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Evaluation of the Equivalent Slip Modulus of Nailed Connections for Application in Linear Analysis of Plywood Timber Beams

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The study of the stiffness of laterally loaded semi-rigid connections in plywood-timber beams is justified by the influence that the deformation of the connection has on the overall displacements of the structure. Semi-rigid connections are characterized by the occurrence of a slip between the connected pieces. The characterization of a connection is usually based on an isolated single connector behavior, which is described by its load-slip relationship expressed by the slip modulus, and so it is extended to the group of connectors. Although the method of analysis is well established, the concept of equivalent slip modulus, defined as the slip modulus per unit length of a connection, has not been totally explored. In this study, we focus on the experimental determination of the equivalent slip modulus for mechanically analyzing plywood-timber beams with continuous connections. The results demonstrated that the test is suitable for obtaining experimental values of the equivalent slip modulus.

Keywords: *semirigid connections, stiffness, nails, plywood-timber beams*

1. Introduction

In plywood timber beams assembled with semi-rigid connections, the deformation of the connection has an important influence on the stiffness of the beam. The quantification of that influence depends on the knowledge of the behavior of the connection, expressed by its load-slip relationship.

In the linear-elastic range, load and slip are related by a constant denominated slip modulus. The method for the design of plywood-timber beams is usually based on linear-elastic model. The non linearity is accounted by means of different values of slip modulus according to the level of loading.

In general, that model is well established. On the other hand, the concept of equivalent slip modulus, defined as the slip modulus per unit length of a connection, has not been totally evaluated. In this study, we focus on the experimental determination of the equivalent slip modulus for mechanically analyzing plywood-timber beams with continuous connections.

As an example of application, we direct the application of the slip modulus to the linear analysis of nailed plywood timber beams with a box section. In Brazil, this type of beam has been increasingly investigated and used, following a trend of rationalization of wooden structures. However, we can say that the large spread of its use is still obstructed by the lack of details in the technical code for the design of wooden structures - NBR7190/1997[1] (ABNT, 1997).

Figure 1 shows a schematic of a beam of this type. In the United States, this type of beam is documented by the APA[2] (American Plywood Association, 1985), which provides some standardized sections. The “I” section is another shape of the same type of beam. The box section and the “I” section have basically the same structural behavior under bending and differ for some construction details. The box section allows the potential to increase the thicknesses of the webs in the portions with higher shear forces and the “I” section allows the potential to increase the widths of the flanges in the portions with higher moments. The box section has a slight constructive advantage due to the use of nails with shorter length.

For the design of nailed plywood timber beams, accounting for the slip of the connection in the methods of calculation of displacements, several technical codes around the world provide specifications. One of them is the Eurocode 5[3] (European Committee for Standardization, 2004), whose specifications where approached by Kreuzinger[4] in 1995.

Considering now the behavior of a semi-rigid connection, it is expressed by its load-slip relationship. This behavior may be considered linear elastic or non-linear elastoplastic, depending on the level of loading. In the linear elastic range, load and slip are related by a linear relationship whose constant is denominated slip modulus. On the other hand, if the connection is designed to attain its non-linear range, the structure will have a non-linear behavior. Thus, the behavior of the connection will determine the behavior of the structure.

Along 20th century, analysis of plywood timber beams and timber composite beams in general has been made based on both ranges of behavior, linear and non-linear.

In spite of the advances of non-linear analysis since the spreading of computers, the technical code specifications are based only on linear approaches.

By the other hand, technical codes are very superficial in what concerns the calculation of the slip modulus value. Many technical codes are superficial even in the specifications for plywood timber beams and timber composite beams in general, what justifies the development of a more accurate method for the calculation of these kinds of beams by designers.

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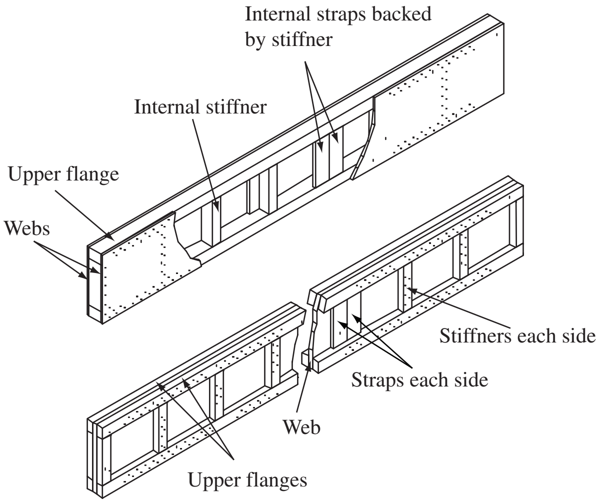


Figure 1. Scheme of Plywood timber beams with box and "I" section.

