PHYSICAL REVIEW C **78**, 034909 (2008)

Phase transition of the nucleon-antinucleon plasma at different ratios

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(Received 26 November 2007; revised manuscript received 23 July 2008; published 11 September 2008)

We investigate phase transitions for the Walecka model at very high temperatures. As is well known, depending on the parametrization of this model and for the particular case of a zero chemical potential (μ), a first-order phase transition is possible [J. Theis, G. Graebner, G. Buchwald, J. A. Maruhn, W. Greiner, H. Stocker, and J. Polonyi, Phys. Rev. D **28**, 2286 (1983)]. We investigate this model for the case in which $\mu \neq 0$. It turns out that, in this situation, phases with different values of antinucleon-nucleon ratios and net baryon densities may coexist. We present the temperature versus antinucleon-nucleon ratio as well as the temperature versus the net baryon density for the coexistence region. The temperature versus chemical potential phase diagram is also presented.

DOI: 10.1103/PhysRevC.78.034909 PACS number(s): 21.65.−f, 12.38.Mh, 21.30.−x, 24.10.Jv

Finite nuclei and nuclear matter properties have been reasonably well described by the Walecka model [2]. This renormalizable model employs nucleons and mesons (σ and ω) as the degrees of freedom. The sources for the fields are the scalar (ρ_s) and vector (ρ) densities associated with the Lorentz scalar (S) and vector (V) interactions. It is well known that this model, after the fitting of the experimental infinite nuclear matter binding energy at the saturation density, gives for this system a high bulk incompressibility, as well as a high spin-orbit splitting energy, when applied for finite nuclei. To overcome this problem, nonlinear relativistic hadronic models which include cubic and quartic scalar mesonic self-couplings have been proposed [3]. The preliminary success of this kind of nonlinear Walecka model, estimulated the proposal of different parametrizations to better describe nuclear matter properties [4]. Still aiming at refining its predictions for nuclear matter and neutron stars properties, new kinds of models including scalar(vector) self-coupling interactions added to usual nonlinear Walecka models [3,4] have been constructed [5]. Effective hadronic models with density-dependent coupling constants have also been proposed [6].

In general, (non-)linear Walecka models models [2–6] have been investigated under extreme density and temperature regimes. Most of such studies focus on the hadronic quark-gluon plasma phase transition, which is not the aim of our present work. Here, our purpose is to extend the very interesting work of Theis *et al*. [1]. In their study [1], the Walecka model was investigated in the extreme high-temperature regime in which the chemical potential (μ) is zero. This leads to a situation in which the net baryon density (ρ) also vanishes. In this case, the number of nucleon and antinucleon are the same. This study showed that, in this nucleon-antinucleon plasma at very high temperature, a phase transition exists.

In this work, we will investigate the Walecka model under an extreme temperature regime, but allowing the number of antiparticles and particles to be different in the two phases it can lead to. The question we pose here is whether by allowing different ratios of nucleons-antinucleons a phase transition scenario is still present, as in the particular case $\rho = 0$, still in the so-called "no-sea" approximation. That is, we consider explicitly only the valence Fermi and Dirac sea states. Let us here remark that such an approximation, as investigated carefully by the Ohio group [7], becomes good at low energies, where the contributions of the Dirac sea can be renormalized in effective coupling constants. Of course, a rigorous way to perform our investigation would be to include vacuum polarization effects in the Walecka model for high excitation energies, where the antiparticles play an essential role. This treatment, however, is beyond the scope of this work and we perform an exploratory investigation even though the validity of the "no-sea" approximation in this regime may not be granted.

Nowadays, high-energy experiments reveal evidence of nuclear systems with very small baryonic density in the study of particle yields measured in central Au-Au collision at RHIC. Different experimental analysis (STAR, PHENIX, PHOBOS, BRAHMS) furnish the antiproton-proton ratio $\bar{p}/p \approx 0.65$ for a temperature of 174 MeV and a chemical baryonic potential of 46 MeV estimated from thermal models to fit antiparticle-particle ratios [8]. High energy Pb-Pb collisions show that this ratio reach values around 0.9 [9]. If relativistic hadronic models are to be used in the description of the multiplicity observed today in ultrarelativistic heavy-ion collisions [10,11], the behavior of these models at high temperature regimes may be of importance.

