratory experiments[1–3] that exhibit nearly sonic flow of the ions along the magnetic field lines at the edge and also in the scrape off layer of a tokamak device. Strong radial gradients of the parallel flow in the vicinity of a poloidal shear layer have also been detected.[4] Similarly, in naturally occurring plasmas, equilibrium plasma flow velocities are found to be spatially inhomogeneous in a direction perpendicular to the ambient magnetic field lines of force. Thus, the velocity shear can have both parallel and perpendicular components and either of these may excite electrostatic waves. For example, when equilibrium plasma flows have a gradient perpendicular to the ambient magnetic field, the excited modes are of the Kelvin–Helmholtz-type.[5] On the other hand, when the magnetic field-aligned ion plasma flows are sheared, we have the possibility of exciting shorter scale electrostatic waves.[6] These results suggest that the PIVS modes could be one of the potential candidates for explaining the salient features of anomalous transports that are caused by saturated electrostatic turbulence in laboratory and space plasmas.

In the past, several authors[6–8] have presented a detailed instability analysis of low-frequency (in comparison with the ion gyrofrequency ω_{ci}) electrostatic waves in fully ionized collisionless magnetoplasmas in the presence of the ion velocity gradient. The latter produces a phase lag between the magnetic field-aligned ion flow velocity perturbation and the wave potential, which lead to an instability. Three-dimensional simulations of this parallel ion velocity shear instability (PIVS) have been recently carried out by McCarthy *et al.*[9] in order to understand the nonlinear mode structure and the anomalous momentum transport in tokamak edges.[4] Furthermore, it has also been shown[10–13] that the PIVS can also cause the instability of electrostatic waves in partially ionized collisional magnetoplasmas. Experimental observations[12] verify the theoretical predictions[11,12] of the collisional parallel velocity shear (PVS) instability.

However, in most of the laboratory and space plasmas, the plasma beta ($\beta=8\pi n_0 T/B_0^2$, where n_0 is the plasma number density and B_0 the strength of the ambient magnetic field) could exceed the electron to ion mass ratio. Accordingly, it becomes necessary to incorporate the electromagnetic effects on the PIVS modes.

In this paper, we present an investigation of low-frequency (in comparison with ω_{ci}) electromagnetic waves in the presence of equilibrium sheared plasma flows in nonuniform magnetoplasmas. The two-fluid model is used to derive a set of nonlinear fluid equations. The latter include the continuity and parallel momentum equations for the electron and ion fluids, in which the parallel component of the drift fluid velocities are inserted. In the linear limit, we obtain a local dispersion relation which admits a new electromagnetic instability. On the other hand, the newly derived nonlinear equations can be written as a set of six coupled mode equations, representing a generalization of Lorenz[14] and Stenflo[15] equations. The latter admit chaotic trajectories.

The manuscript is organized in the following fashion: In

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Sec. II, we present a derivation of the nonlinear mode coupling equations. Section III contains local linear dispersion relations which predict electromagnetic instability both in collision-dominated and collisionless regimes. The chaotic behavior of nonlinearly interacting finite amplitude electromagnetic waves is discussed in Sec. IV. A brief summary and conclusions are presented in Sec. V.

II. DERIVATION OF NONLINEAR PVS MODE EQUATIONS

We consider the nonlinear propagation of low-frequency ($\omega \ll \omega_{ci} = eB_0/m_i c$) electromagnetic waves in nonuniform magnetoplasmas which have the equilibrium density gradient $\partial n_{j0}/\partial x$ and the equilibrium velocity gradient $\partial v_{j0}/\partial x$. Here, e is the magnitude of the electron charge, B_0 is the strength of the external magnetic field which is directed along the z-axis, m_i the ion mass, c the speed of light, $n_{j0}(x)$ the unperturbed number density of the particle species j (j equals e for the electrons and i for the ions), and $v_{j0}(x)$ the magnetic field-aligned unperturbed plasma flow velocity. Both the equilibrium density and velocity gradients are along the x-axis, which is transverse to $B_0\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector along the z-axis.

