# Reduced load approximation for WDM rings with wavelength continuity constraint

**Divanilson R. Campelo**

*Optical Networking Laboratory, Universidade Estadual de Campinas (UNICAMP), Caixa Postal 6101, Campinas, São Paulo, Brazil*
*dcampelo@decom.fee.unicamp.br*

**Raul C. Almeida, Jr.**

*Departamento de Eletrônica e Sistemas, Universidade Federal de Pernambuco (UFPE), 50740-530 Recife, Pernambuco, Brazil*
*r.c.almeida.jr@gmail.com*

**Helio Waldman**

*Optical Networking Laboratory, Universidade Estadual de Campinas (UNICAMP), Caixa Postal 6101, Campinas, São Paulo, Brazil*
*waldman@decom.fee.unicamp.br*

Received May 22, 2006; revised August 8, 2006; accepted September 24, 2006; published November 7, 2006 (Doc. ID 71065)

A reduced load approximation for WDM rings with a wavelength continuity constraint under Poissonian, homogeneous traffic with any spatial profile is presented. The model accounts for interactions among successive links in a linear network, correcting the overestimation of the blocking probability caused by the link independence assumption. Comparisons with simulation results show that the proposed reduced load approximation is a better substitute for the well-known reduced load approximation that uses the link independence assumption for blocking computation. It is shown that the proposed model provides accurate estimates of blocking probabilities in WDM rings with a wavelength continuity constraint. © 2006 Optical Society of America

*OCIS codes:* 060.4510, 060.4250.

## 1. Introduction

Methods for calculating blocking probabilities of calls have been a foremost concern in emerging optical networks, which are expected to be circuit-switched at first. This framework is characterized by new peculiarities, of which the most severe is the wavelength continuity constraint when wavelength conversion is not allowed. This restriction forces a light path to use the same wavelength on all the links from the origin node to the destination node. Moreover, the high granularity of traffic of optical networks and the currently high costs of optical ports indicate that such networks tend to be much less connected when compared with the classical circuit-switched networks. Linear networks (chains of nodes and rings) are the extreme case, for which link interactions among successive links are dominant.

In this paper we present a reduced load approximation that accounts for link interactions in WDM rings. Reduced load approximations are important from the user's point of view because they take into account the correlation among channels, providing a better estimation of blocking probabilities in networks with multiple channels. The term "reduced load approximation" reflects the fact that the load offered to a link may be reduced (or thinned) by blocking on other links. Most reduced load approximation techniques assume that blocking occurs independently from link to link [1–3], which overestimates the blocking probability for networks with a high link load correlation among subsequent links. Moreover, most techniques were proposed for highly connected, classical circuit-switched networks, for which blocking on a route happens only if at least one link of the route is in a blocking state. A reduced load approximation that considers load correlation was proposed in Ref. [4], but the complexity of the calculations is considerable for good accuracy in sparse networks.

The reduced load approximation we introduce in this paper replaces the link independence assumption with a better substitute, called the object independence assumption. This assumption has been proved to be asymptotically true for single-wavelength infinite line networks and also to be a very good approximation for finite rings with any size [5]. In this paper we address a new approach to WDM rings by the combination of the object independence assumption with the Erlang loss model applied to an isolated link. As a result, the load offered to a link may be reduced by accounting for blocking on other links on the route without overestimating the blocking probability.

The paper is organized as follows. Section 2 discusses the independent object assumption for a single-wavelength linear network. The Erlang model applied to a single link is discussed in Section 3. The reduced load approximation is presented in Section 4, and the model performance is presented in Section 5. Finally, we conclude our work in Section 6.

