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P-representable subset of all bipartite Gaussian separable states

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P-representability is a necessary and sufficient condition for separability of bipartite Gaussian states only for the special subset of states whose covariance matrix are $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ locally invariant. Although this special class of states can be reached by a convenient $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ transformation over an arbitrary covariance matrix, it represents a loss of generality, avoiding inference of many general aspects of separability of bipartite Gaussian states.

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In the recent years the question to whether a given quantum state is separable or entangled has become central to the quantum information and to the quantum optics communities. Mostly because fault-tolerant quantum information protocols, such as quantum computation and quantum teleportation are completely dependent on the ability to prepare pure (or close to pure) entangled states [1]. Recent attention however has been centered on continuous variable versions of quantum communication protocols, such as the unconditional quantum teleportation [2,3], whose efficiency rests on the ability to generate entangled states of systems with infinite dimensional Hilbert spaces. Bipartite systems with finite Hilbert space have been exhaustively investigated in order to achieve a precise quantification of entanglement. Peres [4] and Horodecki [5] demonstrated that a necessary and sufficient condition for separability of bipartite systems with Hilbert spaces of dimension $\leqslant 2\otimes 3$ is the positivity of the partial transpose of the system density matrix. On the other hand, algebraic similarities between bipartite states with Hilbert space of dimension $2\otimes 2$ and bipartite Gaussian states (described by 4×4 covariance matrices) allow the extension of the positivity criterion to those special continuous variable states as developed in Refs [6,7], and considered afterwards in discussions on entanglement of Gaussian bipartite states (e.g., Refs. [8–18]).

Of particular importance is the connection between Glauber *P* representability of a bipartite quantum state and separability [10]. A *P*-representable state is the one that is represented by a positive Glauber *P*-distribution function $P(\alpha,\beta)$, which is less (or equally) singular than the delta distribution, such as

$$\rho = \int d\alpha^2\, d\beta^2\, P(\alpha,\beta)|\alpha,\beta\rangle\langle\alpha,\beta|. \tag{1}$$

Under this condition $P(\alpha,\beta)$ assumes the structure of a legitimate probability distribution function over an ensemble of states, allowing the connection between separability and classicality. However, although any *P*-representable bipartite state is separable, as can be immediately seen by the *P*-representation definition, the inverse is not necessarily true. *P* representability and separability are completely equivalent only for Gaussian states with locally $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ invariant covariance matrices. Since any covariance matrix can be brought to this invariant form under appropriate $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ transform, *P* representability and separability have been misleadingly accepted as one-to-one equivalent properties of bipartite Gaussian states. The purpose of the present paper is to give a complete classification of the set of all bipartite Gaussian separable states (BGSS). Particularly we show that *P*-representable Gaussian bipartite states form a subset of BGSS with locally $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ invariant form. We begin by revising some necessary properties of bipartite Gaussian states and give the necessary and sufficient conditions for the state to be separable. Next we discuss the *P* representability of those states and show that they actually form a subset of the separable states. We then provide the unitary $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ map connecting the two sets.

Any bipartite quantum state ρ is Gaussian (see, e.g., Refs. [17,19]) if its symmetric characteristic function is given by $C(\boldsymbol{\eta})=\mathrm{Tr}[D(\boldsymbol{\eta})\rho]=e^{-\frac{1}{2}\boldsymbol{\eta}^\dagger\mathbf{V}\boldsymbol{\eta}}$, where $D(\boldsymbol{\eta})=e^{-\boldsymbol{\eta}^\dagger\mathbf{E}\mathbf{v}}$ is a displacement operator in the parameter four-vector $\boldsymbol{\eta}$ space, with $\boldsymbol{\eta}^\dagger=(\eta_1^*,\eta_1,\eta_2^*,\eta_2)$, $\mathbf{v}^\dagger=(a_1^\dagger,a_1,a_2^\dagger,a_2)$, being a_1 $(a_1^\dagger)$ and a_2 $(a_2^\dagger)$ the annihilation (creation) operators for party 1 and 2, respectively. Here,