We assume that the difference between the electron and ion equilibrium drifts lead to an equilibrium plasma current, which produces a small shear component of the equilibrium magnetic field. The latter, however, can be neglected for laboratory and space applications, as the main component of the magnetic field is a thousand times larger than the sheared equilibrium magnetic field component. Furthermore, the plasma is assumed to be isothermal and that the equilibrium is maintained by some external sources.

In the low-frequency electromagnetic fields, the electron and ion fluid velocity perturbations,[13–16] are

$$\mathbf{v}_e \approx \mathbf{v}_{EB} + \mathbf{v}_{De} + (v_{e0} + v_{ez})\mathbf{B}_\perp / B_0 + \hat{\mathbf{z}} v_{ez} \tag{1}$$

and

$$\mathbf{v}_i \approx \mathbf{v}_{EB} + (v_{i0} + v_{iz})\mathbf{B}_\perp / B_0 + \hat{\mathbf{z}} v_{iz} + \mathbf{v}_{pi}, \tag{2}$$

where $\mathbf{v}_{EB} = (c/B_0)\hat{\mathbf{z}} \times \nabla \phi$ and $\mathbf{v}_{De} = -(cT_e/eB_0 n_e)\hat{\mathbf{z}} \times \nabla n_e$ are the usual $\mathbf{E}\times\mathbf{B}_0$ and $\mathbf{v}_{De}$ the electron diamagnetic drift velocities, respectively, and $\mathbf{v}_{pi} = -(c/B_0\omega_{ci})(\partial_t + \nu_i + v_{i0}\partial_z + \mathbf{v}_{EB}\cdot\nabla + v_{iz}\partial_z)\nabla_\perp \phi$ the ion polarization drift velocity, T_e the electron temperature, n_e the electron number density, v_{iz} the parallel component of the ion velocity perturbation, ν_i the ion collision frequency, and the z-component of the electron fluid velocity perturbation v_{ez} is obtained from Ampère's law, yielding

$$n_e v_{ez} = n_i v_{iz} + \frac{c}{4\pi e}\nabla_\perp^2 A_z. \tag{3}$$

Furthermore, n_i is the ion number density, $\mathbf{E} = -\nabla \phi - c^{-1}\partial_t A_z \hat{\mathbf{z}}$ and $\mathbf{B}_\perp = \nabla A_z \times \hat{\mathbf{z}}$ are the electric and magnetic field vectors, where ϕ and A_z are the scalar and the z-component of the vector potentials, respectively. The compressional magnetic field perturbation has been neglected in view of the low-β approximation (viz., $m_e/m_i \ll \beta \ll 1$), where m_e is the electron mass. For simplicity, the ions are assumed to be cold.

Substituting for the z-component of the electric field into the parallel component of the electron momentum equation and using (1), we obtain

$$(\mathcal{L}_t^e + \mathbf{v}_{De0}\cdot\nabla - \eta_e \nabla_\perp^2)A_z + c(\partial_z + \mathbf{S}_{v0}^e\cdot\nabla)\phi - \frac{cT_{e0}}{en_{e0}}\mathcal{L}_z n_{e1} = 0, \tag{4}$$

where $\mathcal{L}_t^j \equiv \partial_t + \mathbf{v}_{EB}\cdot\nabla + v_{jz}\partial_z$, $\mathcal{L}_z \equiv \partial_z + (1/B_0)\nabla A_z \times \hat{\mathbf{z}}\cdot\nabla$, n_{e1} $[= n_e - n_0(x) \ll n_e]$ is the electron number density perturbation, $\eta_e \equiv 0.51\nu_{ei}\lambda_e^2$ is the plasma resistivity, ν_{ei} the electron–ion collision frequency, $\lambda_e = c/\omega_{pe}$ the collisionless electron skin depth, ω_{pe} the electron plasma frequency, $\mathbf{S}_{v0}^j = \hat{\mathbf{z}} \times \nabla v_{j0}/\omega_{cj}$, and ω_{cj} the gyrofrequency of the particle species j.

