NMR linewidth and skyrmion localization in quantum Hall ferromagnets

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The nonmonotonic behavior of the NMR signal linewidth in the 2D quantum Hall system is explained in terms of the interplay between skyrmions localization, due to the influence of disorder, and the nontrivial temperature dependent skyrmion dynamics.

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I. INTRODUCTION

The ground state of the two-dimensional electron gas (2DEG) at filling factor one ($\nu=1$) is a spin-polarized state[1] in which all electrons completely fill the lowest Landau level with spin up polarization (quantum Hall ferromagnet). The low lying charged excitation of the 2DEG is called skyrmion, or charged spin texture, and carries an unusual spin distribution.[2] Pictorially, this distribution can be viewed as a configuration in which the spin field points down at a given position and smoothly rotates as one moves radially outwards from that point, until all the spins are polarized as in the ground state. These nontrivial objects are topologically stable, with its size—the number of reversed spins—fixed by the competition between the Coulomb and Zeeman interactions.[3] The existence of this many body electronic state was confirmed by Barrett and coworkers[4] who performed optically pumped nuclear magnetic resonance (OPNMR) measurements of the ^{71}Ga nuclei located in an *n*-doped GaAs multiple quantum well. As the nuclei of the quantum wells are coupled to the the spins of the 2DEG via the isotropic Fermi contact interaction, the Knight shift (K_s) from the NMR signal gives information about the spin polarization of the 2DEG. It was reported that K_s symmetrically drops around $\nu=1$, showing that the charged excitation of the quantum Hall ferromagnet involves a large number of spin-flips[4] in contrast with the expected behavior for noninteracting electrons. Since then, heat capacity, magnetoabsorption and magnetotransport measurements[5–7] have been performed in order to understand the main features of this novel state, however, valuable information can be still extracted from the NMR experiments.

Recently, new OPNMR measurements[8] of the ^{71}Ga nuclei in *n*-doped GaAs/$Al_{0.1}Ga_{0.9}$As multiple quantum wells were performed at temperatures ($T\approx 0.3$ K) lower than the ones considered in Ref. 4. In Ref. 8, Khandelwal *et al.* observed that the Knight shift data around $\nu=1$ consist of a "tilted plateau," in contrast to the symmetrical fall at higher temperatures. As NMR is a local probe, it was argued that the existence of the tilted plateau is related to the localization of the skyrmions. In addition, the full width at half maximum (FWHM) of the OPNMR signal has a nonmonotonic temperature dependence at $\nu\approx 1$, increasing when the temperature is lowered up to a critical value and then, falling down. It is well known[9] that the linewidth of the NMR measurements is affected by the dynamics of the nuclei (*motional narrowing effect*). However, as the ^{71}Ga nuclei are spatially fixed in the GaAs quantum wells, any observed dynamical effect must be related to the dynamics of 2DEG. In particular, if $\nu\approx 1$, those effects should be related to the skyrmion dynamics. In this scenario, the behavior of the FWHM can be understood as an evolution of the skyrmion dynamics from the *motional-narrowed* to the *frozen* regimes, as the temperature decreases.[8] This nonmonotonic behavior of the linewidth was previously observed by Kuzma *et al.*[10] when the filling factor of the 2DEG was slightly changed around $\nu=1/3$. Later on, in order to investigate the evolution of the spectral line shape as function of T, Sinova and coworkers[11] simulated the NMR signal of ^{71}Ga nuclei when $\nu\approx 1$. As at low T it is expected that the short length scale correlations contain the ground state features, the 2DEG in Ref. 11 was described by a square skyrmion lattice[12] that randomly jumps—as a whole—over an average distance. Despite the fact that the formation of a pinned Wigner crystal has never been observed in the range of temperatures where the experiments[8,10] have been performed, the approach used in Ref. 11 reproduced the qualitative behavior of the FWHM near the completely frozen situation. However, as far as we know, no explicit correspondence has been established between the temperature dependence of the skyrmion dynamics, for the whole range of experimentally available temperatures, and the line shape of the NMR experiments. This is precisely our main goal in this paper.