$$\mathbf{E}=\begin{pmatrix}\mathbf{Z} & \mathbf{0}\\ \mathbf{0} & \mathbf{Z}\end{pmatrix},\quad \mathbf{Z}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{2}$$

and $\mathbf{V}$ is the Hermitian 4×4 covariance matrix with elements $V_{ij}=(-1)^{i+j}\langle\{v_i,v_j^\dagger\}\rangle/2$,

$$\mathbf{V}=\begin{pmatrix}\mathbf{V_1} & \mathbf{C}\\ \mathbf{C}^\dagger & \mathbf{V_2}\end{pmatrix}=\begin{pmatrix}n_1 & m_1 & m_s & m_c\\ m_1^* & n_1 & m_c^* & m_s^*\\ m_s^* & m_c & n_2 & m_2\\ m_c^* & m_s & m_2^* & n_2\end{pmatrix}, \tag{3}$$

where $\mathbf{V_1}$ and $\mathbf{V_2}$ are Hermitian matrices containing only local elements while $\mathbf{C}$ is the correlation between the two parties. Any covariance matrix must be positive semidefinite $(\mathbf{V}\geqslant\mathbf{0})$, furthermore the generalized uncertainty principle, $\mathbf{V}+\frac{1}{2}\mathbf{E}\geqslant\mathbf{0}$, must hold. Those general positivity criteria can be decomposed into block using matrix positivity properties.

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A reliable and convenient way to check the covariance matrix positivity is through the following block Schur decomposition [20]: Any Hermitian matrix is positive if and only if any principal block matrix is also positive, or, if its upper left block and the block's Schur complement are also positive. So that for the covariance matrix (3), $\mathbf{V}\geqslant\mathbf{0}$ only if

$$\mathbf{V_1}\geqslant\mathbf{0}, \tag{4}$$

and the Schur complement of $\mathbf{V_1}$,

$$S(\mathbf{V_1})\equiv\mathbf{V_2}-\mathbf{C}^{\dagger}(\mathbf{V_1})^{-1}\mathbf{C}\geqslant\mathbf{0}. \tag{5}$$

It is interesting to observe that the Schur complement of block matrices representing Gaussian states covariances, such as above, embodies a manifestation of a physical operation when considering partial projections onto Gaussian states [21,22].

Through the Schur decomposition the physical positivity criterion applies only if

$$\mathbf{V_1}+\tfrac{1}{2}\mathbf{Z}\geqslant\mathbf{0} \tag{6}$$

and

$$\left(\mathbf{V_2}+\tfrac{1}{2}\mathbf{Z}\right)-\mathbf{C}^{\dagger}\left(\mathbf{V_1}+\tfrac{1}{2}\mathbf{Z}\right)^{-1}\mathbf{C}\geqslant\mathbf{0}. \tag{7}$$

Explicitly, the generalized uncertainty (6) and (7) further simplify to

$$n_1\geqslant\sqrt{|m_1|^2+\tfrac{1}{4}} \tag{8}$$

and

$$n_2\geqslant\frac{s}{d}+\sqrt{\frac{1}{4}\left[\frac{\left||m_c|^2-|m_s|^2\right|}{d}-1\right]^2+|m_2-c|^2}, \tag{9}$$

respectively, with $s=n_1(|m_c|^2+|m_s|^2)-m_cm_sm_1^*-m_c^*m_s^*m_1$, $c=2n_1m_s^*m_c-m_c^2m_1^*-(m_s^*)^2m_1$, $d=n_1^2-\frac{1}{4}-|m_1|^2$.