The thermodynamics of the Walecka model may be given, for finite temperature, in terms of the energy density and pressure functionals,

$$\mathcal{E} = \frac{C_\omega^2}{2M^2}\,\rho_b^2 + \frac{2C_\sigma^2}{2M^2}\,\rho_s^2 + \frac{\gamma}{2\pi^2}\int k^2 dk\, E^*(k)(n_k + \bar{n}_k) \quad (1)$$

and

$$p = \frac{C_\omega^2}{2M^2}\,\rho_b^2 - \frac{2C_\sigma^2}{2M^2}\,\rho_s^2 + \frac{\gamma}{6\pi^2}\int \frac{k^4 dk}{E^*(k)}(n_k + \bar{n}_k), \quad (2)$$

where

$$\rho_b = \rho - \bar{\rho}, \quad (3)$$

$$\rho = \frac{\gamma}{2\pi^2}\int k^2 dk\; n_k, \quad (4)$$

$$\bar{\rho} = \frac{\gamma}{2\pi^2}\int k^2 dk\; \bar{n}_k, \tag{5}$$

$$\rho_s = M^* \frac{\gamma}{2\pi^2}\int \frac{k^2 dk}{E^*(k)}\,(n_k + \bar{n}_k). \tag{6}$$

In the expressions above, γ, is the degeneracy factor ($\gamma = 4$ for nuclear matter and $\gamma = 2$ for neutron matter), n_k and $\bar{n}_k$ stand for the Fermi-Dirac distribution for baryons and antibaryons respectively, with arguments $(E^* \mp \nu)/T \cdot E^*(k)$ is given by $E^*(k) = (k^2 + M^{*2})^{1/2}$, whereas an effective chemical potential, which preserves the number of baryons and antibaryons in the ensemble, is defined by $\nu = \mu - V$, $V = C_\omega^2 \rho_b / M^2$, where μ is the thermodynamic chemical potential. The solution for the equation of state is obtained explicitly through the minimization of $\mathcal{E}$ relative to the scalar field, or equivalently to $m^* = M^*/M$,

$$f\left(\frac{M^*}{M}\right) = 1 - \frac{M^*}{M} - \frac{C_s^2}{M^3}\,\rho_s = 0. \tag{7}$$

This equation, known as the gap equation, has to be solved self-consistently with Eqs. (1) and (2) and provides the basis for obtaining all thermodynamic quantities in the mean-field approach we are using.

Usually, the constants C_σ^2 and C_ω^2 are determined in favor of the experimental nuclear matter binding energy (16 MeV) at the saturation density ($\rho_b = \rho_o = 0.15$ fm^{-3}) at $T = 0$. At finite temperature, this model predicts a critical liquid-gas behavior as a van der Waals equation of state. Its critical temperature is around 18 MeV [2]. For this temperature, the antinucleon-nucleon ratio, given by $R = \bar{\rho}/\rho$ is negligible. Only when the temperature gets higher, the antinucleon density starts to take significant values.

In a very interesting work [1], the Walecka model was studied in the extreme situation in which $R = 1$. In this case, $\rho_b = 0$ and $\nu = \mu = 0$. This study showed that, in this nucleon-antinucleon plasma at very high temperature, a phase transition exists. It is also remarkable to note that the order of the phase transition itself becomes dependent on the C_σ^2 versus C_ω^2 space parameter. By small changes on these parameters (which means to change by a few percent the nuclear matter binding energy and the saturation density), the phase transition changes from first to second order. If we take the values of $C_\sigma^2 = 359.35$ and $C_\omega^2 = 275.12$, fitting the infinite nuclear matter binding energy as 16 MeV at a density of 0.15 fm^{-3}, the phase transition is of first order. The findings of Ref. [1] may be summarized in Fig. 1.

In Fig. 1 we see that the nucleon effective mass decreases abruptly at $T \approx 184$ MeV. Because all other thermodynamic quantities are dependent on m^*, this effect also manifests itself in energy density, pressure, and specific heat, calculated as the temperature first derivative of the energy density. The fast increase of the entropy is a clear signal of a first-order phase transition. Because the baryonic density is kept constant $\rho_b = 0$, the order parameter should be the entropy. It is a very curious constrained system, interpreted as a nucleon-antinucleon plasma [1].

