

Light propagation and Anderson localization in disordered superlattices containing dispersive metamaterials: Effects of correlated disorder

D. Mogilevtsev,^{1,2} F. A. Pinheiro,³ R. R. dos Santos,³ S. B. Cavalcanti,⁴ and L. E. Oliveira²

¹*Institute of Physics, NASB, F. Skarina Ave. 68, Minsk, 220072, Belarus*

²*Instituto de Física, UNICAMP, Campinas-SP, 13083-859, Brazil*

³*Instituto de Física, Universidade Federal do Rio de Janeiro, Rio de Janeiro-RJ, 21941-972, Brazil*

⁴*Instituto de Física, Universidade Federal de Alagoas, Maceió-AL, 57072-970, Brazil*

(Received 16 June 2011; published 21 September 2011)

We have investigated the effects of disorder correlations on light propagation and Anderson localization in one-dimensional dispersive metamaterials. We consider and compare the cases where disorder is uncorrelated to situations where it is totally correlated and anticorrelated. The photonic gaps of the corresponding periodic structure are not completely destroyed by the presence of disorder, which leads to minima in the localization length. In the vicinities of a gap, the behavior of the localization length depends crucially on the physical origin of the gap (Bragg or non-Bragg gaps). Within a Bragg gap, the localization length increases as the degree of disorder increases, an anomalous behavior that only occurs for the uncorrelated and completely correlated cases. In these cases, minima of the localization length at the positions of Bragg gaps are shifted by increasing disorder, which does not occur for the anticorrelated case, where the positions of the minima remain unaltered. Minima in the localization length corresponding to non-Bragg gaps are not shifted by increasing disorder, albeit the widths of these minima are changed. We have found that the asymptotic behavior for the localization length $\xi \propto \lambda^6$ for disordered metamaterials is not affected by correlations. Finally, we have investigated the role of absorption on the delocalized Brewster modes and argue that it could be mitigated in light of the state-of-the-art of metamaterials research.

DOI: [10.1103/PhysRevB.84.094204](https://doi.org/10.1103/PhysRevB.84.094204)

PACS number(s): 42.25.Dd, 78.67.Pt, 72.15.Rn

I. INTRODUCTION

After more than 50 years of the pioneer work of Anderson,¹ strong localization remains a lively topic of research. This can be partly explained by its interdisciplinary character and by the many aspects remaining to be understood. On the one hand, the concept of Anderson localization, an interference wave phenomenon originally conceived as the vanishing of electronic diffusion in disordered systems, was extended to classical wave systems. Without being hampered by interactions, as in electronic systems, Anderson localization has been observed in classical wave systems, such as microwaves, light, and ultrasound.² The impact of disorder on light propagation and localization in photonic crystals has been investigated both theoretically and experimentally.³ In quantum systems, Anderson localization has recently been observed in Bose-Einstein condensates.² As a result, Anderson localization is today a truly interdisciplinary topic, and important contributions have emerged from different areas, ranging from condensed matter, photonics, acoustics, atomic physics, and seismology. On the other hand, a complete understanding of Anderson localization is still far from being achieved, especially in higher dimensions ($d > 1$), both in quantum and classical systems. While in the former the interplay between interaction and localization effects remains a challenge, in the latter, the roles of absorption,⁴⁻⁶ gain,⁷ and polarization⁸ are the subject of intense research.

The interest in Anderson localization has also been aroused by the development of new materials supporting (classical and quantum) wave propagation in disordered systems. For electronic systems, the investigation of Anderson localization in graphene has revealed a wealth of new physics and

stimulated a lively debate.⁹ For classical systems, and for light in particular, the rapid progress in the new field of metamaterials also leads to interesting peculiarities in Anderson localization. Metamaterials, which are artificial structures with electromagnetic properties that can be engineered, can exhibit negative refraction,¹⁰ resolve images beyond the diffraction limit,¹¹ exhibit optical magnetism,^{12,13} act as an electromagnetic cloak,^{14,15} and yield slow light propagation.¹⁶ The presence of negative refraction in one-dimensional (1D) disordered metamaterials strongly suppresses Anderson localization due the lack of phase accumulation during wave propagation, thus weakening interference effects necessary for localization;^{17,18} this mechanism leads to an unusual behavior of the localization length ξ at long wavelengths λ , $\xi \propto \lambda^6$ instead of the well-known¹⁹ $\xi \propto \lambda^2$ asymptotic behavior.