Here, instead of trying a fully quantum mechanical approach for the Fermi contact interaction between ^{71}Ga nuclei and the 2DEG for $\nu\approx 1$ we will use a semiclassical theory, in which the skyrmion center of mass is treated as a true dynamical variable[13] to which a temperature dependent diffusion constant is associated.[14] This way of facing the problem will allow us to study the whole range of temperature, namely from the complete delocalized situation to the frozen limit.

We organize this paper as follows. In Sec. II the model used to describe the skyrmion dynamics and the localization by disorder are presented and the dependence of the linewidth with the temperature is calculated. Sec. III is devoted to analyzing the change in the energy of the skyrmions when impurities are taken into account. Finally in Sec. IV we present the final results and our conclusions.

II. THE LINEWIDTH TEMPERATURE DEPENDENCE

To start with we recall that the FWHM in the magnetic resonance experiment is proportional to the induced phase shift experienced by the nuclei every time they interact with a fluctuating field. More precisely, the FWHM is equal to T_2^{-1}, where T_2 is the *transversal relaxation time*.[9] For the Ga nuclei in the quantum wells, the fluctuating field is given by the deviation from the completely polarized state provided by moving skyrmions. Therefore, the spins of the ^{71}Ga nuclei will precess with an extra phase over its normal precession (in the presence of the completely polarized state) every time a skyrmion passes near by. This change in phase can be calculated as $\delta\phi = \pm\gamma_n \overline{n}\,\tau$,[9] where γ_n is the gyromagnetic ratio for the ^{71}Ga nuclei, $\overline{n}$ is the "fluctuating" field associated to the skyrmion magnetization and τ is the interaction time. The mean square dephasing $\langle\Delta\phi^2\rangle$ after n τ-intervals of time will be $n\,(\gamma_n\overline{n}\tau)^2$. Here $n=2DtN_{fs}/S$ is the number of skyrmions passing by the nucleus in a time t, with N_{fs} being the number of free skyrmions, D the diffusion constant and S the quantum well surface area. As a result,

$$\langle\Delta\phi^2\rangle = \frac{2DN_{fs}}{\mathcal{S}}(\gamma_n\overline{n}\tau)^2 t. \tag{1}$$

Now, if we define T_2 as the time for a group of spins in phase at $t=0$ get about one radian out of step[9] we find that

$$T_2^{-1} = \frac{2DN_{fs}}{\mathcal{S}}(\gamma_n\overline{n}\tau)^2. \tag{2}$$

On the other hand, the time interval τ can be estimated from the diffusion constant assuming that the length in which the skyrmions effectively interact with the nucleus is about one lattice constant a. In this way $\tau\sim a^2/2D$ and

$$T_2^{-1} = \frac{a^4}{2SD}N_{fs}(\gamma_n\overline{n})^2. \tag{3}$$

Now, in order to make the temperature dependence of expression (3) explicit, we need to know how the skyrmion diffusion constant changes with the temperature. From the semiclassical point of view the study of the skyrmion dynamics was performed[13,14] starting from a generalized nonlinear sigma model, which has the skyrmion as the lowest lying charged excitation.[2] Within this approach the origin of the nontrivial skyrmion dynamics can be understood in terms of its interaction with the thermal bath of spin waves. This kind of interaction can be correctly treated using the well known collective coordinate method.[15] This technique promotes the center of mass of the skyrmion to a true dynamical variable and, as a final result, provide us with a Hamiltonian in which the skyrmion momentum is coupled to the generalized spin wave momentum. The effective equation of motion for a single skyrmion was obtained using the Feynman-Vernon functional approach[16] (by tracing out the magnons' degrees of freedom) and corresponds to a Brownian particle with temperature dependent damping and diffusion constants. The explicit temperature dependence of the skyrmion diffusion coefficients[14] is given by

$$D = \overline{D}T^3 \exp(-2g/T), \tag{4}$$

where the Zeeman gap $(g=g^*\mu_B B)$ will be measured in Kelvin.