By mapping the positivity necessary and sufficient condition for dimension $2\otimes 2$ to bipartite systems of infinite dimension, Simon [6] has discovered an elegant geometrical interpretation of separability in terms of the Wigner distribution function for the density operator. The Peres-Horodecki separability criterion in the Simon framework reads: *if a bipartite density operator is separable, then its Wigner distribution necessarily goes over into a Wigner distribution under a phase space mirror reflection*. The separability criterion can be understood as a valid Wigner-class-conservative quantum map under local time reversal. Following Ref. [6] a necessary and sufficient condition for a Gaussian quantum state to be separable, $\rho=\Sigma_k p_k\rho_k^A\otimes\rho_k^B$, is that its covariance matrix must satisfy $\widetilde{\mathbf{V}}+\frac{1}{2}\mathbf{E}\geqslant\mathbf{0}$, under a partial phase space mirror reflection (partial Hermitian conjugation) $\widetilde{\mathbf{V}}=\mathbf{TVT}:(\mathbf{Tv})^{\dagger}=\mathbf{v}_T^{\dagger}=(a_1^{\dagger},a_1,a_2,a_2^{\dagger})$, with

$$\mathbf{T}=\begin{pmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{X}\end{pmatrix},\quad \mathbf{X}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \tag{10}$$

otherwise the state is entangled. Similarly to the generalized uncertainty decomposition, the separability condition is satisfied if and only if (6) and

$$\left(\mathbf{XV_2X}+\tfrac{1}{2}\mathbf{Z}\right)-\mathbf{XC}^{\dagger}\left(\mathbf{V_1}+\tfrac{1}{2}\mathbf{Z}\right)^{-1}\mathbf{CX}\geqslant\mathbf{0}, \tag{11}$$

are both satisfied, which explicitly implies in (8) and

$$n_2\geqslant\frac{s}{d}+\sqrt{\frac{1}{4}\left[\frac{\left||m_c|^2-|m_s|^2\right|}{d}+1\right]^2|m_2-c|^2}, \tag{12}$$

respectively. We call the set of states ρ that fall inside (8) and (12) the set BGSS of all bipartite Gaussian separable states $\mathbb{S}$. Any state that does not fall inside the region bounded by those inequalities is entangled, being it pure or not. Purity is only reached when the equalities in (8) and (9) hold.

The very definition of a separable state can be written in a coherent state representation through the Glauber P-function (1), but it is not obvious that $P(\alpha,\beta)$ is a legitimate probability distribution function. That is only true if the state is P representable, i.e., if the P function is non-negative and less (or equally) singular than the delta distribution. In terms of the covariance matrix, a quantum state is P-representable [19] if

$$\mathbf{V}-\tfrac{1}{2}\mathbf{I}\geqslant\mathbf{0}, \tag{13}$$

which in terms of the upper left block matrix and its Schur complement writes as

$$\mathbf{V_1}-\tfrac{1}{2}\mathbf{I}\geqslant\mathbf{0} \tag{14}$$

and

$$\left(\mathbf{V_2}-\tfrac{1}{2}\mathbf{I}\right)-\mathbf{C}^{\dagger}\left(\mathbf{V_1}-\tfrac{1}{2}\mathbf{I}\right)^{-1}\mathbf{C}\geqslant\mathbf{0}. \tag{15}$$

Thus

$$n_1\geqslant|m_1|+\tfrac{1}{2} \tag{16}$$

and

$$n_2\geqslant\frac{s'}{d'}+\frac{|m_2-c'|}{d'}+\frac{1}{2}, \tag{17}$$

with $s'=\left(n_1-\frac{1}{2}\right)(|m_c|^2+|m_s|^2)-m_cm_sm_1^*-m_c^*m_s^*m_1$, $c'=2\left(n_1-\frac{1}{2}\right)m_s^*m_c-m_c^2m_1^*-(m_s^*)^2m_1$, $d'=\left(n_1-\frac{1}{2}\right)^2-|m_1|^2$. States that follow (16) and (17) form the set of all bipartite P-representable Gaussian states $\mathbb{P}$.