More recently, it was established²⁰ that other routes to suppress Anderson localization of light in one-dimensional disordered metamaterials are possible through a combination of oblique incidence and dispersion. The effects of polarization and oblique incidence on light propagation in disordered metamaterials were also treated in Ref. 21. On the one hand, oblique incidence allows a polarization-induced delocalization effect, known as Brewster anomalies,²² to set in for one-dimensional disordered structures; on the other hand, metamaterials are necessarily dispersive to ensure positiveness of the electromagnetic energy density.²³ Reference 20 demonstrates that, indeed, for specific combinations of angle of incidence and frequency, Brewster anomalies are present (for both states of polarization, and not necessarily in the negative refraction regime), thus effectively delocalizing light. It was also established that the λ^6 asymptotic behavior of the localization length does not hold for sufficiently large angles of incidence. Finally, it was

demonstrated that delocalization associated with the Brewster anomaly may occur in the frequency region corresponding to the very edge of a band gap of the periodic structure.²⁰

Notwithstanding the wide range of new aspects of Anderson localization of light, unveiled by the presence of metamaterials, consideration of the effects of correlation on disorder has not been addressed in this context so far. Correlated disorder is generally known to induce an anomalous delocalization in 1D systems,^{24–26} for which the vast majority of states is localized in the case of uncorrelated disorder.¹⁹ While the effect of correlations in disordered photonic crystals has been investigated in Ref. 3, the impact of disorder correlations in metamaterials exhibiting negative refraction remains, as far as we know, unexplored.

The purpose of this paper is thus to investigate Anderson localization and light propagation through randomly perturbed 1D photonic heterostructures composed of alternating layers of nondispersive right-handed (RH) materials (labeled *A*) and dispersive left-handed (LH) metamaterials (label *M*). By allowing for dispersion of both dielectric permittivity and magnetic permeability of the *M* layers, together with oblique incidence, we will establish the impact of disorder correlations and anticorrelations in light propagation through disordered metamaterials exhibiting negative refraction.

This paper is organized as follows. Section II is devoted to the description of the model and of the methodology. In Sec. III, the results are presented and discussed, while Sec. IV is reserved for a summary of conclusions.

II. MODEL

We model our system as a stack of alternating layers of air ($\epsilon_A = \mu_A = 1$) and of a Drude-type metamaterial, with responses for the dielectric permittivity and magnetic permeability of the *M* layer given as²⁷

$$\epsilon_M(\omega) = \epsilon_0 - \frac{\omega_e^2}{\omega^2}, \quad \mu_M(\omega) = \mu_0 - \frac{\omega_m^2}{\omega^2}, \quad (1)$$

such that $v_e = \omega_e/(2\pi\sqrt{\epsilon_0})$ and $v_m = \omega_m/(2\pi\sqrt{\mu_0})$ are the frequencies associated with the electric and magnetic plasmon modes, respectively. The electric and magnetic plasma frequencies are $\omega_p^e = \omega_e/\sqrt{\epsilon_0}$ and $\omega_p^m = \omega_m/\sqrt{\mu_0}$, respectively. We have followed previous work²⁷ and use $\epsilon_0 = 1.21$ and $\mu_0 = 1.0$ in Eq. (1). Disorder is introduced by allowing the widths of the *A* and *M* components at the *j*th layer to fluctuate around their respective mean values *a* and *b*: $a_j = a + \delta_j^A$ and $b_j = b + \delta_j^M$, where the random variables $\delta_j^{A,M}$ are homogeneously distributed in the interval $[-\Delta/2, \Delta/2]$. Here, we take $a = b = 12$ mm and $\omega_e = \omega_m = 6\pi$ GHz. The localization length ξ is calculated numerically using the standard definition¹⁹

$$\xi^{-1} = - \lim_{L \rightarrow \infty} \left\langle \frac{\ln |T|}{L} \right\rangle, \quad (2)$$

where *T* is the transmission coefficient and *L* is the total stack length $L = \sum_{j=1}^N (a_j + b_j)$, with *N* being the total number of double layers; $\langle \dots \rangle$ denotes configurational average.

The transmission coefficient is given by²⁸

$$T = \frac{2Z}{Z(M_{22} + M_{11}) - Z^2 M_{12} - M_{21}}, \quad (3)$$

where $Z = \cos \theta$, with θ being the angle of incidence, and the elements of the transfer matrix M_{ij} are defined as

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \quad \mathbf{M} = \prod_{j=1}^N \mathbf{M}_j^{(A)} \mathbf{M}_j^{(M)}, \quad (4)$$

with

$$\mathbf{M}_j^{(x)} = \begin{bmatrix} \cos q_x x & i f_x^{-1} \sin q_x x \\ i f_x \sin q_x x & \cos q_x x \end{bmatrix}, \quad x = A, M \quad (5)$$