Similarly, the z-component of the ion momentum equation can be written as

$$(\mathcal{L}_t^i + \nu_i)v_{iz} \simeq -\frac{e}{m_i}\left[(\partial_z + \mathbf{S}_{v0}^i\cdot\nabla)\phi + \frac{1}{c}\mathcal{L}_t^i A_z\right]. \tag{5}$$

The remaining nonlinear equations for the electrons and the ions in the presence of electromagnetic fields can be obtained by substituting (1) and (2) into the electron and ion continuity equations. We have

$$(\mathcal{L}_t^e + v_{i0}\partial_z - D_e\nabla_\perp^2)n_{e1} - \frac{c}{B_0}\hat{\mathbf{z}}\times\nabla n_{e0}\cdot\nabla\phi - \frac{1}{eB_0}\hat{\mathbf{z}} \times \nabla J_{e0}\cdot\nabla A_z = -n_{e0}\mathcal{L}_z\left(v_{iz} + \frac{c}{4\pi e n_{e0}}\nabla_\perp^2 A_z\right) \tag{6}$$

and

$$(\mathcal{L}_t^i - D_i\nabla_\perp^2)n_{i1} - \frac{c}{B_0}\hat{\mathbf{z}}\times\nabla n_{i0}\cdot\nabla\phi - \frac{cn_{i0}}{B_0\omega_{ci}}(\mathcal{L}_t^i + \nu_i)\nabla_\perp^2\phi = -\frac{1}{eB_0}\hat{\mathbf{z}}\times\nabla J_{i0}\cdot\nabla A_z - \mathcal{L}_z(n_i v_{iz}), \tag{7}$$

where $J_{j0} = q_j n_{j0} v_{j0}$ is the unperturbed plasma current density, $n_{i1}[= n_i - n_0(x) \ll n_0(x)]$ is the ion number density perturbation, and $q_i = e$, and $q_e = -e$.

Equations (3)–(7) with $n_{e1} = n_{i1} \equiv n_1$ are the desired nonlinear equations for electromagnetic waves in nonuniform collisional magnetoplasmas with equilibrium density gradient and sheared plasma flows.

III. LINEAR DISPERSION RELATION

In this section, we present the local dispersion relation for electromagnetic modes by neglecting the nonlinear terms and assuming that the perturbation wavelength is much smaller than the velocity and density gradient scale-lengths. Equations (3)–(7) are then Fourier transformed by assuming that all the perturbed quantities are proportional to exp($i\mathbf{k}\cdot\mathbf{r}$

$-i\omega t)$, where $\mathbf{k}$ and ω are the wave vector and the frequency, respectively. Equations (3)–(6) yield, respectively,

$$n_1=\frac{en_0}{T_{e0}}\left[S_{*e}\phi-\frac{1}{ck_z}(\Omega_e-\mathbf{k}\cdot\mathbf{v}_{De0}+i\eta_e k_\perp^2)A_z\right], \tag{8}$$

$$(\Omega_i+i\nu_{in})v_{iz}\simeq\frac{ek_z}{m_i}\left[S_{*i}\phi-\frac{\Omega_i}{ck_z}A_z\right], \tag{9}$$

$$(\Omega_e-k_z v_{i0}+iD_e k_\perp^2)n_1+\frac{ck_y d_x n_0}{B_0}\phi$$
$$+\left(\frac{ck_z k_\perp^2}{4\pi e}+\frac{k_y d_x J_{e0}}{eB_0}\right)A_z-n_0 k_z v_{iz}=0, \tag{10}$$

and

$$(\Omega_i+iD_i k_\perp^2)n_1-n_0 k_z v_{iz}-\frac{k_y d_x J_{i0}}{eB_0}A_z$$
$$=-\left[\frac{ck_y d_x n_0}{B_0}+\frac{c^2k_\perp^2}{4\pi e v_A^2}(\Omega_i+i\nu_{in})\right]\phi, \tag{11}$$

where $\Omega_j\equiv\omega-k_z v_{j0}$ and $S_{*j}=(1+\mathbf{k}\cdot\mathbf{S}_{v0}^j/k_z)$.

