Superconducting fluctuations for three-dimensional anisotropic superconductors in the presence of a magnetic field with arbitrary direction

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A nonperturbative method for evaluation of thermodynamic scaling functions in the critical region of type-II superconductors, appropriate for high-temperature superconductors, is extended for the case of external magnetic fields with arbitrary angles with respect to the c axis for the case of three-dimensional anisotropic superconductors. An explicit scaling function for the magnetization is presented, discussed, and compared with experimental data from measurements with applied fields along the ab planes. [S0163-1829(97)01334-9]

I. INTRODUCTION

In the last years interest has grown in the study of properties of strongly type-II superconductors such as high-temperature superconductors (HTSC's), fullerene superconductors, superconducting superlattices, etc. The behavior in the vicinity of $H_{c2}(T)$ is strongly influenced by the thermodynamic fluctuations and many experimental[1–6] as well as theoretical[7–14] results have been reported in relation to the critical behavior arising as a consequence of the thermal fluctuations in these systems. For ordinary superconductors, the superconducting transition is very well described by the Ginzburg-Landau theory (GL), which was shown by Gorkov to be equivalent to the BCS theory in the limit $T \rightarrow T_c$. This is a direct manifestation of mean-field behavior in the strongest form, i.e., where both the order parameter and coefficients of the GL theory can be calculated from the microscopic mean-field theory. However, mean-field theory does not account for most second-order phase transitions. These departures from classical behavior are generally attributed to thermal fluctuations which are neglected in the mean-field approach. Standard estimates of the critical region show that for the case of conventional superconductors the range of temperatures around T_c within which fluctuations are important is so small that is not in general experimentally accessible.[7] The new HTSC's however display the effects of fluctuations in a wide range of temperatures around T_c, the mean-field transition temperature. The effect of fluctuations in HTSC's changes the phase diagram of these materials and new phase boundary lines appear. The true thermodynamic superconducting transition line differs from the mean-field $H_{c2}(T)$ that becomes a crossover line. In fact, Gammel *et al.*[1,2] found experimental evidence for a melting transition from an ordered phase into a high-temperature flux liquid in $YBa_2Cu_3O_{7-\delta}$ single crystals. The strongly temperature-dependent correlations involving up to 10^5 vortices, obtained near the transition, suggest that only many-particle theories displaying a phase transition will be able to explain those results. Ullah and Dorsey[8] have used the time-dependent GL theory to study both transverse and longitudinal transport properties of a layered superconductor in a magnetic field perpendicular to the layers near to the mean-field transition temperature $T_c(H)$.

In an experiment performed by Welp *et al.*,[3] high-precision measurements of magnetization and resistivity of $YBa_2Cu_3O_{7-\delta}$ crystals were performed, near the superconducting transition in magnetic fields applied perpendicular to the CuO_2 layers. These data showed a scaling behavior in the variable $[T-T_c(H)]/(TH)^{2/3}$ which is consistent with the GL fluctuation theory for a three-dimensional (3D) system in a high magnetic field. Li *et al.*[4] noticed that the magnetization of highly anisotropic $Bi_2Sr_2Ca_2Cu_3O_{10}$ crystals near the critical temperature may be described by the 2D version of the scaling function in the variable $[T-T_c(H)]/(TH)^{1/2}$. In a recent work, Tesanovic *et al.*[9–14] have developed a nonperturbative theory of critical behavior for anisotropic superconductors for both 2D and 3D systems. In Ref. 9 Tesanovic considers a strong magnetic field H applied in the direction of anisotropy of the crystal (c axis) and in this approach the critical behavior is described by means of an interacting particle system with long-ranged multiple-body forces (dense vortex plasma). The superconducting transition corresponds to the liquid-solid transition in the dense vortex plasma (DVP). Recently measurements have been reported for the magnetization in $YBa_2Cu_3O_{7-\delta}$ single crystals for applied magnetic fields parallel to the ab planes (perpendicular to the c axis).[5,6] These experimental results are the motivation to generalize Tesanovic's results in the limit of the anisotropic 3D superconductor for the case of an arbitrary direction of the applied magnetic field.