Englert and Wódkiewicz [10] have recently stated that P representability is equivalent to the separability condition, for the specific symmetric situation where $m_1=m_2=m_s=0$, $n_1=n_2=n$, and $m_c=m$, which indeed set $\mathbb{P}\rightleftharpoons\mathbb{S}$ as we see below. The generality of their statement is justified only if $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ local operations are used to bring those parameters to the special symmetric class mentioned above (see also Ref. [19]). However, this particular situation does not represent total equivalence between $\mathbb{S}$ and the set of all P-representable states. In general the P-representability conditions, (16) and (17), are more restrictive than the separability ones, (8) and (12), respectively, as we now investigate.

First observe that (8) is less restrictive than (16), equaling only for $|m_1|=0$ or $|m_1|\to\infty$, being enough to check if (17) dominates over (12) for the simplest $|m_1|=0$ situation. For that we make use of the knowledge that (9) is always stronger than (17), including the situation where $d=0$, i.e., $n_1=1/2$. In such a case, the comparisons of the (17) lower

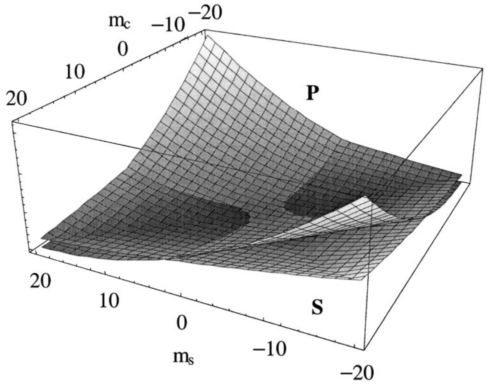

FIG. 1. Typical separability (**S**) and *P*-representability (**P**) boundaries for $m_1=0.5$ and $m_2=1$. The shaded area where the **P**-fold is lower than the **S**-fold does not represent physical quantum states.

bound to (12) and to (9) are equivalent and thus if (12) is violated so is (9). These inequalities must satisfy

$$(|m_2| + |m_c|^2)(|m_2| + |m_s|^2) \geqslant 0, \tag{18}$$

and since the quantities involved are always strictly positive the criterion (18) is always satisfied. The equality however occurs only if $|m_2|=|m_c|^2=0$ or $|m_2|=|m_s|^2=0$, which then set the equivalence $\mathbb{P}\rightleftharpoons\mathbb{S}$ for the two following special $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ invariant forms for $\mathbf{V}$ of Eq. (3).

Invariant form 1,

$$\mathbf{V}_i = n_i\mathbf{I}, \;\; i=1,2; \text{ and } \mathbf{C} = \begin{pmatrix} 0 & m_c \\ m_c^* & 0 \end{pmatrix}. \tag{19}$$

Invariant form 2,

$$\mathbf{V}_i = n_i\mathbf{I}, \;\; i=1,2; \text{ and } \mathbf{C} = \begin{pmatrix} m_s & 0 \\ 0 & m_s^* \end{pmatrix}. \tag{20}$$

Special forms 1 and 2 are locally $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ invariant covariance matrices that form the $\mathbb{P}\rightleftharpoons\mathbb{S}$ subset. The separability and thus *P*-representability criterion is then reduced to $(n_1-\frac{1}{2})(n_2-\frac{1}{2})\geqslant|m_{(c,s)}|^2$, for the special form 1 or 2, respectively, while the physical condition of existence of a general bipartite Gaussian state of the form 1 or 2 writes as $(n_1-\frac{1}{2})(n_2+\frac{1}{2})\geqslant|m_{(c,s)}|^2$, respectively.

Remark 1: There are *P*-representable Gaussian operators that violate (9), which however do not represent any valid positive definite quantum state. As an example Fig. 1 shows the comparison of the limiting bounds (12) and (17), assuming all real coefficients and setting $m_1=0.5$ and $m_2=1$. Only those states that lay in or above the separable class boundary are valid *P*-representable separable states.