## 2. Independent Object Assumption

Let only active paths (groups of successive, simultaneously active links) and free links be taken as objects in a linear network with $N$ links. Paths of size $i\in I$ are requested on any given route, where $I$ is the set of path sizes for which there is nonzero traffic demand. If $\rho$ is the network utilization rate (since the traffic is assumed to be homogeneous, although with any spatial profile, $\rho$ is also the link utilization rate), each path size $i$ will have a partial utilization rate $\rho_i$, with $\rho=\sum_{i\in I}\rho_i$. The independent object assumption consists in assuming that, as you walk along a linear network, the next object to be found is independent of the previous ones. The assumption is used to calculate the blocking probability of a request for an $i$-link path. For the $i$-link path request not to be blocked, the following events must occur: the first link must be free, which will happen with probability $(1-\rho)$, and the remaining $(i-1)$ requested links must also be free—each of these events will happen with probability $(1-\rho)/(1-\rho+\rho/\bar{H})$ for large $N$, where $\bar{H}$ is the mean active path length [5]. Since all such events are assumed to be independent, the blocking probability of the $i$-link path request will be

$$p_{bi}=1-(1-\rho)\left(\frac{1-\rho}{1-\rho+\rho/\bar{H}}\right)^{i-1}, \tag{1}$$

where

$$\bar{H}=\frac{\rho}{\sum_{i\in I}\rho_i/i}. \tag{2}$$

The partial utilization rates $\rho_i$ may be obtained by solving the $|I|$-dimensional system of equations given by the equilibrium equation:

$$\nu_i=\frac{\rho_i}{i(1-p_{bi})}=\frac{\rho_i}{i}\frac{\left[1-\sum_{j\in I}\left(1-\frac{1}{j}\right)\rho_j\right]^{i-1}}{[1-\sum_{j\in I}\rho_j]^i},\quad i\in I, \tag{3}$$

where $\nu_i$ is the offered load per node in erlangs for paths of size $i$. The overall blocking probability is given by

$$P_b=\frac{\sum_{i\in I}\nu_i p_{bi}}{\sum_{i\in I}\nu_i}. \tag{4}$$

Equation (1) was first introduced in Ref. [6] based on a heuristic argument motivated by the observation that it yields very good approximations for finite rings of any size, correcting the overestimation of blocking probabilities due to the independent

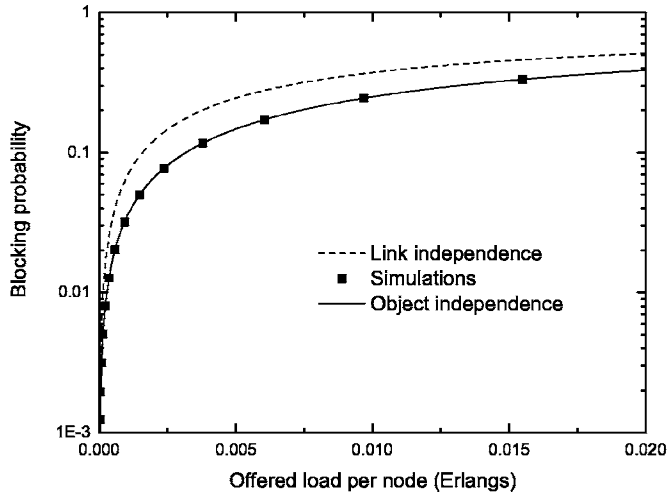


Fig. 1. Link independence, simulations, and object independence in a single-wavelength 13-node ring.

link assumption. A mathematical proof of the asymptotical validity of Eq. (1) when the number of network links $N$ is taken to infinity has been provided in Ref. [5], given that calls are Poissonian.

Figure 1 depicts the comparison of the link independence assumption, the object independence assumption, and simulations on a single-wavelength 13-node ring with shortest-path routing and $I=\{1,2,\ldots,6\}$. The superior fit of the object independence assumption to the simulation results motivates its extension to rings with multiple wavelengths. In this paper, we extend this result to WDM rings with the wavelength continuity constraint under the random wavelength assignment algorithm. We first discuss in the next section the Erlang loss model applied to a single link.