In expression (3) not only the diffusion constant but also the number of free, or moving, skyrmions are temperature dependent quantities. In fact, as it was pointed out by Nederveen and Nazarov,[17] the donors situated on a layer located at a distance d from the 2DEG (inside the potential barriers) generate a random attractive potential (disorder) in which the skyrmions start to localize below some temperature. For higher temperatures the number of skyrmions will be only determined by the deviation of the filling factor from $\nu=1$. Therefore, around any additional electron or hole (whose number is N_0) introduced in the completely polarized state, a new spin texture will be formed. In this case the number of free, or moving, skyrmions (N_{fs}) exactly coincides with N_0, which is related to the electronic density of the 2DEG n_0 by $N_0=n_0\mathcal{S}|1-\nu|$.

As it can be seen our model assumes that as the temperature is lowered the number of free skyrmions decreases. At first sight if the unbinding of skyrmions from the disordered centers is a thermally activated process, the number of free spin textures will be roughly given by $N_{sf}=n_0\mathcal{S}|1-\nu|\exp(-U/T)$, where U is some average value of the attractive potential induced by the disorder. However, this simple assumption implies that all particles localize at the same temperature, in contradiction with the different patterns observed in the Knight shift measurements for different values of ν. In order to be more realistic in modeling the disorder, a certain degree of correlation between them should be included. The main effect of this correlation is nothing but a distribution of potentials with different depths in such a way that as the temperature decreases the skyrmions start to localize in the deepest wells and this process continues until the "last skyrmion" localizes in the shallowest well. If this distribution is assumed to be Gaussian around some specific value U_o, the number of potentials with depth U can be written as

$$n(U) = \mathcal{N}\exp\big(-(U-U_0)^2/\Delta^2\big), \tag{5}$$

where the constant $\mathcal{N}$ is related to the total number of donors localized in the barriers between the GaAs quantum wells. As all the electrons in the 2DEG arise from the donors their number should be equal, i.e.,

$$n_0\mathcal{S} = \mathcal{N}\int_0^\infty dU\, e^{-(U-U_0)^2/\Delta^2} = \frac{\sqrt{\pi}}{2}\mathcal{N}\Delta\big(1+\mathrm{erf}(U_0/\Delta)\big), \tag{6}$$

where $\mathrm{erf}(x)$ is the error integral.[18]

Now, using expressions (4) and (5) the temperature dependence of the transversal relaxation time (3) can be generalized to be a mean value over the potential distribution as

$$T_2^{-1} \sim \frac{e^{2g/T}}{T^3}\int_{U_{min}}^\infty dU\, e^{-(U-U_0)^2/\Delta^2} e^{-U/T}. \tag{7}$$

Here, the value of U_{min} is related to the total number of skyrmions in the 2DEG through

$$n_0|1-\nu|\mathcal{S}=\mathcal{N}\int_{U_{min}}^{\infty}dU\;e^{-(U-U_0)^2/\Delta^2}$$
$$=\frac{\sqrt{\pi}}{2}\mathcal{N}\Delta\left[1-\mathrm{erf}\left(\frac{U_{min}-U_0}{\Delta}\right)\right]. \qquad (8)$$

Notice that as the number of skyrmions increases with $|1-\nu|$, the value of U_{min} should decrease in order to accommodate all skyrmions in the localization centers.

Finally, integrating Eq. (7) and using the expressions (6) and (8), it is possible to show that

$$T_2^{-1}\sim\frac{e^{2g/T+(\Delta^2-4TU_0)/4T^2}}{T^3}\left[1-\mathrm{erf}\left(\frac{\Delta}{2T}+\phi\right)\right], \qquad (9)$$

where ϕ is given by

$$\phi=\mathrm{erf}^{-1}\left(1-|1-\nu|\left[1+\mathrm{erf}\left(\frac{U_0}{\Delta}\right)\right]\right),$$

and $\mathrm{erf}^{-1}(x)$ is the inverse function of $\mathrm{erf}(x)$.