$q_x = (\omega/c)u_x(\omega, \theta)$, $u_x(\omega, \theta) \equiv \sqrt{\epsilon_x(\omega)\mu_x(\omega) - \sin^2 \theta}$; for incident transverse electrical (TE) and transverse magnetic (TM) waves, the coefficients f_x are

$$f_x^{\text{TE}} = \frac{u_x(\omega, \theta)}{\mu_x(\omega)}, \quad f_x^{\text{TM}} = \frac{u_x(\omega, \theta)}{\epsilon_x(\omega)}. \quad (6)$$

For an infinitely periodic structure without disorder, Eqs. (5) and (6) lead to the dispersion relation

$$\cos(kd) = \cos(q_A a) \cos(q_M b) - \frac{F_{\pm}}{2} \sin(q_A a) \sin(q_M b), \quad (7)$$

where $F_{\pm} = (f_A/f_M) \pm (f_M/f_A)$, with f_x corresponding to TM or TE waves; $d = a + b$ is the period of the system and *k* is the Bloch wave vector along the direction of the axis of the periodic photonic crystal.

The numerical simulations are supplemented by a generalization (to the case of oblique incidence) of an analytic expression for ξ derived by Izrailev and Makarov (IM) for bilayered photonic structures,²⁹ valid for weak disorder (small fluctuating widths in our case)

$$\xi^{-1} = \frac{F_{\pm}^2}{8d \sin^2(kd)} \left[q_A^2 \sigma_A^2 \sin^2(q_M b) + q_M^2 \sigma_M^2 \sin^2(q_A a) - 2q_A q_M \sigma_{AM} \sin(q_A a) \sin(q_M b) \cos(kd) \right], \quad (8)$$

where

$$\sigma_A^2 = \langle \delta_A^2 \rangle, \quad \sigma_M^2 = \langle \delta_M^2 \rangle, \quad \sigma_{AM} = \langle \delta_A \delta_M \rangle. \quad (9)$$

It is important to mention that Eq. (8) was obtained by the inverse of the Lyapunov exponent.²⁹ For the vast majority of one-dimensional disordered systems, the localization length and the inverse Lyapunov exponent coincide. A unique exception so far is a random structure composed of alternate layers of positive and negative refractive materials, where it has been recently shown that the inverse Lyapunov exponent and the localization length may differ by a numerical prefactor in the long-wavelength limit.¹⁸ For homogeneous random perturbations with the same amplitude on both layers, one has $\sigma_A^2 = \sigma_M^2 = \Delta^2/12$. In the case of uncorrelated disorder, $\sigma_{AM} = 0$. For completely correlated disorder, where the widths of both layers of the double stack are simultaneously changed by the same amount, $\sigma_{AM} = \Delta^2/12$. For completely anticorrelated disorder, when the total width of the double stack is constant, we take $\sigma_{AM} = -\Delta^2/12$.

III. RESULTS AND DISCUSSIONS

First, it is instructive to compare the analytical IM approximation for the localization length ξ [Eq. (8)] with the numerical simulations [Eq. (2)]. Figure 1 exhibits the normalized localization length as a function of frequency, calculated using both methods, for the cases of uncorrelated and correlated disorder, and incidence angle $\theta = \pi/6$. It is important to mention that we have confirmed in the numerical simulations that the system is self-averaging, i.e., the behavior of ξ calculated from a single realization of disorder for a system made up of a sufficiently large number of layers does not differ significantly from that obtained through Eq. (2), considering many disorder realizations for a system not so large. We have considered 100 realizations of the system consisting of 5000 double layers.

In Ref. 20, we mentioned briefly that the IM approximation fails in the cases of moderate to strong disorder, as expected; we now take a closer look at this. From Fig. 1(a), we see that the IM approximation leads to dips in the localization length at three different frequencies. The lowest one corresponds to