## 3. Erlang Model on a Single Link

Consider the Erlang model applied to an isolated link comprising $W$ wavelengths. Calls arrive at the link as a Poisson process with traffic intensity $\tau$. Call holding times have a finite mean, and they are independent of one another and of the arrival times. The link is in state $k$ if it is busy in $k$ wavelengths, $0 \le k \le W$, and each state transits from and to its neighbors only. The classic Erlang loss formula gives exactly the proportion of calls that are lost in such a circuit-multiplexed link [1]:

$$p_W = \frac{\tau^W}{W!}\left[\sum_{n=0}^{W}\frac{\tau^n}{n!}\right]^{-1}. \tag{5}$$

## 4. Reduced Load Approximation

The independent object assumption is asymptotically valid for a single-wavelength linear network with multiple links, whereas the Erlang model presents the exact blocking characterization for one link with $W$ wavelengths under Poissonian arrivals. In this paper the proposed reduced load approximation consists of combining both the independent object assumption and the Erlang model. The proposed model considers a given link of the chosen route as the reference link, and it is able to reduce the load offered to that particular link by accounting for the blocking on its adjacent links through the independent object assumption.

Consider a request for a path of size $i$ that contains a given link. Suppose also that this link is free in a given wavelength plane. According to Section 2, the probability that this request cannot be accommodated (i.e., be blocked) on that wavelength plane in the $(i-1)$ remaining steps will be

$$r_i = 1 - \left( \frac{1-\rho}{1-\rho+\rho/\bar{H}} \right)^{i-1}. \tag{6}$$

The probability $r_i$ may be called the lateral blocking probability, since it is associated with activity in the vicinity of the reference link.

Let $W$ be the number of wavelengths in the network. In the random algorithm, the wavelength assigned to a path is chosen randomly from the set of the available wavelengths. The reference link will be in state $k$ when it is busy in $k$ wavelengths, $0 \leq k \leq W$. When the reference link is in state $k$, a request of size $i$ will be accepted if at least one of the $(W-k)$ available wavelengths on the reference link does not present lateral blocking, which has a probability $(1-r_i^{W-k})$. The transition rate $T_k$ from state $k$ to state $(k+1)$ depends on $k$, and it must account for the contributions from all path requests that have the reference link as their member. Since the reference link may be contained in one 1-link path request, two 2-link path requests, three 3-link path requests and so on, we have

$$T_k = \sum_{i \in I} i\lambda_i (1 - r_i^{W-k}), \tag{7}$$

where $\lambda_i$ is the arrival rate per node of $i$-link path requests. The death rate $k\mu$ is proportional only to the population of active calls $k$. Notice that if the requests were only for paths with unit size, the transition rates would be the same for all states, i.e., the request would never be blocked due to the occupation of adjacent links ($r_i=0$), which reduces to the Erlang model on a single link (5). The state diagram for transitions in the reference link is shown in Fig. 2.

Let $q(k)$ be the temporal steady-state probability of state $k$. Then the blocking probability of a request for a path of size $i$ will be

$$p_{bi} = \sum_{k=0}^{W} q(k) r_i^{W-k}. \tag{8}$$

Notice that, since it represents a modulated Poisson process, then $q(k)$ may be achieved by the following closed-form expression:

$$q(k) = \frac{\prod_{j=0}^{k-1} T_j / k!\, \mu^k}{\sum_{u=0}^{W} \prod_{j=0}^{u-1} T_j / u!\, \mu^u}, \tag{9}$$

where $T_{-1}=1$. The definition of $T_{-1}=1$ does not alter the product, and it must be provided because in Eq. (9) the upper limit of the products in the numerator and the denominator is $-1$ when $k$ or $u$ are zero, respectively.

Since on the average the random wavelength assignment algorithm distributes the offered load evenly among the $W$ wavelength planes, the occupancy of $i$-link paths, $\rho_i$, will be the same for each wavelength plane. To calculate the occupancies, we will consider the average traffic offered to each wavelength plane.