The work is organized as follows: In Sec. II the lowest-Landau-level expansion for the order parameter is generalized for arbitrary directions of the applied magnetic field. In Sec. III it is obtained the partition function of the problem.

Section IV presents the scaling for the magnetization for applied magnetic fields parallel to the ab planes for an anisotropic 3D superconductor. In Sec. V we discuss results of the angular dependence of the coupling constant and the theoretical scaling function for the magnetization is compared to data for $YBa_2Cu_3O_{7-\delta}$. Finally, the conclusions of the work are presented.

II. THE EXPANSION FOR THE ORDER PARAMETER

The problem of fluctuations near $H_{c2}(T)$ for large applied magnetic fields can be represented in terms of the GL theory on a degenerate manifold spanned by the lowest-Landau levels (LLL) for the Cooper pairs. In the GL-LLL description the order parameter for the superconductor state may be written in the form[9–11]

$$\Psi(x,y,z)=(2\pi l^2)^{1/2}\sum_{\nu=0}^{N} b_\nu(z)\phi_{0\nu}(x,y), \tag{1}$$

where $\{\phi_{0\nu}(x,y)\}$ are the LLL orbitals, $N=\Omega/2\pi l^2$, Ω is the sample cross-sectional area and $l=(c/2eB)^{1/2}$ is the magnetic length. The problem is then reduced to the study of fluctuations in $\{b_\nu\}$ as done in perturbation theory.

The LLL orbitals are eigenfunctions of the operator

$$\tilde{H}=\sum_\mu \frac{1}{2m_\mu^*}\left(-i\partial_\mu+\frac{2e}{c}A_\mu\right)^2, \tag{2}$$

where $\nabla\times\mathbf{A}=\mathbf{B}$ and m_μ^* is the effective mass in the μ direction. For layered superconductors $m_x^*=m_y^*=m$, $m_z^*=M$ and the anisotropy ratio, ε, is defined by $\varepsilon^2=m/M<1$. For a magnetic field along the z direction, described by means of the vector potential $\mathbf{A}=B/2(-y,x,0)$, we obtain

$$\tilde{H}=\frac{1}{2ml^2}\left[-(\partial_{x'}^2+\partial_{y'}^2)+i(y'\partial_{x'}-x'\partial_{y'})+\frac{x'^2+y'^2}{4}\right]-\frac{1}{2M}\partial_z^2, \tag{3}$$

where $x'=x/l$ and $y'=y/l$. Since $[P_z,\tilde{H}]=[-i\partial_z,\tilde{H}]=0$ then $P_z=q$ is a constant of the motion and the eigenfunctions of $\tilde{H}$ take the form

$$\phi_{q\nu}(w,z)\propto e^{iqz}w^\nu\exp(-|w|^2/4), \tag{4}$$

with $w=x'+iy'=(x+iy)/l$.

Starting with Eq. (1) it is possible to obtain[9]

$$\Psi(x,y,z)=(2\pi l^2)^{1/2}\sum_{\nu=0}^{N} b_\nu(z)\phi_{0\nu}(w) =\Phi(z)\prod_{i=1}^{N}[w-w_i(z)]\exp(-|w|^2/4), \tag{5}$$

in which $\{w_i(z)\}$ are the positions of the vortices and Φ describes the overall amplitude fluctuations. The variables Φ and $\{w_i\}$ are the natural way of representing arbitrary linear combinations of LLL orbitals.