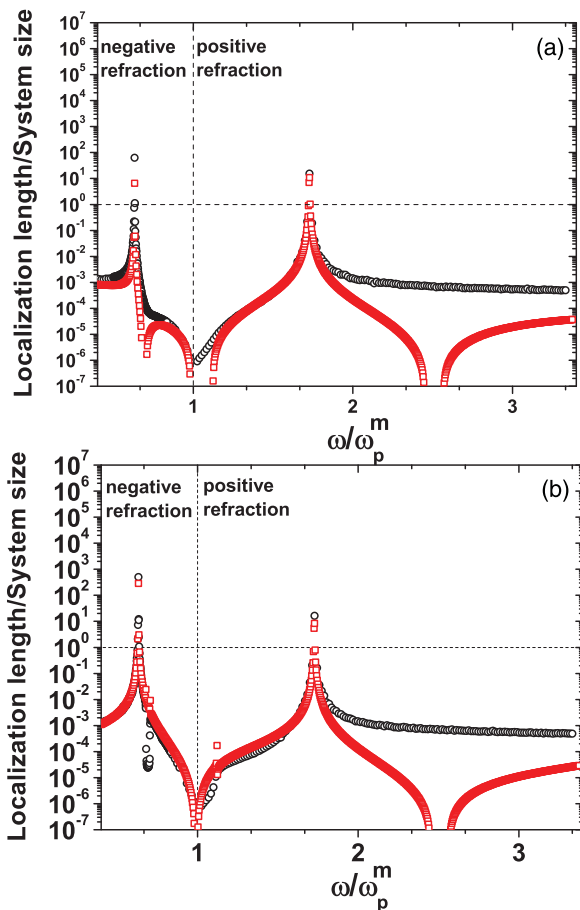


FIG. 1. (Color online) Comparison between the localization length (in units of the system size), given by the IM approximation [Eq. (8)] (open red squares) and numerical simulations with $\Delta = 24$ nm (open black circles), as a function of normalized frequency for an angle of incidence $\theta = \pi/6$ and TE polarization: (a) uncorrelated, and (b) correlated disorder. The horizontal dashed line marks the transition from Anderson localized to delocalized situations.

the vanishing of the average refraction index of the periodic structure $\langle n(\omega) \rangle = 0$, which occurs when

$$an_A(\omega) + bn_M(\omega) = 0, \quad (10)$$

where $n_x = \sqrt{\epsilon_x(\omega)\mu_x(\omega)}$ ($x = A, M$) is the refractive index of each stack composing the double layer; from now on, we will refer to this as the $\langle n(\omega) \rangle = 0$ gap. For the specific case of Fig. 1(a), this dip occurs at $\omega/\omega_p^m \approx 0.67$. Our numerical simulations show that this behavior is an artifact of the approximation, as it becomes a shoulder; in the correlated case [Fig. 1(b)], the IM approximation predicts a smooth behavior at this frequency, while the simulations show a dip. The localization length also dips at the so-called plasmon-polariton gaps, which correspond to the excitation of electric and magnetic (for incident TM and TE waves, respectively) plasmon polaritons;³⁰ they only show up at oblique incidence ($\theta \neq 0$) at frequencies for which $\mu_M(\omega) = 0$ for TE waves, and $\epsilon_M(\omega) = 0$ for TM waves, so that this takes place at $\omega/\omega_p^{m,e} = 1$. For these gaps, we note that the agreement between the IM approximation and the simulations is good for both uncorrelated and correlated disorder, even for an extremely large random perturbation ($\Delta = 24$ nm). The most serious disagreement between the IM approximation and simulations lies in the behavior at, and near, the Bragg gap, which in the present case occurs at $\omega/\omega_p^m \approx 2.5$; we will discuss their behavior with disorder strength below. The peaks in the localization length correspond to Brewster modes, which, for dispersive magnetic metamaterials, can occur for both TM and TE modes, and at the vicinities of a band gap.²⁰ Figure 1 indicates that the IM approximation correctly predicts the positions in frequency for the occurrence of a Brewster mode at a given incidence angle. Within the IM approximation, the Brewster anomalies correspond to the situation where the localization length diverges, which occurs at the zeros of the function F_- , defined below Eq. (7). Since the IM approximation breaks down near two of the gaps (even for small values of the disorder strength), from now on we calculate the localization length using solely numerical simulations; nonetheless, the fact that the IM approximation correctly predicts the positions of the Brewster anomalies justifies the use of Eq. (8) to discuss this effect.

In Fig. 2, we show the localization length ξ as a function of $2\pi c/\omega$ [which from now on we will refer to as the (vacuum) wavelength λ] for normal incidence $\theta = 0$ and for four different values of the disorder strength Δ . Figure 2 illustrates that the asymptotic behavior $\xi \propto \lambda^6$, in the long-wavelength limit, remains valid for the cases of totally correlated disorder and anticorrelated disorder, similar to what occurs for the uncorrelated case.^{17,18,20} This anomalous behavior is in contrast to what is generally expected for disordered systems,¹⁹ and results from the lack of phase accumulation as light propagates through stacks alternating positive and negative refraction.^{17,18,20} It is important to emphasize that this asymptotic behavior does not occur for sufficiently large incidence angles.²⁰

In Fig. 3, the localization length for uncorrelated disorder and oblique incidence is shown as a function of frequency for different disorder strengths Δ for both TM and TE