As it can be checked expression (9) is a nonmonotonic function of the temperature. In fact in the high temperature limit, where the localization effects are negligible, the NMR linewidth (T_2^{-1}) goes to zero in agreement with the motional narrowed effect observed in the experiments. If we look in the opposite direction (low temperatures) where the slow motion of the skyrmions tends to increase the linewidth, the exponentially small number of free skyrmions cancels out this effect and the linewidth decreases again. This is precisely the frozen limit found in the NMR measurements. Therefore the interplay between skyrmion dynamics and localization induced by disorder in the system reproduces some of the major features of the transversal relaxation time near $\nu=1$. It is worth pointing out that depending on the disorder strength the 2DEG, away from $\nu=1$, can evolve to different phases[19] which in principle can influence the linewidth measurements. However, these physics were not included in our very simple model.

III. SKYRMION IN THE PRESENCE OF DISORDER

At this point nothing is left but to estimate the value of U_0 due to the impurities localized outside the quantum well. This can be done computing the change in energy of an isolated skyrmion due to the presence of a positive charged ion. In order to do that the skyrmions will be described by an effective generalized nonlinear sigma model[2] in terms of an unit vector field $\mathbf{m}(\mathbf{r})$ associated to the electronic spin orientation. The Lagrangian density which describes these objects in the presence of an external magnetic field $\mathbf{B}$ can be written as

$$\mathcal{L}_0=T(\mathbf{m})-V(\mathbf{m}), \qquad (10)$$

where

$$T(\mathbf{m})=\frac{\hbar\rho}{4}\mathbf{A}(\mathbf{m})\cdot\partial_t\mathbf{m}, \qquad (11)$$

and

$$V(\mathbf{m})=\frac{1}{2}\left[\rho_s\;(\nabla\mathbf{m})^2-g^*\overline{\rho}\mu_B\;\mathbf{m}\cdot\mathbf{B}+\frac{e^2}{\epsilon}\int dr'^2\frac{q(r)q(r')}{|\mathbf{r}-\mathbf{r}'|}\right]. \qquad (12)$$

In the kinetic term (10) ρ denotes the electronic density and $\mathbf{A}[\mathbf{m}]$ corresponds to the vector potential of a unit monopole, i.e., $\epsilon^{ijk}\partial_jA^k=m_i$. On the other hand in the potential energy density (12), ρ_s is the spin stiffness, g^* is the effective Landé factor, $\overline{\rho}=1/(2\pi l^2)$ is the uniform electronic background density and ϵ is the dielectric constant of the background semiconductor. The deviation of the physical density from the uniform value $\overline{\rho}$ determines the skyrmion density $q(r)$, which can be explicitly written as

$$q(r)=\frac{1}{8\pi}\epsilon_{\mu\eta}\;\mathbf{m}(\mathbf{r})\cdot(\partial_\mu\mathbf{m}(\mathbf{r})\times\partial_\eta\mathbf{m}(\mathbf{r})), \qquad (13)$$

and whose spatial integral is the topological charge.

Instead of directly trying to solve (10)–(12), Sondhi *et al.*[2] considered the soliton solution of the nonlinear σ model obtained by Belavin and Polyakov[20] with a fixed size λ, which is set by the competition between the Coulomb and Zeeman interactions included in (12). Therefore, within this approach, the static solution of (10) can be written as

$$m^0_{x(y)}=\frac{4\lambda r\cos\theta(\sin\theta)}{r^2+4\lambda^2},\quad m^0_z=\frac{r^2-4\lambda^2}{r^2+4\lambda^2}, \qquad (14)$$

which describes a skyrmion of unit topological charge localized at the origin. The skyrmion size is given by $\overline{\lambda}=0.558l_0(\tilde{g}|\ln\tilde{g}|)^{-1/3}$, where l_0 stands for the magnetic length and $\tilde{g}=g\mu_BB\epsilon/e^2$. In this approximation the skyrmion density is given by $q(r)=4\lambda^2/\pi(4\lambda^2+r^2)^2$.