Subtracting (11) from (10) and using (8) and (9), we obtain

$$\left[\{i(D_i-D_e)k_\perp^2+k_z v_{e0}\}S_{*e}+\frac{c^2k_\perp^2\lambda_{De}^2}{v_A^2}(\Omega_i+i\nu_i)\right]\phi$$
$$=\left[\{i(D_i-D_e)k_\perp^2+k_z v_{e0}\}(\Omega_e-\mathbf{k}\cdot\mathbf{v}_{De0}+ik_\perp^2\eta_e)\right.$$
$$\left.+k_z^2c^2k_\perp^2\lambda_{De}^2+\frac{k_y k_z c_s^2 d_x J_{ie}}{en_0\omega_{ci}}\right]\frac{A_z}{k_z c}, \tag{12}$$

where $J_{ie}\equiv J_{i0}+J_{e0}$, $v_A=B_0/(4\pi n_0 m_i)^{1/2}$, and $c_s=(T_e/m_i)^{1/2}$ are equilibrium current, the Alfvén and ion acoustic velocities, respectively. Equations (8)–(10) yield the following result:

$$\left[\{\Omega_e-k_z v_{i0}+ik_\perp^2 D_e\}S_{*e}+\frac{k_y c_s^2}{L_n\omega_{ci}}-\frac{c_s^2k_z^2}{(\Omega_i+i\nu_{in})}S_{*i}\right]\phi$$
$$=\left[(\Omega_e-k_z v_{i0}+ik_\perp^2 D_e)(\Omega_e-\mathbf{k}\cdot\mathbf{v}_{De0}+i\eta_e k_\perp^2)\right.$$
$$\left.-k_z^2c^2k_\perp^2\lambda_{De}^2-\frac{k_y k_z c_s^2 d_x J_{e0}}{en_0\omega_{ci}}-\frac{k_z^2c_s^2\Omega_i}{(\Omega_i+i\nu_{in})}\right]\frac{A_z}{k_z c}. \tag{13}$$

Equations (12) and (13) are two coupled equations in ϕ and A_z which yield the following dispersion relation:

$$\left[\{i(D_i-D_e)k_\perp^2+k_z v_{e0}\}S_{*e}+\frac{c^2k_\perp^2\lambda_{De}^2}{v_A^2}(\Omega_i+i\nu_{in})\right]$$
$$\times\left[(\Omega_e-k_z v_{i0}+ik_\perp^2 D_e)(\Omega_e-\mathbf{k}\cdot\mathbf{v}_{De0}+i\eta_e k_\perp^2)\right.$$
$$\left.-k_z^2c^2k_\perp^2\lambda_{De}^2-\frac{k_y k_z c_s^2 d_x J_{e0}}{en_0\omega_{ci}}-\frac{k_z^2c_s^2\Omega_i}{(\Omega_i+i\nu_{in})}\right]$$
$$=\left[\{i(D_i-D_e)k_\perp^2+k_z v_{e0}\}(\Omega_e-\mathbf{k}\cdot\mathbf{v}_{De0}+ik_\perp^2\eta_e)\right.$$
$$\left.+k_z^2c^2k_\perp^2\lambda_{De}^2+\frac{k_y k_z c_s^2 d_x J_{ie}}{en_0\omega_{ci}}\right]\left[\{\Omega_e-k_z v_{i0}\right.$$
$$\left.+ik_\perp^2 D_e\}S_{*e}+\frac{k_y c_s^2}{L_n\omega_{ci}}-\frac{k_z^2c_s^2}{(\Omega_i+i\nu_{in})}S_{*i}\right]. \tag{14}$$

We now discuss analytical solutions of (14) for two cases. First, for homogeneous and collisionless plasmas with $v_{i0}\gg v_{e0}$, Eq. (14) takes the form

$$\Omega_i^2\left[\omega\Omega_i-k_z^2c^2k_\perp^2\lambda_{De}^2+k_z^2c_s^2-\frac{k_y k_z c_s^2 d_x J_{e0}}{en_0\omega_{ci}}\right]$$
$$=k_z^2v_A^2\delta_0\left[\Omega_i^2+\Omega_i\frac{k_y c_s^2}{L_n\omega_{ci}}+k_z^2c_s^2\left(\frac{k_y d_x v_{i0}}{k_z\omega_{ci}}-1\right)\right], \tag{15}$$

where $\delta_0=1+c_s^2k_y d_x J_{i0}/(en_0\omega_{ci}k_z c^2k_\perp^2\lambda_{De}^2)$. Equation (15) predicts an instability.