Here, following [1], we start to study the Walecka model in different constrained cases. The main question is to understand the behavior of some thermodynamical quantities

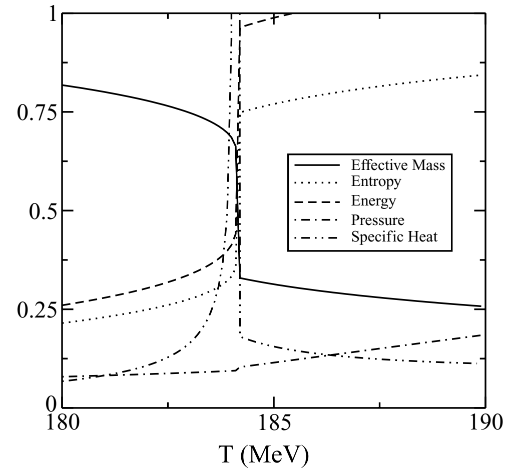

FIG. 1. The effective mass, entropy, energy density, pressure, and the specific heat for $R = 1$ as a function of the temperature. The mass is in units of the nucleon mass and the entropy, energy, and pressure are in Stefan-Boltzmann units.

when the ratio $R = \bar{\rho}/\rho$ varies. Now, all quantities will have contribution from the baryonic density, contrary to Fig. 1 where $\rho_b = 0$.

Let us now remark that we have fixed numerically the ratio within a precision of one part in a thousand. It means that the set of Eqs. (1)–(7) is solved self-consistently with $R - R/1000 \leqslant R \leqslant R + R/1000$. In Fig. 2 we present M^*/M as a function of the temperature for different values of R.

For the same values of R investigated in Fig. 2, the entropy behavior as a function of the temperature is shown in Fig. 3.

Both the nucleon effective mass and the entropy follow the same abrupt decreasing (increasing) behavior one sees for the particular case $R = 1$. From these figures we see that, as

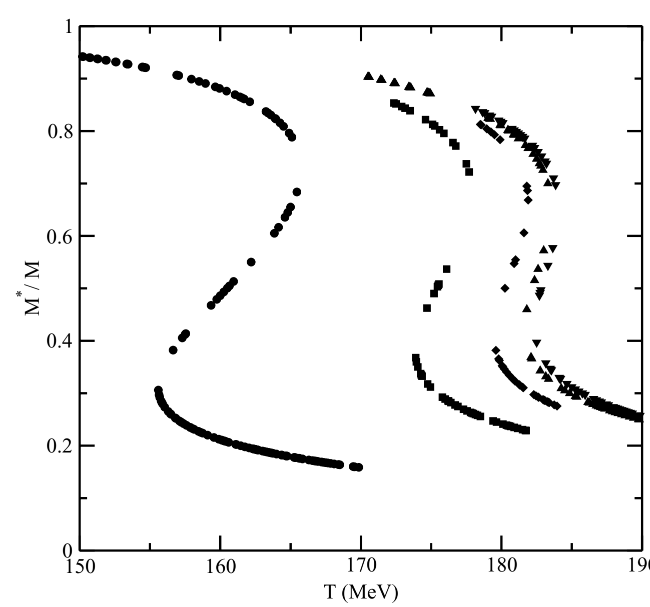

FIG. 2. The effective mass as a function of the temperature for several values of R. From the left to the right: $R = 0.1, 0.3, 0.5, 0.7, 0.9$.

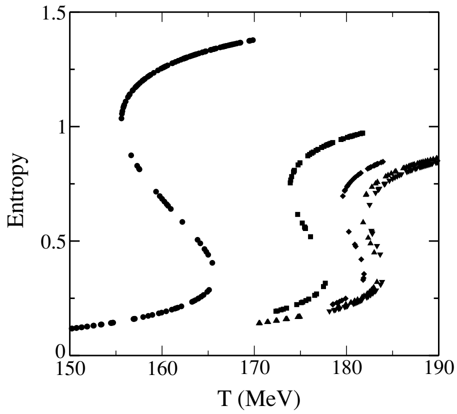

FIG. 3. The entropy, in Stefan-Boltzmann units, as a function of the temperature for several values of R. The values of R are the same as those in Fig. 2.