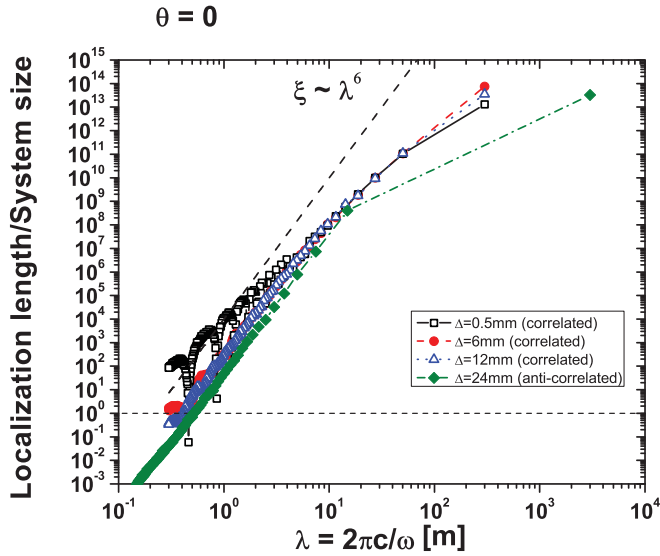


FIG. 2. (Color online) Localization length ξ (in units of the system size) as a function of the vacuum wavelength λ , obtained from numerical simulations, for different values of the disorder strength $\Delta = 0.5, 6,$ and 12 mm. The case of anticorrelated disorder is also shown ($\Delta = 24$ mm). The dashed line corresponds to the asymptotic behavior $\xi \propto \lambda^6$. The horizontal dashed line marks the transition from Anderson-localized to delocalized situations.

polarizations. It is clear that the gaps corresponding to perfectly periodic systems are not completely washed out, even for significantly large disorder strengths (e.g., $\Delta = 12$ mm), resulting in dips of the localization length. However, for the strongly disordered case $\Delta = 24$ mm, the Bragg gap is completely destroyed. The overall effect of increasing Δ is to decrease ξ , as expected, except at the vicinities of photonic band gaps. For the Bragg gaps occurring in the region of positive refraction of the metamaterial, one can see that the increase of Δ results in an increase of ξ , in contrast to what is generally expected. This anomalous effect has already appeared in Ref. 3; we attribute it to the fact that, within the Bragg gap of the periodic structure, disorder weakens the interference effect, which suppresses light transmission. As a result, light transmission is enhanced within Bragg gaps and consequently ξ increases. It is also important to notice that the position of the Bragg gaps shifts toward smaller frequencies as Δ increases. This phenomenon occurs because modifying Δ implies changing the width of the double stack, hence altering the interference conditions necessary for the formation of a Bragg gap. As a result, the Bragg gaps will show up at different frequencies for different values of Δ . By comparing Figs. 2(a) and 2(b), one can notice that this shift is more evident for TM polarization than for the TE one. This fact can be explained since, for TM polarization, the localization length maxima associated with the Brewster anomaly occur far from the Bragg gap.

For gaps whose origin does not rely on the geometry of the structure, such as the plasmon-polariton gap, the effect of disorder on ξ is different. Indeed, from Fig. 3, it is clear that the position of the plasmon-polariton gap at $\omega/\omega_p = 1$ is not modified by the presence of disorder since, within our model, disorder is only incorporated into the widths of the stacks, and not on the optical properties.