Second, in a collisional-dominated plasma without the electron diffusion and electron shear flows, we readily obtain from (14)

$$\Omega_i=\frac{1}{\delta_1}\left[-\frac{\delta_0k_z^2v_A^2k_n k_y c_s^2}{\omega_{ci}\nu_{in}}-i\nu_{in}k_z^2c^2k_\perp^2\lambda_{De}^2\right.$$
$$\left.+i\frac{k_z^4v_A^2c_s^2\delta_0}{\nu_{in}}\left(\frac{k_y d_x v_{i0}}{k_z\omega_{ci}}-1\right)\right], \tag{16}$$

where $\delta_1=\nu_{in}\nu_{ei}k_\perp^2\lambda_e^2+k_z^2[v_A^2\delta_0+c_s^2]$ and $k_n=1/L_n$. For $k_\perp^2\lambda_{De}^2\ll 1$, Eq. (16) admits an oscillatory instability in collisional plasmas with parallel ion velocity gradient whose increment for $\partial v_{i0}/\partial x=d_x v_{i0}>k_z\omega_{ci}/k_y$ is

$$\gamma=\frac{k_z^4v_A^2c_s^2\delta_0}{\delta_1\nu_{in}}\left|\left(\frac{k_y d_x v_{i0}}{k_z\omega_{ci}}-1\right)\right|. \tag{17}$$

In order to estimate the growth rate of PVS mode, we have solved Eq. (14) numerically by choosing some typical parameters[10] of Earth's auroral F-region at an altitude of 350 km. The magnetic field strength $B_0\simeq 0.4$ G, the electron number density $n_{e0}\simeq 2\times 10^{11}\,\mathrm{m}^{-3}$, the neutral number density $n_{n0}\simeq 2.5\times 10^{14}\,\mathrm{m}^{-3}$, the ion temperature $T_i\simeq 4000$ K, and the electron temperature $T_e\simeq 2T_i$. Hence, we obtain for the electron thermal velocity $v_{te}\simeq 5\times 10^5$ m/s, the ion thermal velocity $v_{ti}\simeq 2\times 10^3$ m/s, the ion sound velocity $c_s\simeq 2\times 10^3$ m/s, the ion gyroradius $\rho_s\simeq 8.5$ m, $\nu_{ii}/\omega_{ci}\simeq 7.3\times 10^{-4}$, $\nu_{ei}/\omega_{ce}\simeq 2.2\times 10^{-6}$, $\nu_{in}/\omega_{ci}\simeq 1.25\times 10^{-3}$, $\nu_{en}/\omega_{ce}\simeq 1.6\times 10^{-6}$, the plasma $\beta\simeq 3.5\times 10^{-5}$ with the ion-neutral collision frequency $\nu_{in}=0.3$. The normalized growth rate γ/ν_{in} of PVS mode as a function of $k_\perp\rho_s$ is shown in

Fig. 1 for a fixed value of $k_\parallel c_s/\nu_{in}=0.2$ by ignoring electron and ion diffusion terms and for different values of the sheared parameter $\alpha_s=|\partial v_0/\partial x|/\nu_{in}$. Our numerical results show that the sheared-driven flow (with positive gradient) always destabilizes the plasma for L_n (10 m–100 km). The growth rate of the present electromagnetic instability is found to be larger than the electrostatic mode.[10] It may be noted here that these results are only valid for long perturbation wavelength in which $k_\perp \rho_s<1$. Therefore, for short wavelength modes (in comparison with ρ_s), one has to use kinetic treatment for the ions.

IV. CHAOTIC BEHAVIOR OF ELECTROMAGNETIC TURBULENCE

In order to study the temporal behavior of nonlinearly interacting finite amplitude two-dimensional electromagnetic waves in collisional magnetoplasmas without the density gradient, we follow the approach of Lorenz[14] and Stenflo,[15] and look for solutions having the following form

$$\phi=\phi_1(t)\sin(K_x x)\sin(K_y y), \tag{18}$$

$$n=n_1(t)\sin(K_x x)\sin(K_y y), \tag{19}$$

$$A_z=A_1(t)\sin(K_x x)\cos(K_y y)-A_2(t)\sin(2K_x x), \tag{20}$$

and

$$v_z=v_1(t)\sin(K_x x)\cos(K_y y)-v_2(t)\sin(2K_x x), \tag{21}$$

where K_x and K_y are constant parameters, and ϕ_1, n_1, A_1, A_2, v_1, and v_2 are some time-dependent amplitudes.