the ratio $\bar{\rho}/\rho$ decreases, the temperature in which an abrupt behavior arises also decreases but keeps the same character. In principle, there is an important difference between the system described by Fig. 1 and those of Figs. 2 and 3. In the first, $R = 1$ and $\rho_b = 0$ along the temperature variation. In the second, the ratios are kept constant while ρ_b varies. Visually, Figs. 2–3 suggest phase coexistence at some temperature range. According to the Gibbs criteria, if one has phases 1 and 2, a phase coexistence arises when $p_1 = p_2, \mu_1 = \mu_2$, and $T_1 = T_2$. The critical temperature is achieved when, above that, no more phase coexistence is possible. For the case $R = 1$, a phase coexistence exists at $T = 183.25$ MeV and $\mu = 0$. It means that, if the temperature increases or decreases from this value, the phase coexistence disappears as we can see in Fig. 4,

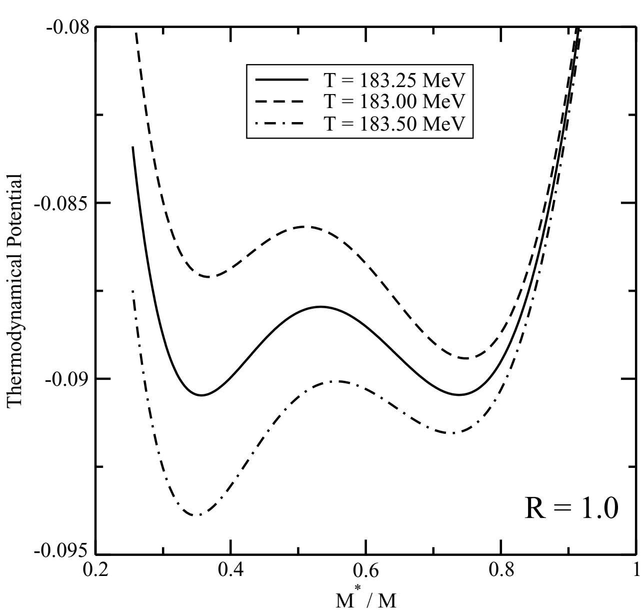

FIG. 4. Thermodynamical potential, in units of the Stefan-Boltzmann pressure, as a function of the nucleon effective mass, for temperatures around the critical temperature in the case where $\mu = 0$ ($R = 1$).

where the minima of the thermodynamical potential have different values for the same temperature.

This kind of investigation was performed, for example, by Asakawa and Yazaki [12] when working with the Nambu and Jona-Lasinio model at finite temperature, where a phase transition is also present. Following their procedure, we can verify whether the system exhibits phase coexistence.

Surprisingly, despite the signals of phase coexistence shown by Figs. 2–3, we did not find phase coexistence for a single value of R when $R \neq 1$. In this case, when the system was in thermal ($T_1 = T_2$) and mechanical ($p_1 = p_2$) equilibrium, chemical equilibrium ($\mu_1 = \mu_2$) was not found. However, along the apparent phase coexistence region signalized in Figs. 2–3 by three different values of M^*/M and entropy for the same temperature, we found a stable phase (global minimum of the thermodynamical potential). It indicates the presence of stable phases up to the start of the backbending of the curves and new stable phases by any increase of the temperature, but with a dramatic decrease (increase) in the effective mass (entropy). Such a phase transition without phase coexistence is rare in physics. Therefore, we decided to leave the fixed ratio scenario to analyze what happens if the system becomes free of such constraint.

Next, we proceed to investigate the phase coexistence in the Walecka model still at high temperature but without any fixed R constraint. In Fig. 5 we display the thermodynamical potential density $-p$ and $f(M^*/M)$ given by Eq. (2) and Eq. (7), respectively.

In the curves presented, the Gibbs criteria $T_1 = T_2$, $p_1 = p_2$, and $\mu_1 = \mu_2$ are clearly satisfied. As we continue to