As pointed out earlier, the disorder effect in the 2DEG is related to the donors situated in a layer whose distance from the 2DEG is d. In order to include those effects in the skyrmion dynamics, we add an extra term to the the Lagrangian density (10). If we consider that the skyrmion density $q(\mathbf{r})$ is coupled to the disorder potential via Coulomb interaction, our model for the skyrmion-impurity system is given by the Lagrangian density,

$$\mathcal{L}=\mathcal{L}_0+\frac{e^2q(r)}{\epsilon|d^2+(\mathbf{r}-\mathbf{r}_0)^2|}, \qquad (15)$$

where e is the electron charge and $\mathbf{r}_0$ denotes the donor coordinate at the 2DEG plane. Once we are assuming a very simple model to describe the system, we will not consider the fact that the disorder potential can be screened by the 2DEG as pointed out by Efros *et al.*[21]

From expressions (10)–(12) and (15), the energy functional for a single static skyrmion can be written as

$$E=\int\left[V[\mathbf{m}]-\frac{e^2q(\mathbf{r})}{\epsilon|d^2+(\mathbf{r}-\mathbf{r}_0)^2|}\right]dr^2, \qquad (16)$$

where $V(\mathbf{m})$ is the interacting potential related to the Lagrangian density (10)–(12) and the integral of the second term is performed over the whole plane containing the 2DEG. If we assume that the presence of the disorder does not modify the skyrmion form and size, the energy of the

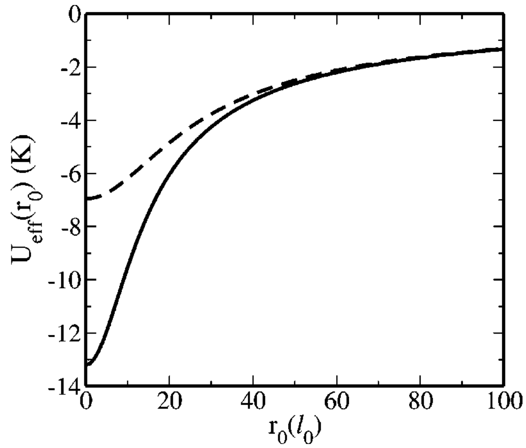

FIG. 1. Effective potential in K as a function of $\mathbf{r}_0$ (the skyrmion-donors distance in the 2DEG). The continuous line corresponds to $d=900$ and the dashed line to $d=1800$.

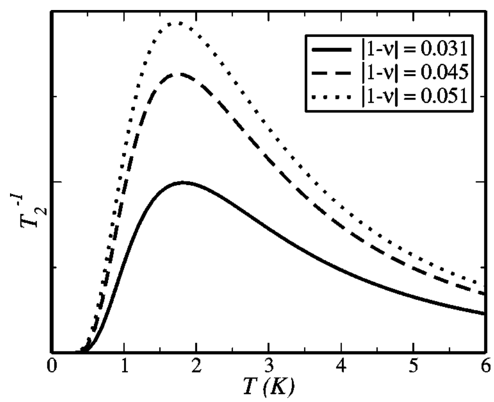

FIG. 2. T_2^{-1} (FWHM) as a function of temperature for $U_0 = 7$ K, $\Delta=2$ K, $g=2$ K and different filling factor values.

system will be given by Eq. (16) with $\mathbf{m}(\mathbf{r})=\mathbf{m}^0(\mathbf{r})$. Therefore, the change in energy of a single skyrmion due to the disorder potential is given by $U_{eff}(\mathbf{r}_0)=E-E_0$, where $E_0 = \int V(\mathbf{m}^0(\mathbf{r},\overline{\lambda}))dr^2$.