Thus, it is essential the correct determination of the slip modulus and its correct insertion in the beam model.

In 2002 Santana[5] and Santana and Mascia[6] demonstrated that the deformation of the connectors has a significant influence on the displacements of the structure in theoretical and experimental terms. In these studies, the authors concluded that the deformation of the mechanic pinned-shaped connection has a significant influence on the displacements of the structure, although that influence is not important on the stresses (the stresses are not so sensible to the behavior of the connection).

In this context, this paper deals with the experimental determination of the equivalent slip modulus of nailed connections, aiming the application on the design of nailed plywood timber beams. We made the theoretical development upon nailed connection, but all conclusions may be extended for other kinds of pin-shaped connectors, such as bolts.

The objectives of this paper are:

- to review a model for the analytical determination of the slip modulus of a nailed connection;
- to present concepts related to the insertion of the slip modulus and the equivalent slip modulus into the analysis of nailed plywood timber beams; and
- to approach the determination of the geometrical and material properties involved in its analytical determination.

With this work, we expect to provide a contribution to the development of a simple analytical method for the calculation of the slip modulus and particularly the equivalent slip modulus aiming the application in the design methods of timber structures with semi-rigid connection.

2. General Theory

2.1. The basic operation of a single nail connection

The relationship among the longitudinal force transmitted by the nail, P, and the slip, Δ, is generally of the kind:

$$P = P(a_o, a_1, ..., a_n, \Delta) \tag{1}$$

where a_o, a_1,..., a_n are parameters related to geometrical and material properties.

In the linear-elastic range, Equation 1 becomes:

$$P = K\Delta \tag{2}$$

where K is denominated "slip modulus" of the connection. The slip modulus, consequently, depends on the material and geometrical properties of the wood and of the nail.

The performance of the connection depends on the mechanical and physical properties of wood, steel properties, geometrical characteristics of the nail, and geometrical properties of the connection. In this paper, we note that failure modes or criteria will be not emphasized.

Johansen[7] published a work that was probably the first attempt to describe the connection behavior theoretically. His work was the basis for the yield model, which describes the failure modes of the connection and which substantiated the Eurocode 5[3] method for design of connections. But his model can not describe the behavior of a connection along loading, because it assumes perfectly plastic behavior both for wood and steel.

Kuenzi[8] developed a research on nailed connections that was innovative by modeling the nail as a linear-elastic beam on elastic foundation and based on this theory. He determined the slip modulus and also the internal efforts acting on the nail. His model was restricted to the proportionality limit. Figure 2 shows the scheme used by Kuenzi[8].

Kuenzi[8] presented the following governing differential equation of the beam on elastic foundation and the solution for the case of single and double shear:

$$EI\frac{dv^4}{dx^4}(x) + kv(x) = 0 \tag{3}$$

where: k is the elastic modulus of foundation, E is the modulus of elasticity of the nail(steel), I is the moment of inercia of the nail section and v is the displacement.

After obtaining the solution, it is possible to find the relationship between load and slip:

$$\Delta = P\left[2(L_1 + L_2) - \frac{(J_1 - J_2)^2}{K_1 + K_2}\right] \tag{4}$$

with L_1, L_2, J_1, J_2, K_1 and K_2 defined by:

$$L_1 = \frac{\lambda_1}{k_1}\left[\frac{\sinh\lambda_1 t_1 \cosh\lambda_1 t_1 - \sin\lambda_1 t_1 \cos\lambda_1 t_1}{\sinh^2\lambda_1 t_1 - \sin^2\lambda_1 t_1}\right] \tag{5a}$$

$$L_2 = \frac{\lambda_2}{k_2}\left[\frac{\sinh\lambda_2 t_2 \cosh\lambda_2 t_2 - \sin\lambda_2 t_2 \cos\lambda_2 t_2}{\sinh^2\lambda_2 t_2 - \sin^2\lambda_2 t_2}\right] \tag{5b}$$

$$J_1 = \frac{\lambda_1^2}{k_1}\left[\frac{\sinh^2\lambda_1 t_1 + \sin^2\lambda_1 t_1}{\sinh^2\lambda_1 t_1 - \sin^2\lambda_1 t_1}\right] \tag{5c}$$

$$J_2 = \frac{\lambda_2^2}{k_2}\left[\frac{\sinh^2\lambda_2 t_2 + \sin^2\lambda_2 t_2}{\sinh^2\lambda_2 t_2 - \sin^2\lambda_2 t_2}\right] \tag{5d}$$

