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A class of stationary nonlinear dusty plasma equilibria

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It is shown that nonparallel density and temperature gradients can produce magnetic fields in dusty plasmas. Spontaneously created magnetic fields can be maintained if there exists plasma vorticity. In order to understand this phenomena, a self-consistent dusty plasma equilibrium model is constructed by employing a kinetic description and invoking the Hamiltonian approach. Stationary nonlinear dusty plasma equilibria contain specific profiles for the plasma number density, the plasma current, and the magnetic field. The relevance of this investigation to low-temperature laboratory dusty and space plasmas is discussed. *© 1999 American Institute of Physics.* [S1070-664X(99)01407-X]

It is well known[1–3] that the presence of charged dust grains in an electron–ion plasma introduces novel collective effects. The latter include dust-acoustic[1] and dust ion-acoustic[2] waves and their instabilities.[4] as well as coherent nonlinear structures.[5,6] New types of dusty plasma waves and vortex structures have been observed[7,8] in low-temperature space and laboratory dusty plasmas. Specifically, recent laboratory observations have conclusively demonstrated the formation of dusty plasma vortices without and with magnetic fields. Dusty plasma vortices are thought to be produced by sheared flows.[9]

In this Brief Communication, we consider the generation and maintenance of magnetic fields in a dusty plasma. First, it is shown that when the equilibrium ion pressure gradient and the gradient of the dust charge density $Z_d n_d$ are nonparallel to each other, we have the possibility of spontaneous generation of magnetic fields in dusty plasmas. Second, we discuss a class of self-consistent dusty plasma equilibria in which spontaneously created magnetic fields are related with other field quantities, namely, the plasma number density, the electric potential, and the plasma currents, in a nonlinear fashion. In order to understand the stationary dusty plasma equilibria in electromagnetic fields, we have adopted the Hamiltonian and guiding center approaches for the plasma particles and have constructed appropriate distribution functions. The latter are then employed to calculate the plasma number and plasma current densities. The quasineutrality condition and Ampere's law are introduced to deduce the equilibrium density, the equilibrium potential and the stationary magnetic fields. Numerical examples of various field quantities are worked out by choosing typical laboratory dusty plasma parameters. The relevance of our investigation to space and laboratory plasma has been discussed.

Let us consider a multicomponent dusty plasma whose constituents are electrons, ions, and negatively charged micron sized dust particulates. In the unperturbed state, we have $n_{i0}=n_{e0}+Z_d n_{d0}$, where n_{j0} is the unperturbed number density of the particle species j (j equals e for the electrons, i for the ions and d for the dust grains) and Z_d the number of charges residing on the dust grains. When most of the electrons from the background plasma are attached to the dust grains, we might encounter a complete electron depletion, leading to the overall quasineutrality condition $n_{i0}\approx Z_d n_{d0}$. In such a situation, the dusty plasma can be considered as a two-component system.

According to Faraday's Law, the magnetic field $\mathbf{B}$ in a plasma can be generated provided that the curl of the electric field $\mathbf{E}$ remains finite. Thus, the induction equation determines the evolution of the magnetic field

$$\partial_t \mathbf{B} = -\nabla\times\mathbf{E}, \tag{1}$$

where the electric field arising from the charge separation is determined from the ion momentum balance equation

$$\mathbf{E}=\frac{\nabla p_i}{e n_i}-\mathbf{v}_i\times\mathbf{B}+\frac{m_i}{e} d_t \mathbf{v}_i, \tag{2}$$

where $p_i=n_i T_i$ is the ion pressure, n_i and T_i are the ion number density and the ion temperature, respectively, e the magnitude of the electron charge, and $\mathbf{v}_i$ the ion fluid velocity. The latter is obtained from Ampere's Law $n_i\mathbf{v}_i = Z_d n_d \mathbf{v}_d + (1/e\mu_0)\nabla\times\mathbf{B}$, where v_d is the dust fluid velocity. Hence, from (1) and (2) we obtain