Substituting (18)–(21) into (4)–(7), we readily obtain

$$(\dot{\phi}_1+\nu_i\phi_1)\alpha_3 K^2=K^2 n_1+\alpha_1 K_y A_1 - \delta_2(K^2-4K_x^2)K_xK_yA_1A_2, \tag{22}$$

$$\dot{A}_1=-\eta_e K^2 A_1+\beta_0 K_y\phi_1-\beta_1 K_xK_y n_1 A_2 + \frac{c}{B_0}K_xK_yA_2\phi_1, \tag{23}$$

$$\dot{A}_2=-4\eta_e K_x^2 A_2-\frac{c}{2B_0}K_xK_y\phi_1A_1+\frac{\beta_1}{2}K_xK_y n_1 A_1, \tag{24}$$

$$\dot{v}_1=-\nu_{in}v_1+\frac{cK_y^2 d_x v_{i0}}{B_0}\phi_1+\frac{cK_xK_y}{B_0}\phi_1 V_2 + \frac{e}{m_iB_0}K_xK_yA_2\phi_1-\frac{e}{m_ic}\dot{A}_1, \tag{25}$$

$$\dot{v}_2=-\nu_{in}v_2-\frac{e}{m_ic}\dot{A}_2-\frac{c}{2B_0}K_xK_y\phi_1 v_1 - \frac{e}{2m_iB_0}K_xK_y\phi_1A_1, \tag{26}$$

and

$$\dot{n}_1=-D_eK^2n_1+\frac{c(K^2-4K_x^2)K_xK_y}{4\pi eB_0}A_1A_2 - n_0K_xd_x\left(\frac{v_{i0}}{B_0}\right)A_1, \tag{27}$$

where $\alpha_1=d_xJ_{ie}/[eB_0(D_e-D_i)]$, $\alpha_2=c/[4\pi eB_0(D_e-D_i)]$, $\alpha_3=cn_{i0}/B_0\omega_{ci}(D_e-D_i)$, $\beta_0=c(d_xv_{e0})/\omega_{ce}$, $\beta_1=(cT_{e0}/en_{e0}B_0)$, and $\eta_e=0.51\nu_{ei}\lambda_e^2$. The time derivative is defined by a dot on ϕ_1, A_1 and A_2. We note that the terms proportional to $\sin(3K_xx)$ have been dropped in the derivation of (22)–(27).

Equations (22)–(27) can be appropriately normalized so that they can be put in a form which is similar to that of Lorenz,[14] Stenflo,[15] and Mirza and Shukla.[17] We have the following 6×6 matrix:

$$\begin{pmatrix} d_\tau X\\ d_\tau Y\\ d_\tau Z\\ d_\tau V\\ d_\tau U\\ d_\tau W\end{pmatrix}=\begin{pmatrix} -\sigma_0 & \sigma_0 & s_0Y & 0 & s_1 & 0\\ r & -1 & -X & 0 & -s_2Z & 0\\ Y & 0 & -b & 0 & s_3Y & 0\\ -1 & b_1 & s_4U & -\sigma_2 & 0 & s_5X\\ 0 & \sigma_1 & s_1Y & 0 & -\sigma_1 & 0\\ -s_7Y & 0 & s_6 & Y & 0 & -\sigma_2\end{pmatrix} \times\begin{pmatrix} X\\ Y\\ Z\\ V\\ U\\ W\end{pmatrix}, \tag{28}$$