The specific form of U_{eff} as a function of r_0 (the skyrmion-donor distance in the plane of the 2DEG) is illustrated in Fig. 1 for $\lambda=1.2l_0$ (the skyrmion size at $B=7$ T, see the Appendix). As it can be seen, the minimum in energy corresponds to the case where the impurity position in the 2DEG plane exactly coincides with the skyrmion center. Therefore, the parameter U_0 in Eq. (9) can be estimated as the value of U_{eff} at $r_0=0$. If we use a typical value of the distance from the donors to the 2DEG as $d \sim 1.800$,[10] we conclude that $U_0 \sim 7$ K. It is interesting to notice that the value of the estimated U_0 is of the same order of the disordered localizing potential reported in Ref. 17 for the case in which $\nu \approx 1$.

IV. RESULTS AND DISCUSSION

At this point we can turn our attention to the linewidth behavior as a function of the temperature. Figure 2 shows a plot of Eq. (9) as a function of temperature for different values of $|1-\nu|$ with $U_0=7$ K, $\Delta=2$ K and $g=2$ K (the Zeeman gap at $B=7$ T). One can see that T_2^{-1} has the nonmonotonic behavior observed in the OPNMR experiments[8,10] going from the *motional-narrowed* regime at relative high temperatures to the *frozen limit* as the temperature decreases. The agreement between the theory and the experimental data reported in Ref. 8 suggests that the relevant gapless mode that induce short nuclear relaxation times T_1 near $\nu \approx 1$ is precisely the translational mode.[13,14] On the other hand, it can be seen that as $|1-\nu|$ increases the maximum value of $1/T_2$ also increases. On physical grounds this can be expected, because as the number of skyrmions grows, the possibility of finding *free* skyrmions which effectively induce nuclei dephasing, increases. Although for the $\nu \approx 1$ measurements[8] this kind of behavior is almost unclear, in the $\nu \approx 1/3$ situation it can be neatedly observed.[10]

For U_0 equal to the potential depth estimated from the isolated impurity case, the peak position—indicating the change in the dynamical regime—is around 1.7 K which is very close to the reported experimental data.[8] It is possible to show that if U_0 changes from 4 K to 8 K, the peak position varies from 1 K to 2.2 K remaining within the reported results.[8] As it can be seen in Fig. 2, the peak position is almost constant as the filling factor changes, which is in contradiction with the experimental results for $\nu \approx 1$[8] but in agreement with the $\nu \approx 1/3$ case.[10] This discrepancy could be possibly associated with the noninteracting skyrmions approach we have employed, that is more appropriate for the small skyrmion near $\nu=1/3$. This conjecture can be tested by applying an in-plane magnetic field, which does not change the filling factor but increase the Zeeman energy making the skyrmion smaller.

It is worth pointing out that our model for the linewidth of the NMR signal is in agreement with the scenario suggested for the observed tilted plateau in the Knight shift measurements for the 2DEG when $\nu \approx 1$. In this case, the existence of the tilted plateau is also related to the localization of the quasiparticles introduced in the 2DEG as the filling factor deviates from $\nu=1$.[8]

Summarizing, we have developed a very simple theory which describes the evolution of the NMR line profile of the ^{71}Ga nuclei coupled to the 2DEG (near $\nu=1$) as a function of the whole available temperature range which seems to be in fairly good agreement with the experimental data.

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TABLE I. Energy scales.

Energy scales		(K)
ϵ_C	$e^2/\epsilon l_0$	$50.40\sqrt{B}$
g	$g^* \mu_B B$	$0.33B$
ρ_s	$\epsilon_C/(16\sqrt{2\pi})$	$1.25\sqrt{B}$

APPENDIX

In Table I, we show the energy scales for the quantum Hall system at $\nu=1$ in Kelvin. In all expressions, the magnetic field is measured in Tesla. For GaAs quantum wells $\epsilon \approx 13$. Therefore the skyrmion size $\lambda = 0.558 l_0 (\tilde{g}|\ln \tilde{g}|)^{-1/3}$ calculated as described in the text can be written as $\lambda = 3.08(\sqrt{B}|\ln(5.95\times 10^{-3}\sqrt{B})|)^{-1/3}$, where $l_0 = \sqrt{\hbar c/eB} = 256/\sqrt{B}$.

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