When the angle between the applied magnetic field $\mathbf{H}$ and the z axis is $\theta=\pi/2$, a suitable gauge is $\mathbf{A}=B/2(z,0,-x)$ and the Hamiltonian may be written as

$$\tilde{H}=\frac{\varepsilon}{2ml^2}\left[-(\partial_{\bar{x}}^2+\partial_{\bar{z}}^2)+i(\bar{x}\partial_{\bar{z}}-\bar{z}\partial_{\bar{x}})+\frac{\bar{x}^2+\bar{z}^2}{4}\right]-\frac{1}{2m}\partial_y^2, \tag{6}$$

which has the eigenfunctions

$$\phi_{q\nu}(w,y)\propto e^{iqy}w^\nu\exp(-|w|^2/4), \tag{7}$$

where $w=\bar{z}+i\bar{x}=(z/\sqrt{\varepsilon}+i\sqrt{\varepsilon}x)/l$. In this case the order parameter $\Psi(w,y)$ looks like Eq. (5) but with the new definition of w.

In order to generalize these results for an arbitrary angle θ between the microscopic magnetic field $\mathbf{B}$ (vortices) and the z axis, we choose the gauge

$$\mathbf{A}=\frac{B}{2}(z\sin\theta-y\cos\theta,x\cos\theta,-x\sin\theta), \tag{8}$$

and obtain the Hamiltonian

$$\tilde{H}=\frac{1}{2ml^2}\left[\left(-\partial_{x'}^2+\frac{\delta^2x'^2}{4}\right)+i\left(-\partial_{y'}^2+\frac{y'^2\cos^2\theta}{4}\right)+\left(-\varepsilon^2\partial_{z'}^2+\frac{\delta^2z'^2\sin^2\theta}{4}\right)+i\cos\theta(y'\partial_{x'}-x'\partial_{y'})+i\sin\theta(\varepsilon^2x'\partial_{z'}-z'\partial_{x'})-\frac{\sin2\theta}{4}y'z'\right], \tag{9}$$

where $x'=x/l$, $y'=y/l$, $z'=z/l$ and $\delta^2=\cos^2\theta+\varepsilon^2\sin^2\theta$ is the angle-dependent anisotropy parameter. In the general case the vectors $\mathbf{H}$ and $\mathbf{B}$ are not parallel, but for large enough fields the internal angle θ becomes equal to the angle θ_H between the external magnetic field and the z axis.

By introducing the following coordinate transformation:[15]

$$\begin{bmatrix} y_1 \\ z_1 \end{bmatrix}=\begin{bmatrix} \cos\theta & -\sin\theta \\ \varepsilon\sin\theta & \varepsilon^{-1}\cos\theta \end{bmatrix}\begin{bmatrix} y' \\ z' \end{bmatrix}, \tag{10}$$

and carrying out the changes $\bar{x}=\sqrt{\delta}x'$, $\bar{y}=y_1/\sqrt{\delta}$ and $\bar{z}=lz_1$, the Hamiltonian (9) is transformed to

$$\tilde{H}=\frac{\delta}{2ml^2}\left[-(\partial_{\bar{x}}^2+\partial_{\bar{y}}^2)+i(\bar{y}\partial_{\bar{x}}-\bar{x}\partial_{\bar{y}})+\frac{\bar{x}^2+\bar{y}^2}{4}\right]-\frac{\delta^2}{2m}\partial_{\bar{z}}^2, \tag{11}$$

which has the eigenfunctions

$$\phi_{q\nu}(w,\bar{z})\propto e^{iq\bar{z}}w^\nu\exp(-|w|^2/4), \tag{12}$$

with $w=\bar{x}+i\bar{y}$.

It is easy to check that in the special cases for $\theta=0$ and $\theta=\pi/2$, the Hamiltonian (11) becomes equal to Eqs. (3) and (6), respectively, corresponding to the limiting cases studied above. For an arbitrary angle θ between $\mathbf{B}$ and the z axis, the order parameter in the superconducting state may be expanded in the form

$$\Psi(x,y,z)=\Phi(\bar{z})\prod_{i=1}^{N}[w-w_i(\bar{z})]\exp(-|w|^2/4), \quad (13)$$

with the corresponding definition for w. Equation (13) is the generalized form of Eq. (5) for an arbitrary direction of the magnetic field.