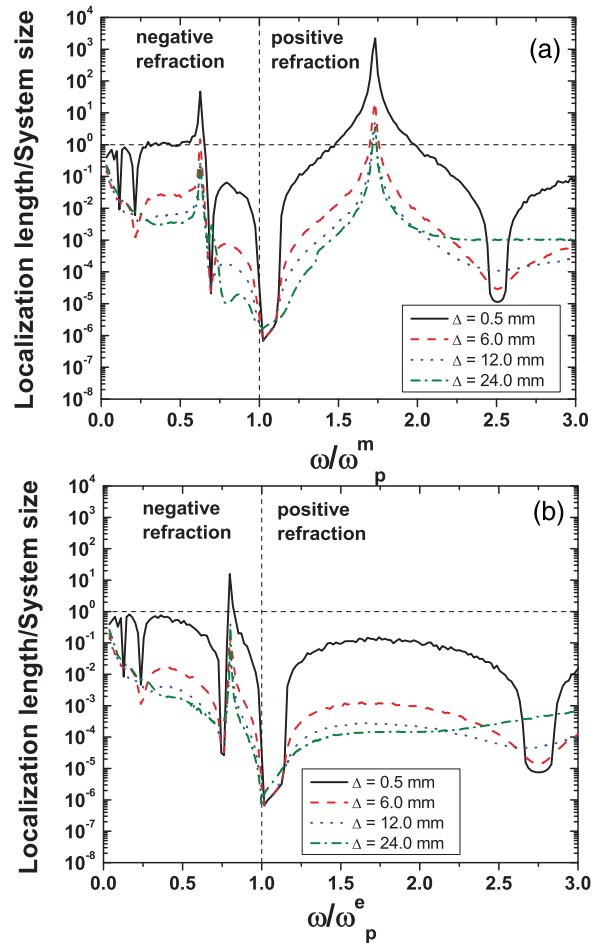


FIG. 3. (Color online) Localization length ξ (in units of the system size), obtained by numerical simulations, as a function of the normalized frequency for uncorrelated disorder and for different values of the disorder strength $\Delta = 0.5, 6, 12,$ and 24 mm for both (a) TE and (b) TM polarizations. The incidence angle is $\theta = \pi/6$. The horizontal dashed line marks the transition from Anderson-localized to delocalized situations.

Except for very large disorder strengths ($\Delta = 24$ mm), the value of ξ is not significantly altered by increasing the degree of disorder within the plasmon-polariton gap. At the plasmon-polariton gap edges, ξ decreases as Δ increases, as expected, leading to a widening of the gap. Similarly, the $\langle n(\omega) \rangle = 0$ gap is not significantly affected by the presence of disorder: it occurs at approximately the same frequencies as for the corresponding perfectly ordered structure, obtained by condition (10). Disorder tends to slightly deform $\langle n(\omega) \rangle = 0$ gaps, making them shallower and wider. Even strong disorder does not obliterate it completely. The reason for the robustness of this non-Bragg gap is exactly the uncorrelated nature of disorder. However, as it will be shown below, the impact of correlated and anticorrelated disorder in the behavior of $\langle n(\omega) \rangle = 0$ gaps is different from the uncorrelated case. From Fig. 3, it is possible to notice that the position of the Brewster modes are not affected by increasing Δ . Again, this can be explained by our model of disorder, in which the randomness is included in the widths of the stacks and not in the optical properties.

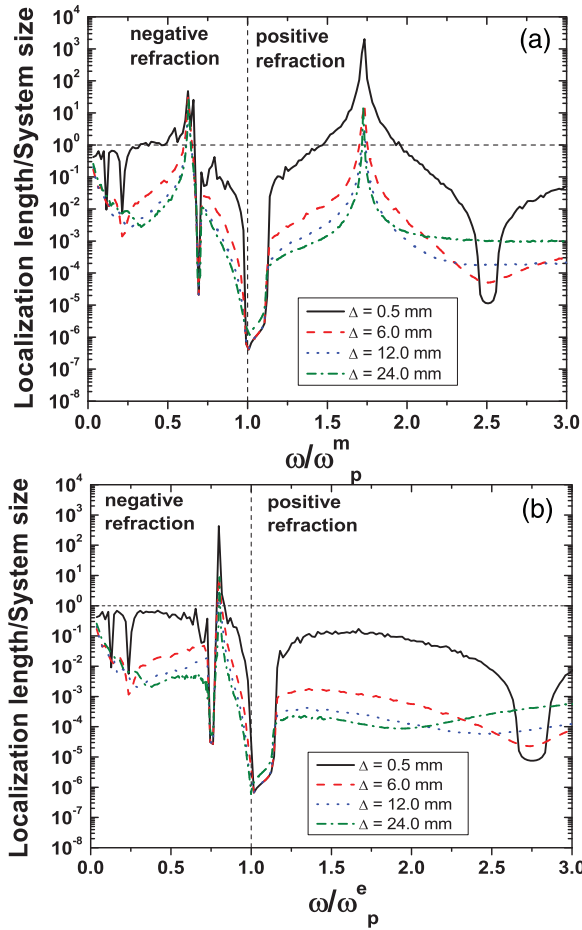


FIG. 4. (Color online) Localization length ξ (in units of the system size), obtained by numerical simulations, as a function of the normalized frequency for correlated disorder and for different values of the disorder strength Δ : (a) TE and (b) TM polarizations; the parameters used are the same as in Fig. 3.