Let $p_i(n)$ be the probability that an $i$-link path request will be offered to a specific wavelength of a set of $n$ free wavelengths in the reference link. Therefore $p_i(n)$ is given by the probability that the specific wavelength will be the first to be chosen plus the probability that it is the second and the first is busy on adjacent links (which happens with lateral blocking probability $r_i$), and so on. Consequently,

$$p_i(n) = \frac{1}{n} + \frac{n-1}{n} r_i \frac{1}{n-1} + \frac{n-1}{n} r_i \frac{n-2}{n-1} r_i \frac{1}{n-2} + \ldots + \frac{n-1}{n} r_i \frac{n-2}{n-1} r_i \ldots \frac{1}{2} r_i, \tag{10}$$

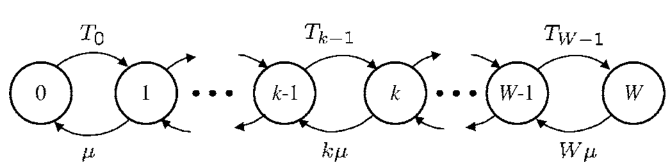


Fig. 2. State diagram for the reference link.

$$\therefore p_i(n) = \frac{1}{n}\left(\frac{1 - r_i^n}{1 - r_i}\right). \qquad (11)$$

Let $\nu_i'(k)$ be the load offered to any wavelength. When the reference link is in state $k$, the offered load will be zero for the fraction $k/W$ of busy wavelengths in the reference link, and will be $p_i(W-k)$ times the original traffic $\nu_i$ for the fraction $(W-k)/W$ of free wavelengths in the reference link. Thus

$$\nu_i'(k) = \nu_i \frac{W - k}{W} p_i(W - k). \qquad (12)$$

Finally, we obtain the average load offered to each wavelength plane,

$$\nu_i' = \sum_{k=0}^{W} \nu_i'(k) q(k), \qquad (13)$$

where $\nu_i'$ has the following equilibrium equation:

$$\nu_i' = \frac{\rho_i}{i(1 - r_i)}. \qquad (14)$$

Notice that Eq. (14) may be used to obtain $\rho_i$ in similar manner as in Eq. (3), such that Eqs. (6)–(14) may be calculated iteratively until the $i$-link path blocking probabilities given in Eq. (8) converge. The overall blocking probability is given by Eq. (4).

## 5. Model Performance

In this section we illustrate with some examples the effectiveness of the proposed model. Figures 3 and 4 show the comparison of the proposed reduced load approximation with simulations and with the reduced load approximation that uses the link independence model. In the latter case, the load offered to a link may be also reduced owing to blocking on other links, but we assume that successive links are available or blocked independently. Simulations were performed with uniform traffic in a 13-node ring with $W=6$ and in a 25-node ring with $W=8$ wavelengths, respectively, both under shortest-path routing and Poissonian arrivals of calls. We have used Poissonian arrivals because the asymptotic proof of the object independence assumption given in Ref. [5] has been obtained under the framework of the classical Erlang model, which assumes Poissonian arrivals of calls. Both figures depict the influence of the object independence assumption on the accuracy of blocking probability estimates in WDM rings. The model has a low complexity and scales linearly with the number of wavelengths, $W$, and with the set of path sizes for which there is nonzero traffic demand, $I$. In addition, the complexity with ring size $N$ has been avoided.

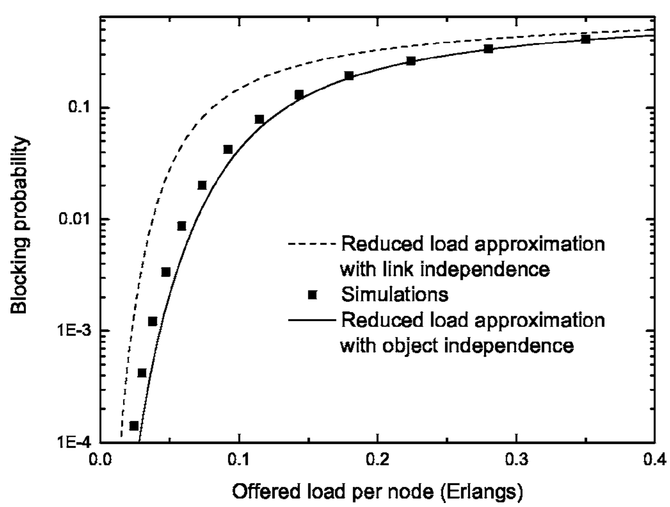


Fig. 3. Reduced load approximation with link independence, simulations, and reduced load approximation with object independence in a 13-node ring with $W=6$ wavelengths.