In order to clarify the significance of the transformation (10), we note that for applied magnetic fields in the direction parallel to the c axis, the eigenfunctions of Eq. (9) describe a free-particle propagation along the field direction and LLL orbitals in the plane perpendicular to the field, which correspond to classical orbits of particles with charge $2e$ in a magnetic field. The plane-wave modes along the field have a continuous spectra with arbitrary small eigenvalues whereas the LLL are separated by gaps and therefore will be only rarely excited. Consequently an anisotropic superconductor in a magnetic field parallel to the c axis will display effectively one-dimensional fluctuations along the magnetic field. For any direction of the magnetic field, different from $\theta=0$ or $\theta=\pi/2$, the Landau orbits are coupled to the phase of the propagation along the field direction. The transformation in Eq. (10) decouples these orbits from the translational motion but now this motion will not be along the field direction. The physical meaning of the solution (12) is that for an arbitrary direction of the external magnetic field the one-dimensional fluctuations will not occur along the field direction but along the $\bar{z}$ axis.

III. THE PARTITION FUNCTION AND COUPLING CONSTANT

If we ignore the fluctuations of the magnetic field (which is an excellent approximation in HTSC's), the essential features of the critical behavior for these superconductors are described by the partition function[9]

$$Z\propto\int D\Psi(\mathbf{r})D\Psi^*(\mathbf{r})\exp\left\{-\frac{1}{k_BT}\int d^3r\left[\bar{\alpha}(T,\theta)|\Psi(\mathbf{r})|^2+\frac{\beta}{2}|\Psi(\mathbf{r})|^4+\frac{n}{2m_\mu^*}\left|\left(\partial_\mu+\frac{2ei}{c}A_\mu\right)\Psi(\mathbf{r})\right|^2\right]\right\}, \quad (14)$$

where $\bar{\alpha}(T,\theta)=\alpha(T)[1-H/H_{c2}(T,\theta)]$, α and β are the GL coefficients, n is the electron density, and the functional integral is to be taken over the subspace spanned by the LLL. It is supposed that high Landau levels are taken into account only by the renormalization of the GL parameters $\bar{\alpha}(T,\theta)$ and β, and these parameters differ from the original ones for the GL model for $H=0$ due to the contribution of high Landau levels. For large enough fields this contribution is small.

By introducing expansion (13) for the order parameter in the partition function (14), we obtain the expression

$$Z\propto\frac{1}{N!}\int\prod_{i=1}^{N}\frac{dw_i(\bar{z})dw_i^*(\bar{z})}{2\pi}\prod_{i<j,\bar{z}}^{N}|w_i(\bar{z})-w_j(\bar{z})|^2\int d\Phi(\bar{z})d\Phi^*(\bar{z})(\Phi(\bar{z})\Phi^*(\bar{z}))^N$$
$$\times\exp\left\{-\frac{\varepsilon}{\delta^2}\frac{2\pi l^2N}{k_BT}\int d\bar{z}\left[\bar{\alpha}(T,\theta)\langle f^2(\bar{z})\rangle|\Phi(\bar{z})|^2+\frac{\beta}{2}\langle f^4(\bar{z})\rangle|\Phi(\bar{z})|^4+|\bar{\alpha}(T,\theta)|\xi_{\bar{z}}^2\langle|\partial_{\bar{z}}(\Phi(\bar{z})f(\bar{z}))|^2\rangle\right]\right\}, \quad (15)$$

where we have used the definitions

$$f(w|\{w_i(\bar{z})\})=\prod_{i=1}^{N}[w-w_i(\bar{z})],$$

$$\langle f^p(\bar{z})\rangle=\int\frac{dw(\bar{z})dw^*(\bar{z})}{2\pi}\times\exp(-p|w|^2/4)f^p(w|\{w_i(\bar{z})\}),$$

and

$$\xi_{\bar{z}}^2=\frac{\delta^2 n}{2m|\bar{\alpha}(T,\theta)|}.$$

For an explicit evaluation of the partition function, the functional integrals, $\Pi_{\bar{z}}$, $\partial_{\bar{z}}$, etc., may be defined on a set of discrete intervals of size Λ and at the end of the calculation the limit $\Lambda\rightarrow 0$ is to be taken. However it is necessary to define a cutoff length, of the order of the correlation length, because the exact free energy is actually divergent in the $\Lambda\rightarrow 0$ limit.[13]