Data for the case of totally correlated disorder are shown in Fig. 4, for both TM and TE polarizations. Again, it is possible to notice that the photonic gaps are not completely obliterated by the presence of not so large disorder strengths. While the robustness of the plasmon-polariton gap against disorder can be explained by the absence of disorder in the optical properties, the preservation of the $\langle n(\omega) \rangle = 0$ gap is a consequence of the fact that the widths of both metamaterial and air are chosen to be equal. Indeed, the average of the refractive index on the j th air-metamaterial double stack is

$$\langle n(\omega) \rangle^{(j)} = \frac{n_A(\omega)(a + \delta_j^A) + n_M(\omega)(b + \delta_j^M)}{a + b + \delta_j^A + \delta_j^M}, \quad (11)$$

where $n_x(\omega) = \sqrt{\epsilon_x(\omega)\mu_x(\omega)}$. For the completely correlated disorder, one has $\delta_j^A = \delta_j^M$, and taking $a = b$, one immediately gets $\langle n(\omega) \rangle^{(j)} = [n_A(\omega) + n_M(\omega)]/2$, so that the average refractive index of the double stack does not depend on disorder. As in the case of uncorrelated disorder, ξ exhibits an anomalous behavior in the vicinities of the Bragg gap: ξ increases as Δ increases. Far from the original gaps of the corresponding periodic structure, ξ decreases as Δ increases. If compared with the situation of totally uncorrelated disorder

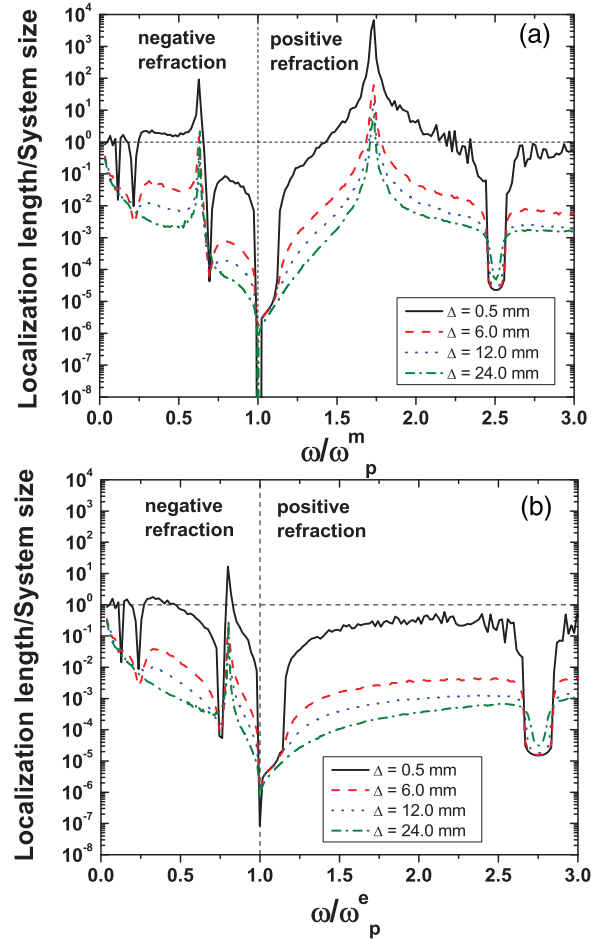


FIG. 5. (Color online) Localization length ξ (in units of the system size), obtained by numerical simulations, as a function of the normalized frequency for anticorrelated disorder and for different values of the disorder strength Δ : (a) TE and (b) TM polarizations; the parameters used are the same as in Fig. 3.

(Fig. 3), one can see that the overall effect of introducing correlations in disorder is to enhance delocalization in regions far from the original gaps, as expected.

In Fig. 5, ξ is displayed as a function of frequency for the case of anticorrelated disorder and for distinct values of the disorder strength Δ . Since $a = b$ and $\delta_A = -\delta_M$, the width of the double-stack air-metamaterial remains constant (equals to $2a$) as Δ varies. As a result, the interference conditions necessary for the formation of a Bragg gap, being sensitive to the width of the double stack, are hardly affected by increasing Δ , even for large Δ ($\Delta = 24$ mm). This also explains why Bragg gaps are more robust against disorder in the anticorrelated case, in comparison with both uncorrelated and completely correlated cases. On the other hand, it is exactly this feature that leads to the destruction of minima corresponding to the non-Bragg $\langle n(\omega) \rangle = 0$ gaps. Figure 5 also shows that, except for very large values of the disorder strength ($\Delta = 24$ mm), both the $\langle n(\omega) \rangle = 0$ and the plasmon-polariton gaps are not completely destroyed by the presence of disorder.^{32–35}

We now discuss how absorption affects the behavior of the localization length. Absorption may be introduced phenomenologically in the current Drude-type model through