Remark 2: The special symmetric situation depicted in Refs. [10,13] for the two-mode thermal squeezed state, where $m_1=m_2=m_s=0$, $n_1=n_2=n$, and $m_c=m$, is a particular example of the specific form 1, and thus a separable state in this case is always *P* representable.

Any general covariance matrix can be mapped into one of those invariant forms under appropriate $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ transform. In other words, it is possible to map $\mathbb{S}$ into $\mathbb{P}$ such that $\rho_{\mathrm{Sp}}=U_L\rho_G U_L^{-1}$ be the state obtained by the local unitary transform $U_L=U_1\otimes U_2$ over a general bipartite Gaussian density operator ρ_G assuming

$$U_L\mathbf{v}U_L^{-1} = \mathbf{S_L}\mathbf{v}, \;\; \mathbf{S_L} = \begin{pmatrix} \mathbf{S_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{S_2} \end{pmatrix}, \tag{21}$$

with the condition $\mathbf{S_L}^{-1}=\mathbf{E}\mathbf{S_L}^{\dagger}\mathbf{E}$. The new symmetric characteristic function writes as $C_{\mathrm{Sp}}(\boldsymbol{\eta})=\mathrm{Tr}[D(\boldsymbol{\eta})\rho_{\mathrm{Sp}}]=\mathrm{Tr}[U_L^{-1}D(\boldsymbol{\eta})U_L\rho_G]=e^{-\frac{1}{2}\boldsymbol{\eta}^{\dagger}\mathbf{V}_{\mathrm{Sp}}\boldsymbol{\eta}}$, with

$$\mathbf{V}_{\mathrm{Sp}} = \mathbf{S_L}^{\dagger}\mathbf{V}\mathbf{S_L}. \tag{22}$$

The transformed covariance matrix writes as (3), but with new block elements

$$\mathbf{V_i'} = \mathbf{S_i}^{\dagger}\mathbf{V_i}\mathbf{S_i}, \;\; \mathbf{C}' = \mathbf{S_1}^{\dagger}\mathbf{C}\mathbf{S_2}. \tag{23}$$

Assuming a local $\mathrm{Sp}(2,R)$ transform as

$$\mathbf{S_i} \equiv \begin{pmatrix} e^{i\phi_i}\cosh\theta_i & e^{i\varphi_i}\sinh\theta_i \\ e^{-i\varphi_i}\sinh\theta_i & e^{-i\phi_i}\cosh\theta_i \end{pmatrix}, \tag{24}$$

the condition to bring $\mathbf{V}$ to the invariant form 1, with non-null elements $(V_i')_{1,1}=(V_i')_{2,2}=\nu_i$, and $C'_{1,2}=(C'_{2,1})^*=\mu_c$, is obtained by setting $\phi_i+\varphi_i=-\mu_i+\pi$ and $\tanh 2\theta_i=|m_i|/n_i=|m_c|/|m_s|$, for $i=1,2$, respectively, where $e^{-i\mu_i}=m_i/|m_i|$, for $|m_c|\geqslant|m_s|$.

Now the condition to bring $\mathbf{V}$ to the invariant form 2, with non-null elements $(V_i')_{1,1}=(V_i')_{2,2}=\nu_i$, and $C'_{1,1}=(C'_{2,2})^*=\mu_s$, is immediately attained if $\phi_i+\varphi_i=-\mu_i+\pi$ also, but now with $\tanh 2\theta_i=|m_i|/n_i=|m_s|/|m_c|$ (assuming $|m_c|\leqslant|m_s|$). Since both $\mathbf{V_1}'$ and $\mathbf{V_1}'$ are proportional to the identity, they do not change under unitary local rotations and the two invariant forms are then connected through those operations. As such, the last two conditions on $|m_c|$ and $|m_s|$ can be waved by appropriate rotations.