which describes the nonlinear coupling between various amplitudes. Here, $\sigma_0=\nu_{ei}/\eta_eK^2$, $\sigma_1=D_e/\eta_e$, $\sigma_2=\nu_{in}/\eta_eK^2$, $r=\beta_0k_ya_1/\eta_eK^2$, $b=4K_x^2/K^2$, $s_0=\alpha_1K_ya_2/a_1\alpha_3\eta_eK^4$, $s_1=a_4/a_1\alpha_3\eta_eK^2$, $s_2=\beta_1K_xK_ya_3a_4/a_2K^2\eta_e$, $s_3=\beta_1K_xK_ya_2a_4/2K^2a_3\eta_e$, $s_4=e\beta_1K_xK_ya_3a_4/cm_iK^2a_5\eta_e$, $s_5=cK_xK_ya_1a_6/B_0K^2a_5\eta_e$, $s_6=4eK_x^2a_3/m_icK^2a_6$, $s_7=cK_xK_ya_1a_5/2K^2\eta_eB_0a_6$, with $K^2=K_x^2+K_y^2$ and $\tau=t/t_0$, where $t_0=\eta_eK^2$.

Next, if we take $K_y^2=4K_x^2$, (28) then reduces to the Lorenz and Stenflo-type equations. However, the normalizations used here are

$$\begin{aligned} \phi_1&=a_1X=\pm\frac{\sqrt{2}\,\eta_eK^2B_0}{cK_xK_y}X,\\ A_1&=a_2Y=\pm\frac{\sqrt{2}\,\eta_e\nu_{in}\alpha_3K^4B_0}{c\alpha_1K_xK_y^2}Y,\\ A_2&=a_3Z=-\frac{\nu_{ei}\eta_eK^4B_0\alpha_3}{c\alpha_1K_xK_y^2}Z,\\ n_1&=a_4U=\pm\frac{\sqrt{2}\,\nu_{ei}\eta_e\alpha_3K^2d_xJ_{ie}}{ce\alpha_1K_y^2D_e}U, \end{aligned} \tag{29}$$

$$V_1=a_5V=\mp\frac{\sqrt{2}B_0d_xv_{i0}}{K_y}V,$$

and

$$V_2=a_6W=\frac{\beta_1\nu_{ei}^2\eta_e\alpha_3^2K^4B_0d_xJ_{i0}}{m_ic^3\alpha_1^2K_y^3D_e}W.$$

Equations (28) are the generalized Lorenz–Stenflo equations, whose properties can be studied both analytically, as well as numerically by means of standard techniques.[18] The equilibrium points of (28) can be obtained by setting time derivative terms equal to zero and solving this nonlinear set of coupled equations. The 3×3 matrix case has been studied in some detail by Mirza and Shukla.[17] It is worth mentioning that a detailed behavior of chaotic motion can be studied by numerically solving (28). However, this investigation is beyond the scope of this paper.

The stability of the stationary states can be studied by a simple linear analysis. Letting $X=X_s+X_1$, $Y=Y_s+Y_1$, $Z=Z_s+Z_1$, $U=U_s+U_1$, $V=V_s+V_1$, and $W=W_s+W_1$, the linearized system is

$$\begin{pmatrix} d_\tau X_1\\ d_\tau Y_1\\ d_\tau Z_1\\ d_\tau V_1\\ d_\tau U_1\\ d_\tau W_1\end{pmatrix} = \begin{pmatrix} -\sigma_0 & \sigma_0 & s_0Y_s & 0 & s_1 & 0\\ r & -1 & -X_s & 0 & -s_2Z_s & 0\\ Y & 0 & -b & 0 & s_3Y & 0\\ -1 & b_1 & s_4U_s & -\sigma_2 & 0 & s_5X_s\\ 0 & \sigma_1 & s_1Y_s & 0 & -\sigma_1 & 0\\ -s_7Y_s & 0 & s_6 & Y_s & 0 & -\sigma_2\end{pmatrix} \times \begin{pmatrix} X_1\\ Y_1\\ Z_1\\ V_1\\ U_1\\ W_1\end{pmatrix}, \tag{30}$$

where $X_1\ll X_s$, $Y_1\ll Y_s$, $Z_1\ll Z_s$, $U_1\ll U_s$, $V_1\ll V_s$, and $W_1\ll W_s$ and $(X_s, Y_s, Z_s, U_s, V_s, W_s)$ represents a stationary state. The corresponding characteristic equation is thus

$$(\sigma_2+\lambda)^2(\lambda+b)[\sigma_0\sigma_1+(\sigma_0(1-r)+2\sigma_1+\sigma_0\sigma_1)\lambda + (1+\sigma_0)\lambda^2+\lambda^3-(\sigma_0-r)\sigma_1s_1]=0, \tag{31}$$

which governs the linear stability of the stationary state. If we set $\sigma_2=\sigma_1=s_1=0$ then we recover the results of our earlier investigation.[17] For example, if we take $r<1$, the origin is a hyperbolic sink and is thus stable. On the other hand,