$$\begin{aligned}\partial_t(1-\lambda_i^2\nabla^2)\mathbf{B} &= -\frac{1}{eZ_d^2 n_d^2}\nabla p_i\times\nabla(Z_d n_d)+\nabla\times(\mathbf{v}_d\times\mathbf{B})+\frac{m_i}{e}\nabla\times\mathbf{v}_i \\ &\quad\times\left(\nabla\times\mathbf{v}_d-\frac{e\lambda_i^2}{m_i}\nabla^2\mathbf{B}\right)+\frac{eZ_d\lambda_d^2}{m_d}\nabla\times[(\nabla\times\mathbf{B})\times\mathbf{B}],\end{aligned} \tag{3}$$

where $\lambda_d = c/\omega_{pd}$ is the dust skin depth, $\omega_{pd} = (Z_d^2 e^2 n_d/m_d\epsilon_0)^{1/2}$ the dust plasma frequency, c the speed of light, and λ_i the ion skin depth. The origin of various terms in (3) is obvious. The first term on the right hand-side of (3) is the baroclinic vector for the dusty plasma, which is the source for the magnetic field and remains finite when the

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ion pressure gradient and the gradient of $Z_d n_d$ are nonparallel to each other. The second term on the right-hand side is a kind of Hall term (the dynamo) involving the dust vorticity, whereas the third and fourth terms are associated with the nonlinear couplings between the dust vorticity and the $\mathbf{J}\times\mathbf{B}$ flow with the ion flows, as well as with the curl of the $\mathbf{J}\times\mathbf{B}$ flow, where $\mathbf{J}$ is the plasma current. The dynamo and the nonlinear terms play very important roles in the evolution of spontaneously generated magnetic fields. The question now arises as how the latter are maintained in dusty plasmas. To answer this question, we follow Ref. 10 and present a class of stationary dusty plasma equilibria in which the plasma density, the electric potential, the plasma current densities, and the magnetic fields are related in a nonlinear fashion.

Let us suppose that the dust particulate dynamics in the electromagnetic fields is governed by the Hamiltonian

$$H_d=\frac{1}{2m_d}\left[\left(\frac{p_\theta}{R}+Z_d eA\right)^2+p_R^2\right]-Z_d e\phi, \tag{4}$$

where m_d is the dust mass, A is the theta component of the vector potential, ϕ the electric potential, and the canonical angular momentum is given by

$$p_\theta=m_d R^2\dot{\theta}+Z_d eRA. \tag{5}$$

Here, $p_R=m_d\dot{R}$, and R and θ are the radial and azimuthal coordinates in the cylindrical geometry.

The corresponding dust distribution function $f_d(p_\theta,p_R,\theta,R)=f_d(H,p_\theta)$ is found to be

$$f_d=\frac{Z_{d0}n_{d0}}{2\pi m_d T_d}\exp\left[-\frac{1}{T_d}\left(H_d+\frac{\alpha_d}{2m_d}p_\theta^2\right)\right], \tag{6}$$

where T_d is the dust temperature and α_d is a constant.

It is now an easy matter to determine the number density of the dust, $n_d(R)$, and the θ component of the current density $J_d(R)$. We have

$$\begin{aligned}n_d&=\int\frac{f_d}{R}dp_R dp_\theta\\&=\frac{Z_{d0}n_{d0}}{(1+\alpha_d R^2)^{1/2}}\\&\quad\times\exp\left[-\frac{1}{T_d}\left(\frac{\alpha_d Z_d^2 e^2 R^2 A^2}{2m_d(1+\alpha_d R^2)}-Z_d e\phi\right)\right],\end{aligned} \tag{7}$$

$$\begin{aligned}\mu_0 J_d&=\mu_0 e\int\frac{R\dot{\theta}f_d}{R}dp_R dp_\theta\\&=-\frac{\omega_{pd}^2}{c^2}\frac{\alpha_d R^2 A}{(1+\alpha_d R^2)^{3/2}}\\&\quad\times\exp\left[-\frac{1}{T_d}\left(\frac{\alpha_d Z_d^2 e^2 R^2 A^2}{2m_d(1+\alpha_d R^2)}-Z_d e\phi\right)\right].\end{aligned} \tag{8}$$