The integration over Φ in Eq. (15) may be carried out and leads to the many-body system of interacting particles $\{w_i(\bar{z})\}$, obeying all the symmetries of the problem. The variable $\Phi(\bar{z})$ is a classical variable whose contribution to Z for 3D systems, in the thermodynamic limit, is controlled by the nontrivial saddle point

$$g\langle f^2(\bar{z})\rangle\Phi(\bar{z})+\frac{1}{2}\langle f^4(\bar{z})\rangle|\Phi(\bar{z})|^2\Phi(\bar{z})-\frac{1}{\Phi^*(\bar{\bar{z}})}$$
$$=|g|\langle f^*(\bar{z})\partial_{\bar{z}}^2(\Phi(\bar{z})f(\bar{z}))\rangle, \quad (16)$$

where the lengths are being measured in units of $\xi_{\bar{z}}$, $\Phi(\bar{z})$ has been suitably rescaled and the coupling constant $g(T,H,\theta)$ is a pure number defined by

$$g(T,H,\theta)=\frac{\sqrt{\epsilon}}{\delta}\left(\frac{2\pi l^2\xi_{\bar{z}}}{2\beta k_BT}\right)^{1/2}\bar{\alpha}(T,\theta). \quad (17)$$

The properties of the many-body system $\{w_i(\overline{z})\}$ are determined by the coupling constant g. The value $g(T,H,\theta)=0$ corresponds to $H_{c2}(T,\theta)$, whereas the solid-liquid transition line may be obtained from the melting value $g(T,H,\theta)=g_M(\theta)<0$.

The coupling constant g may be used for the evaluation of the thermodynamic scaling functions[10,13] in both 2D and 3D cases. In Ref. 13 Tesanovic and Andreev, by introducing a suitable rescaling on the partition function (15) for magnetic fields parallel to the c axis (angle $\theta=0$), obtained simple scaling functions for the free energy, magnetization and specific heat. Their calculations are in a very good agreement with experimental data.[3] Given the angular dependence of the partition function (15), it is always possible to obtain similar scaling functions for any direction of the external magnetic field in the 3D case. The special case of $\theta=\pi/2$, for which there is data available, will be discussed in the next section and compared with those data for $YBa_2Cu_3O_{7-\delta}$ in Sec. V.

In the limiting cases, $\theta=0$ and $\theta=\pi/2$, the coupling constant g is reduced to

$$g(T,H,0)=\left(\frac{2\pi l^2\xi_c}{2\beta k_B T}\right)^{1/2}\overline{\alpha}(T,0), \tag{18a}$$

and

$$g(T,H,\pi/2)=\left(\frac{2\pi l^2\xi_{ab}}{2\beta k_B T}\right)^{1/2}\overline{\alpha}(T,\pi/2). \tag{18b}$$

Equation (18a) for the coupling constant corresponds to Tesanovic's coupling constant for the 3D case for a magnetic field applied along the c axis of the crystal,[9] while Eq. (18b) gives the result for $\theta=\pi/2$ which corresponds to the applied field along the ab planes.

IV. SCALING OF THE MAGNETIZATION FOR MAGNETIC FIELDS ALONG THE *ab* PLANES

For an applied magnetic field parallel to the ab planes ($\theta=\pi/2$) we obtain, from the partition function (15), the expression for the free energy

$$\frac{F(T,H)}{k_BT}=\frac{2\pi l^2N}{k_BT}\int dy\bigg[\overline{\alpha}(T,\pi/2)\langle f^2(y)\rangle|\Phi(y)|^2 + \frac{\beta}{2}\langle f^4(y)\rangle|\Phi(y)|^4 + |\overline{\alpha}(T,\pi/2)|\xi_{ab}^2\langle|\partial_y(\Phi(y)f(y))|^2\rangle\bigg], \tag{19}$$

where $\xi_{ab}(T,H)=[n/2m|\overline{\alpha}(T,\pi/2)|]^{1/2}$ is the correlation length along the ab planes.