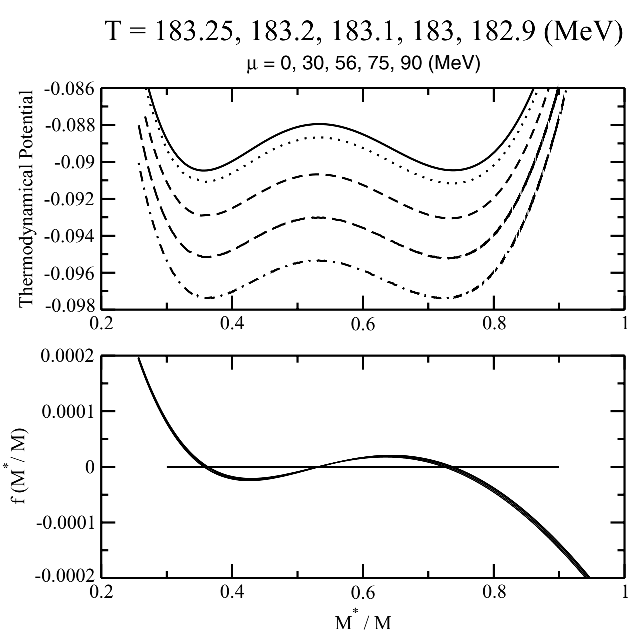

FIG. 5. The same as in the previous figure, but showing the phase coexistence for temperatures between $T = 183.25$ MeV (upper curve in top panel) and $T = 182.90$ MeV (lower curve in botton panel). The values of μ are those that make the two minima to be have the same value for each temperature. Note that now R is not constrained and has a different value for each temperature and chemical potential.

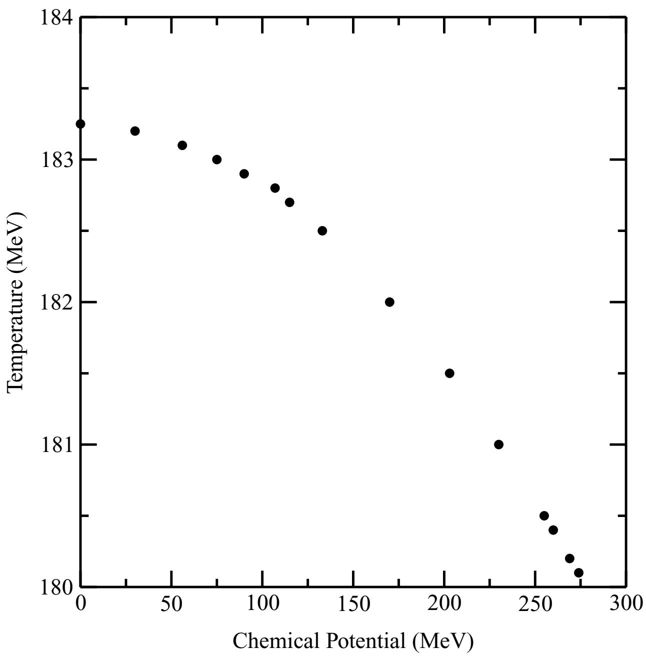

FIG. 6. Temperature versus baryon chemical potential for the coexistence phase region.

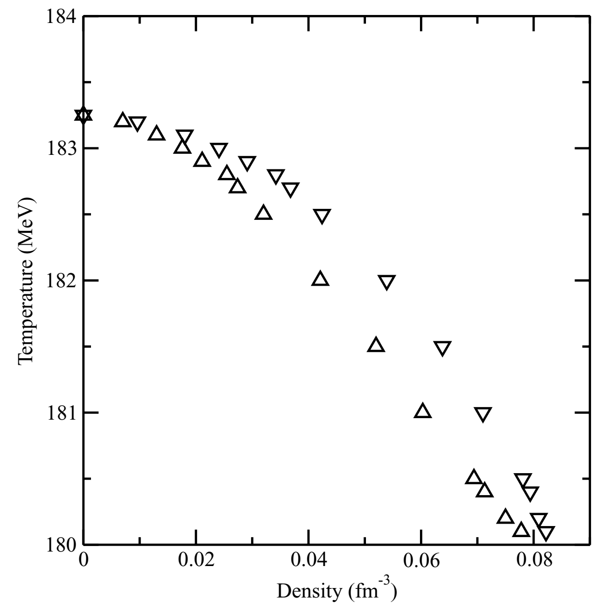

FIG. 7. Temperature versus net baryon density for the phase coexistence region.

decrease the temperature, the Gibbs criteria can be fulfilled because the chemical potential μ increases. The curves for the thermodynamical potential for $T < 182.9$ MeV (not shown) becomes flatter as the temperature decreases. This happens until $T \approx 180$ MeV and $\mu \approx 274$ MeV. Therefore, the coexistence region for the Walecka model, with the coupling constants given previously and without any R constraint, is $180.1 \leqslant T \leqslant 183.25$ MeV with $0 \leqslant \mu \leqslant 274$ MeV. The phase diagram $T \times \mu$ is given in Fig. 6.