Next, we suppose that the Larmor radius of singly charged positive ions is much smaller than the scale size, and therefore, use the guiding-center Hamiltonian for the ions

$$H_i=\mu\omega_{ci}+e\phi, \tag{9}$$

where $\mu(=m_i v_\perp^2/2\omega_{ci})$ is the magnetic moment. The canonical variables are as μ, $\theta=\int\omega_{ci}dt$, R, and Θ. It is to be noted that for ions, R and Θ correspond to guiding center coordinates and H_i and R are assumed as constants of motion. The ion distribution function is taken to be of the form

$$\begin{aligned}f_i(\mu,\theta,R,\Theta)&=f_i(H_i,R)\\&=\frac{n_{i0}\omega_{ci}}{2\pi T_i}\exp\left[-\frac{1}{T_i}(\mu\omega_{ci}+e\phi)+g(R)\right].\end{aligned} \tag{10}$$

Here, $g(R)$ acts in the function at $\mathbf{R}$ larger. It will be determined later.

The ion number density and the ion guiding center and magnetization current densities can be obtained in a straightforward manner. The results are, respectively,

$$n_i=\int f_i(H,R)d\mu d\theta=n_{i0}\exp\left[\frac{e\phi}{T_i}+g(R)\right], \tag{11}$$

$$\begin{aligned}\mu_0 J_i&=-e\mu_0\int R\dot{\theta}f_i d\mu d\theta\\&=\frac{\omega_{pd}^2}{c^2}\frac{1}{\omega_{cd}}\left(\frac{T_i}{Z_d e}\frac{d\ln\omega_{ci}}{dR}+\frac{1}{Z_d}\frac{d\phi}{dR}\right)\\&\quad\times\exp\left[\frac{e\phi}{T_i}+g(R)\right],\end{aligned} \tag{12}$$

and

$$\begin{aligned}\mu_0 J_m&=\frac{e\mu_0}{m_i}\frac{d}{dR}\int\mu f_i d\mu d\theta\\&=-\frac{\omega_{pd}^2}{c^2}\frac{1}{\omega_{cd}}\left(\frac{T_i}{Z_d e}\frac{d\ln\omega_{ci}}{dR}+\frac{1}{Z_d}\frac{d\phi}{dR}\right.\\&\quad\left.-\frac{T_i}{Z_d e}\frac{\eta_d\alpha_d R}{(1+\alpha_d R^2)}\right)\exp\left[\frac{e\phi}{T_i}+g(R)\right].\end{aligned} \tag{13}$$

The first and second terms on the right-hand side of (13) correspond to the $\nabla\mathbf{B}$ and $\mathbf{E}\times\mathbf{B}$ drifts, respectively. We have also used $R\dot{\theta}=(\partial H/\partial R)/eB$ and the quasineutrality condition for the two-component dusty plasma.

Ampere's Law for the vector potential in cylindrical coordinates is

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{d}{dR}A\right)-\frac{A}{R^2}=-\mu_0\mathbf{J}_t, \tag{14}$$

where $\mathbf{J}_t=\mathbf{J}_i+\mathbf{J}_d+\mathbf{J}_m$.

The magnetic field is given by

$$\frac{1}{R}\frac{d}{dr}(RA)=\mathbf{B}. \tag{15}$$

At the time, when a vortex is setup, the centrifugal force of the dust should be balanced by the pressure gradient. The self-generated magnetic field should be zero when $R\to\infty$. The value of density at the heart of vortex is reduced and varies as $n(\neq 0)$ at $R\to\infty$. When $|\mathbf{B}|_\infty\to 0$, A should be bounded there. Then, applying the quasineutrality condition and the potential $[|\phi|_\infty\to T_d/(2Z_d e)\ln(1+\alpha_d R^2)]$, we readily obtain

$$g(R)=\frac{\eta_d}{2}\ln(1+\alpha_d R^2),$$

where $\eta_d=T_d/(Z_d T_i)$. Hence, the density and the potential assume the form