By following the same procedure as that in Ref. 13 for the 3D case and introducing the usual expressions for the GL coefficients, the scaling function for the magnetization may be written in the form

$$\frac{M}{(TH)^{2/3}}\,\frac{2(4\pi)^{1/3}(\phi_0\kappa_{ab}\xi_{ab}(0))^{2/3}T_c|H'_{c2}|}{(k_B\epsilon H^{ab}_{c2}(0))^{2/3}} =\left(g_1+\sqrt{g_1^2+\frac{\tan^{-1}\pi}{\pi U^2}}\right)^{1/3}(GU^2-U\sqrt{G^2U^2+2}), \tag{20}$$

where H^{ab}_{c2} is the upper critical field parallel to the ab planes, H'_{c2} is the slope of that field at $T=T_c$, κ_{ab} is the GL parameter, and g_1 is related to the scaling variable $t=[T-T_c(H)]/(TH)^{2/3}$ by means of the equation

$$g_1\left(g_1+\sqrt{g_1^2+\frac{\tan^{-1}\pi}{\pi U^2}}\right)^{1/3} =\frac{[\phi_0\xi_{ab}(0)\epsilon^2H^{ab2}_{c2}(0)]^{2/3}}{2(16\pi^2\kappa_{ab}^4k_B^2)^{1/3}T_c}t. \tag{21}$$

The quantities U and G depend on g_1 in the form[13]

$$U(g_1)=0.818-0.110\tanh\left(\frac{G+\sqrt{2}}{2\sqrt{2}}\right), \tag{22}$$

and

$$G=g_1+\frac{\pi-\tan^{-1}\pi}{2\tan^{-1}\pi}\left(g_1+\sqrt{g_1^2+\frac{\tan^{-1}\pi}{\pi U^2}}\right). \tag{23}$$

For each value of the scaling variable t results two coupled equations for g_1 and G, which may be decoupled taking for U the mean value $U_0\approx0.8$ wherever it appears on the right-hand side of Eq. (22).

V. RESULTS AND DISCUSSION

In Figs 1 and 2, the angular dependence of $g(T,H,\theta)/g(T,H,0)$ is shown for points located in several

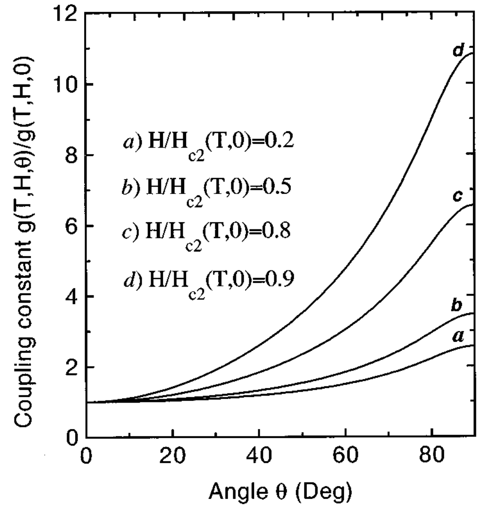

FIG. 1. Angular dependence of the coupling constant for some points below $H_{c2}(T,0)$ in the phase diagram, for the values of $H/H_{c2}(T,0)$: (a) 0.2, (b) 0.5, (c) 0.8, and (d) 0.9.