Now, we have the values of M^*/M that allow the phase coexistence for the phase diagram of Fig. 6. With these, we can extract the net baryon densities ρ as a function of T that allows phase coexistence. The results are presented in Fig. 7.

Following the same procedure, in Fig. 8 we show the different ratios R for which the system affords coexistence. It is interesting to observe that no coexistence exists if one of the ratio is not greater than 1/2. As we can see, only in the particular case $R = 1$, which corresponds to $\rho_b = 0\,\mu$, there is a phase coexistence allowing only one fixed ratio.

In short, we start by studying the Walecka model at high temperature regime constraining the antinucleon-nucleon ratio (R) to be constant but not equal to 1 as has been done by Theis *et al.* [1]. We have seen that the visual signals of phase transition for the effective nucleon mass (M^*) and entropy (S) versus temperature (T) for the cases $R \neq 1$ are typically the same of those obtained in the case $R = 1$. By this we mean an abrupt decrease (increase) of both (M^* and S) for $T > 180$ MeV. Surprisingly, however, we could not find for $R \neq 1$, contrary to the $R = 1$ case, a phase coexistence signature by using the Gibbs criteria. Phase transition without phase coexistence is a rare occurrence in physics. As far as we know, they are theoretically predicted for the anomalous two-dimension Kosterlitz-Thouless phase transition [13], as an example. Because we could not find a clear signature for phase coexistence at fixed $R \neq 1$ we have investigated phase coexistence for nonconstrained values of R. Our study shows that the only phase coexistence regime for a fixed R occurs at $R = 1$ or, what is the same, $\mu = 0$. In the the Walecka model parametrization we have used, coexistence phases occur approximately in the interval $180 \leqslant T \leqslant 183.25$ MeV and different values of R at each phase are needed.

If one uses hadronic models [10,11] to study high-energy heavy-ion collisions, the models face a low-density and high-temperature regime. In the chemical freeze-out,

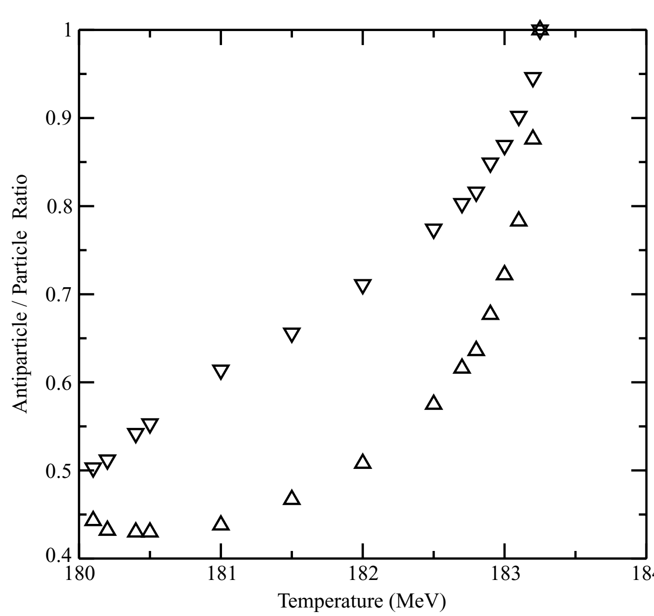

FIG. 8. Antinucleon-nucleon ratio (R) versus temperature for the phase coexistence region.

antinucleon-nucleon are produced at ratios that depend on the center-of-mass energy [14]. Although the Walecka model is too simple to deal with, it may anticipate roughly what can happen with more realistic hadronic models. Theis *et al*. [1] analyzed ($\mu = 0$) that, depending on its parametrization, the Walecka model shows first- or second-order phase transition. The same happens with different hadronic models [15]. Therefore, it is to be expected that models (with $\mu = 0$) that present first-order phase transition would follow a behavior similar to the Walecka model (Figs. 6–8) regarding phase coexistence regions. Therefore, depending on the values of T and μ used to fit the freeze-out data, the chosen hadronic model may be dealing with hadronic phase coexistence region, as depicted in Figs. 6–8.

ACKNOWLEDGMENTS

This work was partially supported by CNPq and CAPES (Brasil). V.S.T. thanks FAPESP and FAEPEX/UNICAMP for financial support.

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