$$Z_d n_d=n_i=n_0\exp\left[-\frac{\eta_d}{1+\eta_d}\frac{\alpha_d Z_d^2 e^2 R^2 A^2}{2m_d T_d(1+\alpha_d R^2)}\right], \tag{16}$$

and

$$\frac{e\phi}{T_i}=\frac{\eta_d}{2}\ln(1+\alpha_d R^2)+\frac{\eta_d}{1+\eta_d}\left[\frac{\alpha_d Z_d^2 e^2 R^2 A^2}{2m_d T_d(1+\alpha_d R^2)}\right]. \tag{17}$$

By substituting (8), (12) and (13) into (14), we obtain

$$\frac{d^2A}{dR^2}+\frac{1}{R}\frac{dA}{dR}-\frac{A}{R^2}$$
$$=-\frac{\omega_{pd}^2}{c^2}\frac{1}{1+\alpha_d R^2}\left(\frac{T_d}{\omega_{cd}Z_d^2 e}\alpha_d R-\frac{\alpha_d R^2 A}{1+\alpha_d R^2}\right)$$
$$\times\exp\left[-\frac{\eta_d}{1+\eta_d}\left(\frac{\alpha_d Z_d^2 e^2 R^2 A^2}{2m_d T_d(1+\alpha_d R^2)}\right)\right]. \tag{18}$$

It is convenient to define $r=R\omega_{pd}/\sqrt{2}c$, $A_0=\sqrt{(1+\eta_d)/\eta_d}\sqrt{2m_d T_d}/Z_d e$, $\alpha_0=2c^2\alpha_d/\omega_{pd}^2$ and introduce the symbols $A^*=A/A_0$, $A_d=\omega_{pd}/(\omega_{cd}cZ_d)\sqrt{\eta_d T_d/m_d(1+\eta_d)}$, $\xi=r^2$, and $u=rA^*$, so that Eq. (18) can be expressed as

$$4\frac{d^2u}{d\xi^2}=\frac{\alpha_0}{1+\alpha_0\xi}(u-A_d)\exp\left[-\frac{\alpha_0 u^2}{2(1+\alpha_0\xi)}\right]. \tag{19}$$

Neglecting the left-hand side of (19), the asymptotic value of its vortex solution is A_d. For this assumption the vector potential becomes constant and the density becomes

$$n_\infty=n_0\exp\left[-\frac{\eta_d}{2(1+\eta_d)}\right]. \tag{20}$$

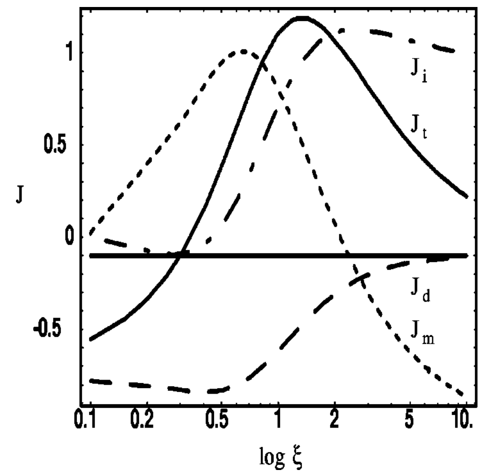

FIG. 2. The variation of the normalized dust current density, $\mathbf{J}_d$ (dashed line), the ion current density, $\mathbf{J}_i$ (dashed–dotted line), the magnetization current density, $\mathbf{J}_m$ (dotted line), and the total current density, $\mathbf{J}_t$ (solid line) against log ξ for $\alpha_d=3$ and $A_d=1$.