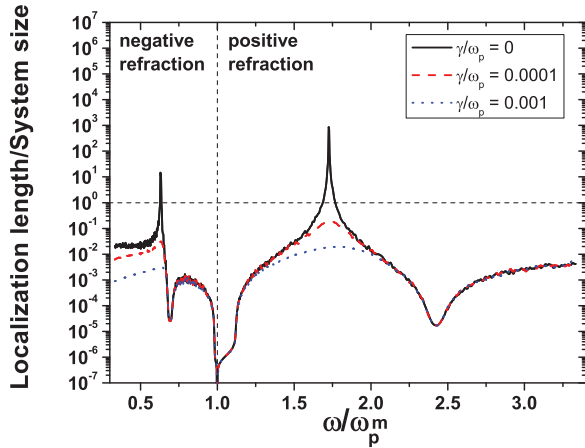


FIG. 6. (Color online) Localization length ξ (in units of the system size), obtained by numerical simulations, as a function of the normalized frequency for uncorrelated disorder, TE polarization, and different values of the absorption coefficient $\gamma/\omega_p = 0, 0.0001$, and 0.001 . The horizontal dashed line marks the transition from Anderson-localized to delocalized situations.

the replacement $\omega^2 \rightarrow \omega(\omega + i\gamma_{e,m})$, where $\gamma_{e,m}$ are the electric and magnetic loss factors. Attributing different, though realistic, values for the absorption loss factors, we resort to numerical simulations to calculate the frequency dependence of ξ ; the results are displayed in Fig. 6. While regions far from Brewster modes are hardly affected by absorption, one can see that absorption is detrimental to the Brewster modes, as well as to the delocalized high-frequency region. Nevertheless, the peaks in the localization length are not washed out completely, especially those near the $\langle n(\omega) \rangle = 0$ gaps. For this reason, we conclude that delocalization effects at the very edge of the band gap might be observable, especially if one considers the existing mechanisms to mitigate losses in metamaterials, such as the inclusion of gain^{36,38} and the use of optical-parametric amplification.³⁷

IV. CONCLUSIONS

We have considered the impact of three kinds of disorder, namely, uncorrelated, correlated, and anticorrelated, on light propagation through 1D disordered structures composed of alternating layers of nondispersive and dispersive metamaterials; the disorder consists in allowing for randomness in the layer widths. In order to appreciate the effects of disorder, we recall that the otherwise nonrandom structure displays the following three gaps for light propagation, in order of decreasing frequency: the usual Bragg gap, the plasmon-polariton gap (which only occurs for oblique incidence, and is a consequence of the excitation of electric and magnetic plasmon polaritons, for incident TM and TE waves, respectively), and the $\langle n(\omega) \rangle = 0$ gap (i.e., the one occurring at a frequency leading to a

vanishing average index of refraction); for frequencies away from these gaps, all modes are propagating (see, e.g., Refs. 30 and 31). In general, the localization length dips at the Bragg gap, but the evolution with disorder strength is quite distinct, depending on the nature of disorder. For both the uncorrelated and the correlated cases, the dips at the Bragg gap broaden as the strength of disorder increases; for the TE modes, the dips hardly shift in frequency, but are completely washed out for sufficiently large disorder, while for the TM modes, the shift toward smaller frequencies is pronounced, but a signature of the dip survives until larger disorder strength. Further, within the Bragg gap in these cases, the localization length increases as the degree of disorder increases, in contrast to what is generally expected. For the anticorrelated case, the dip hardly broadens with increasing disorder, although its depth decreases; also, there is no noticeable shift in frequency, up to the largest values of disorder we considered.

Due to the geometrical nature of the Bragg gap, the frequency at which it occurs is more sensitive to the disorder strength than the nongeometrical ones; however, this does not hold for the anticorrelated case since an overall periodicity of the structure is restored when the double layer is taken as the repeating unit. Indeed, we have found that the non-Bragg gaps are much more robust against disorder of the three kinds considered here: only for the largest values of disorder some broadening becomes noticeable. In summary, the effect of correlated disorder on the behavior of the localization length depends crucially on the physical origin of the gap and on the type of disorder.

On the other hand, delocalization effects occur at the Brewster modes, which result from the vector character of electromagnetic waves. For uncorrelated, correlated, and anticorrelated cases, the frequency at which these anomalies occur does not depend on the disorder strength. This result shows that disorder, intrinsically present in the fabrication processes of photonic structures, does not prevent a possible exploration of the Brewster anomaly in applications. We have also established that correlations in disorder do not modify the asymptotic behavior of the localization length. Finally, we have investigated the role of absorption, and found that it tends to suppress delocalized Brewster modes; nonetheless, we argue that they may not hinder the developments of applications, especially in light of the state-of-the-art of metamaterials research.

ACKNOWLEDGMENTS

We thank E. Reyes-Gómez for useful discussions. Financial support from the Brazilian Agencies CNPq, CAPES, FAPERJ, FAPESP, and FUJB is gratefully acknowledged. D.M. and S.B.C. also acknowledge the warm hospitality of the Institute of Physics at UNICAMP where part of this work was performed. D.M. thanks S. V. Gaponenko for fruitful discussions.

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