$$K_1 = \frac{\lambda_1^3}{k_1}\left[\frac{\sinh\lambda_1 t_1 \cosh\lambda_1 t_1 + \sin\lambda_1 t_1 \cos\lambda_1 t_1}{\sinh^2\lambda_1 t_1 - \sin^2\lambda_1 t_1}\right] \tag{5e}$$

$$K_2 = \frac{\lambda_2^3}{k_2}\left[\frac{\sinh\lambda_2 t_2 \cosh\lambda_2 t_2 + \sin\lambda_2 t_2 \cos\lambda_2 t_2}{\sinh^2\lambda_2 t_2 - \sin^2\lambda_2 t_2}\right] \tag{5f}$$

where: sinh, cosh are the hyperbolic sine and cosine,t_1 is the width of the element 1, t_2 is the penetration of the nail in the element 2, k_1 is the foundation modulus of the element 1, k_2 of the element 2 and with λ_1 and $\lambda_2$defined by:

$$\lambda_1 = \sqrt[4]{\frac{k_1}{4EI}} \tag{6a}$$

$$\lambda_2 = \sqrt[4]{\frac{k_2}{4EI}} \tag{6b}$$

Recalling the definition of the slip modulus, and rearranging Equation 4, it is found that:

$$K = \left[2(L_1 + L_2) - \frac{(J_1 - J_2)^2}{K_1 + K_2} \right]^{-1} \tag{7}$$

Equation 5 is the analytical expression for the slip modulus. Examining this equation, we note that the slip modulus is a function of the geometrical properties of the timber pieces and the nail and of the material properties of wood and steel.

The modulus of elasticity of steel is a known value without much variation. On the other hand, K_1 and K_2 are material properties that need characterization, because of their large variability with the wood species and the grain direction.

Concerning the foundation modulus, Kuenzi[8] proposed for determination of the modulus of foundation:

$$k = E_{w,\alpha} \frac{D}{z_f} \tag{8}$$

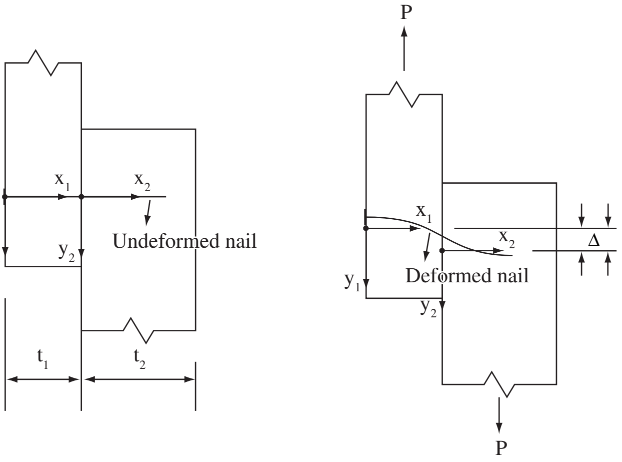
where D is the nail diameter, z_f is the foundation depth, and $E_{w,\alpha}$ is the modulus of elasticity of the wood under compression.

In his work, Kuenzi[8] assumed the foundation depth as being equal to 2,54 cm for examples of calculation. But we suggest that in a connection the depth of the foundation is theoretically limitless. Thus, a new question is the determination of a reasonable value for the depth of the foundation. Wilkinson[9] in 1972 investigated the model developed by Kuenzi[8]. He emphasized the importance of the foundation modulus, and defined the “elastic bearing constant”, as being a parameter k_o such that:

$$k = k_o D \tag{9}$$

From results of an experimental program, Wilkinson[9] observed a relationship between the elastic bearing constant and the density of the wood. Komatsu et al.[10] in 1996 estimated a property named embedding coefficient as a function of the wood modulus of elasticity and the diameter of the drift-pin. Riley and Gebremedhin[11] in 1998 found an empirical relationship between the foundation modulus and the density of the wood, moisture content and angle of grain with respect to load.

All these authors used the model of the beam on elastic foundation in the characterization of connections. In fact, this model is very suitable for the modeling of connections, but requires the characterization of the foundation modulus.



(a) (b)

Figure 2. Deformed shape of the connection and convention of axes.

Other researchers later developed more complete models for the connection, exploring the behavior of the nail under bending and of the wood under embedding. These models which predicts the behavior of both the nail and the wood has been formulated by considering the nail as a beam resting on a elastic or elastoplastic foundation, as reported by Patton-Mallory, Pellicane and Smith[12] in 1997.