We have integrated Eq. (19) radially inwards on a logarithm scale from log $\xi=10$, by choosing typical parameters that are relevant to low-temperature dusty plasma devices. Accordingly, we take $n_d\approx 5\times10^2\,\mathrm{cm}^{-3}$; the dust size of 5 microns; negatively charged grains have $Z_d\sim4000$; whereas the dust mass density is 1 g/cm^3. Moreover, we have taken $m_d\sim10^{-12}$ g; $T_d\sim300$ K; $T_i\sim0.2$ eV; and $T_e\sim2$ eV. The next three figures show examples of stationary nonlinear profiles for the density, the electric potential, the plasma current density, and the magnetic fields in a dusty plasma. Figure 1 is the normalized profiles of the density and the electric potential. Figure 2 depicts the normalized profiles for the current densities, $\mathbf{J}_d$, $\mathbf{J}_i$, $\mathbf{J}_m$, and $\mathbf{J}_t$, whereas Fig. 3 exhibits the normalized magnetic field. All the figures have the normalized profiles as a function of log ξ.

Let us now discuss the effects of some normalized parameters on various plasma and field quantities. First, in Eq. (19), for $\alpha_0>1$, we observe a decrease of n, whereas $\mathbf{B}$ practically does not change (in comparison to the one in Fig.

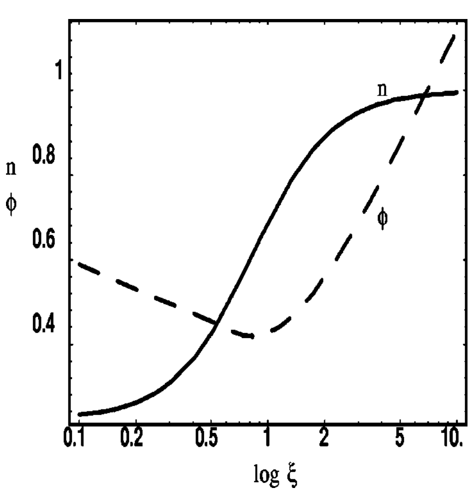

FIG. 1. The variation of the normalized number density, n, (solid line) and a proportional value of the potential 0.4ϕ, (dashed line) against log ξ for $\alpha_d=3$ and $A_d=1$.

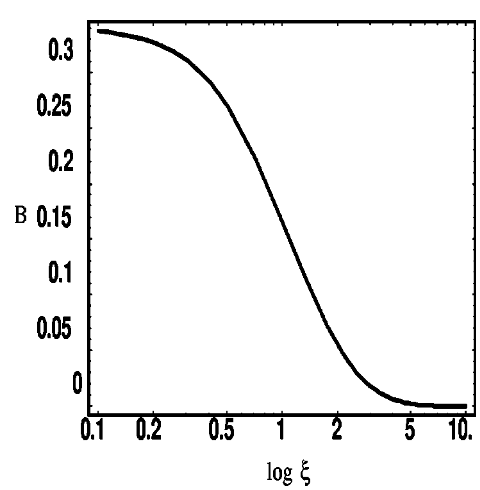

FIG. 3. The normalized magnetic field, $\mathbf{B}$, as a function of log ξ for $\alpha_d=3$ and $A_d=1$.

3), and J_t is dominated by J_d. Second, for $\alpha_0 \gg 1$, we have $n \to 0$, $J_t \to J_d$, and **B** remains unchanged. Third, for $0 < \alpha_0 < 1$, n increases, **B** decreases and J_t is dominated by J_m. Fourth, when $\alpha_0 \ll 1$, $n \to n_0$ and $\mathbf{B} \to 0$. Finally, for $0 < A_d < 1$, n decreases, whereas for $A_d \gg 1$, we have $n \to 0$, $\mathbf{B} \to 0$, and for $A_d \ll 1$, we have $n \to n_0$.

To summarize, we have presented a class of stationary nonlinear dusty plasma equilibria in which the plasma number density, the electric potential, the plasma current density, and the magnetic field are connected in a specific fashion. This problem has been addressed by employing the Hamiltonian and guiding center models and by choosing appropriate dust and ion distribution functions that yield the desired form for a stationary dust vortex which is required for the maintenance of the magnetic fields that could be created by the baroclinic vector in a dusty plasma. The results of our investigation should be useful in understanding not only the dusty laboratory device equilibrium, but also the equilibrium of dusty stars and other astrophysical objects.

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