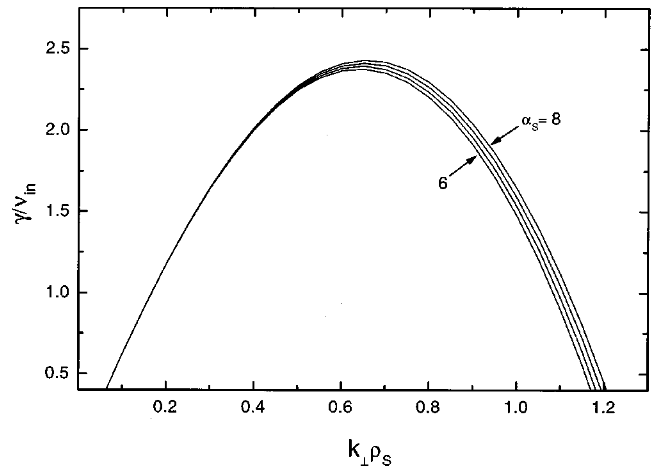

FIG. 1. Normalized local growth rate γ/ν_{in} vs $k_\perp\rho_s$ for $k_\parallel c_s/\nu_{in}=0.2$, $L_n=100$ km, and for different values of α_s. The values of various plasma parameters are given in the text.

for $r=1$, the eigenvalues are $\lambda=-b$ and $\lambda=-(1+\sigma)$, which are always negative. Finally, for $r>1$, the nontrivial stationary points are $X_s^\pm=Y_s^\pm=\pm\sqrt{b(r-1)}$ and $Z_s=r-1$. The eigenvalues of (31) are $\lambda=-(\sigma+b+1)$ and $\pm i\sqrt{2\sigma(\sigma+1)/(\sigma-b-1)}$, so that the stationary states $(X_s^\pm, Y_s^\pm, Z_s)$ are sinks for $r\in(1,r_H)$, where $r_H\equiv\sigma(\sigma+b+3)/(\sigma-b-1)$. A Hopf bifurcation occurs at r_H. For $\sigma>1+b$, imaginary roots are possible and that for $r>r_H$ the nontrivial fixed points are saddles with two dimensional unstable manifolds. Thus, for $r>r_H$ all the three fixed points are unstable but the attractor set still exists.[18] For large r values, further bifurcation may occur leading to chaotic behavior.

V. CONCLUSION AND DISCUSSION

In this paper, we have investigated the linear and nonlinear dynamics of low-frequency electromagnetic waves in nonuniform collisional magnetoplasmas which have equilibrium density gradient as well as sheared plasma flows. It is found that free energy stored in the latter can be coupled to Alfvén-type modes. Specifically, in a collision-dominated magnetoplasma without the density gradient, we have the possibility of a resistive instability of Alfvén-type waves in the presence of equilibrium sheared ion flows. Our numerical studies for the ionospheric parameters also show that the electromagnetic parallel velocity shear (PVS) driven mode grow faster than the electrostatic mode. Furthermore, linearly excited finite amplitude electromagnetic waves interact among themselves and lead to a chaotic state due to the mode couplings. This has been demonstrated by looking for the time-dependent solution of the nonlinear equations that govern the dynamics of finite amplitude electromagnetic waves in a resistive medium. We find that the nonlinear dynamics of electromagnetic turbulence in the presence of sheared plasma flows without the density gradient can be expressed as a set of six coupled mode equations, or simply the generalized Lorenz–Stenflo equations. The latter admit chaotic trajectories under appropriate limits. In conclusion, we stress that the present investigation should be helpful in understanding the salient features of low-frequency electromagnetic turbulence in low-temperature laboratory and space plasmas which contain sheared plasma flows.

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