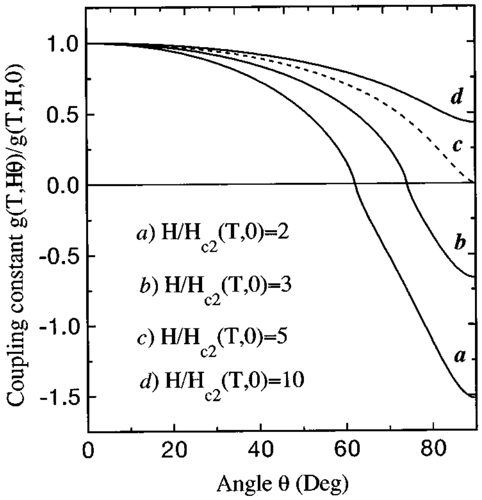

FIG. 2. Angular dependence of the coupling constant for some points above $H_{c2}(T,0)$ in the phase diagram, for the values of $H/H_{c2}(T,0)$: (a) 2, (b) 3, (c) 5, and (d) 10. The dashed line corresponds to points placed on the line $H_{c2}(T,\pi/2)=5H_{c2}(T,0)$.

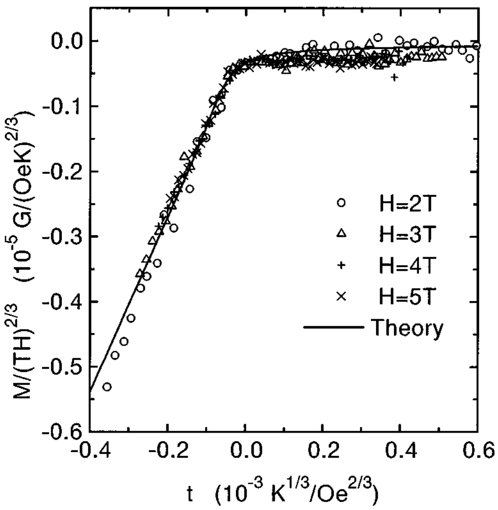

FIG. 3. Experimental magnetization for $YBa_2Cu_3O_{7-\delta}$ for different values of the magnetic field applied parallel to the ab planes (Ref. 5). The data are scaled and plotted with the theoretical expression (solid curve) from Eq. (20), appropriate for 3D scaling.

regions of the phase diagram. It was assumed the usual angular dependence $H_{c2}(T,\theta)=H_{c2}(T,0)/\delta$ for the upper critical field[16] and the value $\epsilon=1/5$ for the anisotropy ratio, which corresponds to the case of $YBa_2Cu_3O_{7-\delta}$.

Figure 1 displays the angular dependence of $g(T,H,\theta)/g(T,H,0)$ for some values of the applied field H. It corresponds to the region below $H_{c2}(T,0)$ in the phase diagram. In this region the sign of $g(T,H,\theta)$ is always negative [the same as $g(T,H,0)$] because these points lie below $H_{c2}(T,0)$ for any direction of the magnetic field. As the angle θ is increased, the absolute value of $g(T,H,\theta)$ rises until reaching its highest value for $\theta=\pi/2$ due to the fact that the upper critical field line $H_{c2}(T,\theta)$ moves away from the point in which $g(T,H,\theta)$ is calculated.

Figure 2 presents $g(T,H,\theta)/g(T,H,0)$ as a function of the angle θ for several values of the applied magnetic field corresponding to points in the region limited by the lines $H_{c2}(T,0)$ and $H_{c2}(T,\pi/2)$, as well as for the region above the line $H_{c2}(T,\pi/2)$ in the phase diagram. For any point in the region limited by $H_{c2}(T,0)$ and $H_{c2}(T,\pi/2)$, the value of $g(T,H,0)$ is positive, and on the other hand, $g(T,H,\theta)$ changes sign from positive to negative, because as θ is increased the line $H_{c2}(T,\theta)$ moves from $H_{c2}(T,0)$ towards $H_{c2}(T,\pi/2)$ and at a certain angle this line crosses that point (curves a and b). In the region above $H_{c2}(T,\pi/2)$ both $g(T,H,0)$ and $g(T,H,\theta)$ are positive and the value of $g(T,H,\theta)$ diminishes as the angle θ is increased since the line $H_{c2}(T,\theta)$ comes near to the point where $g(T,H,\theta)$ is calculated (curve d).