The new transformed elements are

$$\nu_i = \sqrt{n_i^2 - |m_i|^2}, \;\; \mu_s = e^{-i\Phi}\frac{m_s}{|m_s|}\sqrt{|m_s|^2 - |m_c|^2} \tag{25}$$

(for $|m_c|\leqslant|m_s|$), with $\Phi=(\phi_1-\phi_2)$ and

$$\mu_c = e^{-i\Phi'}\frac{m_c}{|m_c|}\sqrt{|m_c|^2 - |m_s|^2}, \tag{26}$$

(for $|m_c|\geqslant|m_s|$), with $\Phi'=(\phi_1+\phi_2)$, which then turn explicit the four invariants of the $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ group: $I_1=\det\mathbf{V_1'}$, $I_2=\det\mathbf{V_2'}$, $I_3=\det\mathbf{C}'$, and $I_4=\mathrm{Tr}[\mathbf{V_1'}\mathbf{Z}\mathbf{C}'\mathbf{Z}\mathbf{V_2'}\mathbf{Z}(\mathbf{C}')^{\dagger}\mathbf{Z}]$.

The general $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ transformation (22) of the (24) form is reached through the squeezing operation $U_L=U_1\otimes U_2$,

$$U_i = e^{i(\hbar t/2)(\kappa_i a_i^{\dagger 2}e^{i\varphi_i} - \kappa_i^* a_i^2 e^{-i\varphi_i})}, \tag{27}$$

over the bipartite Gaussian state ρ_G, with $|\kappa_i|t=\theta_i\equiv 2r_i$, the squeezing parameter associated with the transformation on the mode i and $e^{i\phi_i}=\kappa_i/|\kappa_i|$. An important result is that while all the BGSS set can be mapped into the *P*-representable set by suitable $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ transforms, it is not possible to restitute the original matrices $\mathbf{V_1}$ and $\mathbf{V_2}$ with unitary rotations. That is only reached applying over the squeezing operation. This is immediate from the two invariant forms. Since both covariances reduced matrices $\mathbf{V}_1'$ and $\mathbf{V}_2'$ are proportional to the identity, unitary rotations transform the invariant forms among themselves.

Remark 3: Through the above mapping, we have reached the special subset of locally $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ invariant forms $\mathbb{S}\rightleftarrows\mathbb{P}$. However the BGSS can be set equivalent (in principle) to special P-representable subset $(\mathbb{S}\to\mathbb{P})$ under an appropriate nonlocal operation: Let the separability condition be written in the equivalent form

$$\mathbf{V}+\tfrac{1}{2}\mathbf{TET}\geqslant\mathbf{0}. \qquad (28)$$

Now let U_{NL} be a nonlocal operation, $U_{NL}\mathbf{v}U_{NL}^{\dagger}=\mathbf{Mv}$, where $\mathbf{M}$ is a general transformation matrix, $\mathbf{M}\in\mathrm{Sp}(4,R)$. Such a general $\mathbf{M}$, when acting on (28) must leave $\mathbf{V}$ invariant in form $(\mathbf{V}')$, while $\mathbf{M}^{\dagger}\mathbf{TETM}$ must go necessarily to $-\mathbf{I}$, such that (28) writes as $\mathbf{V}'-\frac{1}{2}\mathbf{I}\geqslant\mathbf{0}$, i.e., the transformed P-representability condition. The Stone-von Neumann theorem provides that if $\mathbf{M}$ exists it must be unitarily implementable [23]. Finding the corresponding U_{NL} operator may not be a simple exercise, however, and we leave this point for future research.

In conclusion, we have derived a complete description of bipartite Gaussian separable states, and have proved that P-representable states form a subset of the set of all bipartite Gaussian separable states, existent only under special symmetry of the covariance matrix. We can state that for positive definite bipartite Gaussian operators, which describe physical quantum states, P representability is a necessary and sufficient condition for separability only for the subset of locally $\mathrm{Sp}(2,R)\otimes\mathrm{Sp}(2,R)$ invariant Gaussian states [24]. In general $\mathbb{P}\subseteq\mathbb{S}$.

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