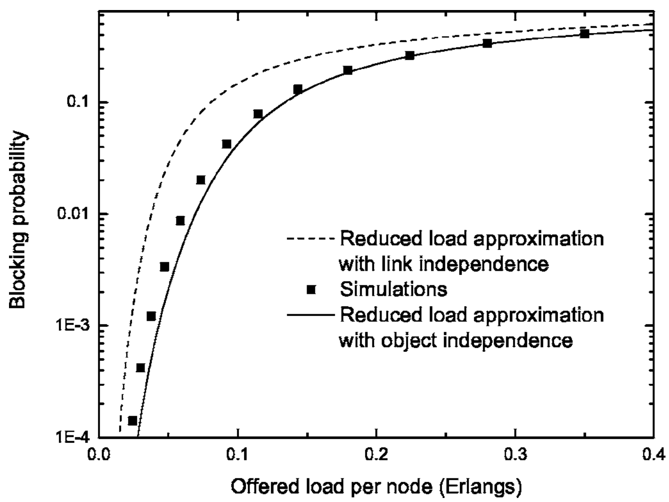


Fig. 4. Reduced load approximation with link independence, simulations, and reduced load approximation with object independence in a 25-node ring with $W=8$ wavelengths.

## 6. Conclusion

A framework for quantifying blocking probability estimates in WDM rings with a wavelength continuity constraint has been presented. The proposed model is based on a simple reduced load approximation that accounts for link interactions in WDM rings. To assess the accuracy of the proposed model, comparisons with simulations have been done under Poissonian arrivals. It has been observed that the proposed reduced load approximation obviates the overestimation of blocking probabilities caused by the link independence assumption in WDM rings.

Some directions for further work include the network performance under the first-fit wavelength assignment algorithm, in which the first available wavelength from a predefined priority list of all network wavelengths is assigned [7]. A more complex, nonreversible Markov chain with $2^W$ states must then replace the state diagram of Fig. 2, so closed-form expression (9) must then be replaced by a matrix equation. Moreover, the partial densities $\rho_i$ will be wavelength specific; so both the density variables $\rho_i$ and the lateral blocking probabilities $r_i$ will be $W$-dimensional vectors. However, the general iterative scheme of Section 4 may still be performed to yield a reduced load approximation based on an improved per-wavelength plane blocking probability estimation.

## Acknowledgments

This work has been supported by Fundação de Amparo à Pesquisa do Estado de São Paulo, Fundação de Amparo à Ciência e Tecnologia do Estado de Pernambuco, and Conselho Nacional de Desenvolvimento Científico e Tecnológico.

## References and Links

1. F. P. Kelly, “Loss networks,” Ann. Appl. Probab. **1**, 319–378 (1991).
2. S. P. Chung and K. Ross, “Reduced load approximation for multirate loss networks,” IEEE Trans. Commun. **41**, 1222–1231 (1993).
3. A. Kumar, D. Manjunath, and J. Kuri, *Communication Networking: an Analytical Approach* (Elsevier/Morgan Kaufmann, 2004).
4. A. Sridharan and K. N. Sivarajan, “Blocking in all-optical networks,” IEEE/ACM Trans. Netw. **12**, 384–397 (2004).
5. H. Waldman and D. R. Campelo, “Asymptotic blocking probabilities in rings with single-circuit links,” IEEE Commun. Lett. **9**, 564–566 (2005).
6. H. Waldman, D. R. Campelo, and R. C. Almeida, Jr., “A new analytical approach for the estimation of blocking probabilities in linear all-optical networks,” in *Proceedings of IEEE GLOBECOM’03* (IEEE, 2003), Vol. 5, pp. 2478–2482.
7. E. Karasan and E. Ayanoglu, “Effects of wavelength routing and selection algorithms on wavelengths conversion gain in WDM optical networks,” IEEE/ACM Trans. Netw. **6**, 186–196 (1998).