For points in the phase diagram far from the region limited by $H_{c2}(T,0)$ and $H_{c2}(T,\pi/2)$, corresponding to values of $H/H_{c2}(T,0)\gg 5$ or $H/H_{c2}(T,0)\ll 1$, the angular dependence of the coupling constant is smoothed and $g(T,H,\theta)$ tends to the limiting value $g(T,H,0)$ in both cases. As one approaches the line $H_{c2}(T,0)$, the value of $g(T,H,0)$ tends to zero and the quantity $g(T,H,\theta)/g(T,H,0)$ rises quickly in absolute value as the angle θ is increased.

The results for the magnetization in applied magnetic fields parallel to the ab planes presented in Sec. IV, are discussed and compared with previously published experimental data.[5] Figure 3 displays the behavior of the magnetization for 3D anisotropic superconductors using Eq. (20). The theoretical expression (20) is plotted and compared with the recent measurements in $YBa_2Cu_3O_{7-\delta}$ for different applied magnetic fields parallel to the ab planes of the crystal.[5] It was assumed the values $H'_{c2}=-8.6$T/K for the slope of the upper critical field.[5] $\xi_{ab}(0)=16$ Å for the in-plane correlation length,[17] and $H_c(0)=\epsilon H_{c2}^{ab}(0)/\sqrt{2}\kappa_{ab}\approx 1.2$ T for the thermodynamic critical field.[18] These data have been analyzed in a previous work,[5] showing 3D scaling behavior. The present analysis shows that the scaled data agree rather well with the theoretical scaling function (20). In order to perform an internal check of the theory, a similar analysis was made

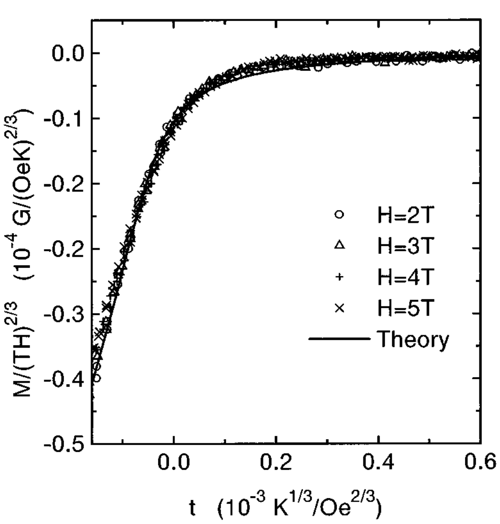

FIG. 4. Experimental magnetization for $YBa_2Cu_3O_{7-\delta}$ for different values of the magnetic field applied parallel to the c axis (Ref. 6). The data are scaled and plotted with the theoretical expression (solid curve) from Ref. 13, appropriate for 3D scaling.

for the experimental data in Ref. 6 for applied magnetic fields parallel to the c axis. In this analysis the scaling function of the magnetization for $\theta=0$ of the present angular-dependent model was used (which is the result previously presented by Tesanovic and Andeev,[13]). Figure 4 shows the corresponding scaling and the experimental data. The good agreement shown is similar to that of Fig 3.

In summary, we have obtained a description of the critical behavior in terms of the dense vortex plasma for any direction of the applied magnetic field and generalized the expression for the coupling constant g, which determines the properties of the DVP, for the case of an arbitrary angle θ. Our result for the coupling constant reduces to that of Tesanovic[9] in the limit $\theta=0$. An important matter to point out is that for an arbitrary direction of the external magnetic field, the anisotropic superconductor displays one-dimensional fluctuations in a direction different from that of the magnetic field. We have also obtained a scaling function for the magnetization in the presence of magnetic fields parallel to the ab planes. Our theoretical results are in good agreement with recent experimental data.

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