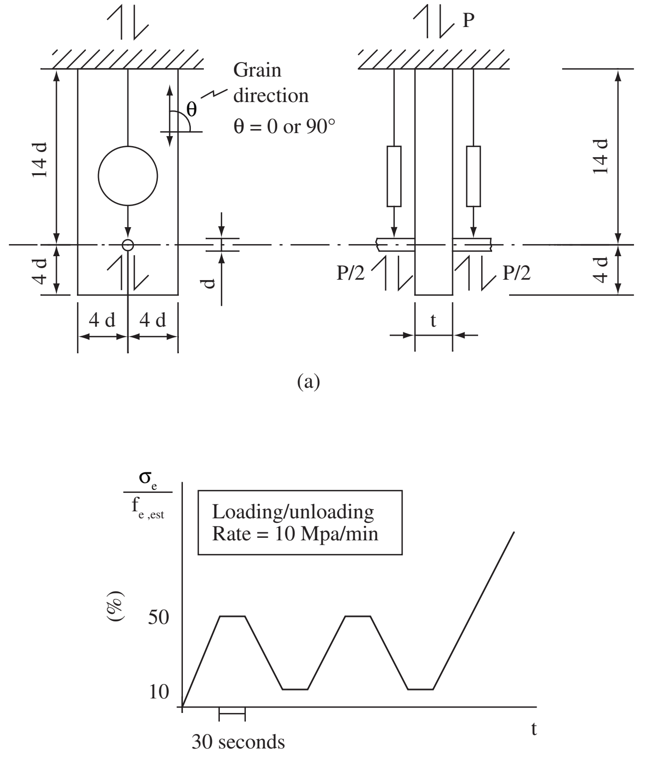
For tropical species, such property is not characterized. In fact, it is necessary that research for the foundation modulus commence. For the purpose of the systematization of that determination, we propose that the foundation modulus of wood be determined through the embedding test, which is a standardized test for the determination of the embedding strength in the codes including the Brazilian code NBR7190/1997[1]. This test is similar to ASTM D5764-97a[13] used in North America. Figure 3 illustrates the embedding test.

From the embedding test, it is possible to obtain a curve of the distributed compressive force along the axis of the nail of diameter d versus the foundation displacement. From the slope of this curve, we obtain the foundation modulus. The knowledge of the wood foundation behavior is a very important property to be investigated, because, once characterized, it allows the analytical determination of the slip modulus.

Furthermore, we can observe that the lack of knowledge about the foundation modulus or the elastic bearing constant of wood is a consequence of the unavoidable loss of information about the real stress distributions around the dowel when simplifications are inserted in analytical models.

2.2. Assumptions and simplifications in connection study

In the study of connections, the following assumptions and simplifications are usually made:



(a)

(b)

Figure 3. Brazilian Code standard embedding test scheme. a) Teste setup scheme; and b) Loading scheme.

- friction forces between the wooden pieces of the connection are disregarded;
- the static scheme of forces acting over the nail is not affected by its deformed configuration, which is equivalent to say that the forces which appear in the direction of the nail axis can be despised;
- the wood is considered as a continuous material, with unidimensional behavior;
- under crushing, the displacement of the wood at any point is assumed to be proportional to the force in the same direction at that point. This is equivalent to say that the wood is considered as a Wrinkler foundation.
- the distribution of stresses around the wall of the hole (tridimensional) is approximated by a distribution of stresses in the plan of the connection (bidimensional), being the plan of the connection the plan which contains the directions of the longitudinal force and the axis of the nail; and
- the head of the nail prevents rotation of this end in the plan of the connection;

In this paper we make another additional assumption:

- there is not clearance between the nail and the hole. According to the Brazilian code, NBR7190[1], the nails must be driven into holes drilled previously, with diameter equal to 98% of the diameter of the nail for hardwoods, and 85% of the diameter of the nail for softwoods.

With the above assumptions, we can schematically describe the scheme of behavior of the connection. Figure 4 shows a scheme of the deformed shape of a nail in a typical two-member connection and the forces acting over the nail and over the crushed wood.

In spite of the advances on the description of the load-slip behavior in the elastoplastic range, the slip modulus of the elastic model (which may be taken as the initial stiffness of the elastoplastic model) have many applications, as we have exposed before.

2.3. Determination of the equivalent slip modulus for application on built-up beam analysis

In the case of a nailed plywood timber beam, the function of the connections is to assemble the parts (the webs and the flanges) and to transmit the shear stresses from bending. The connection extends the length of the beam.

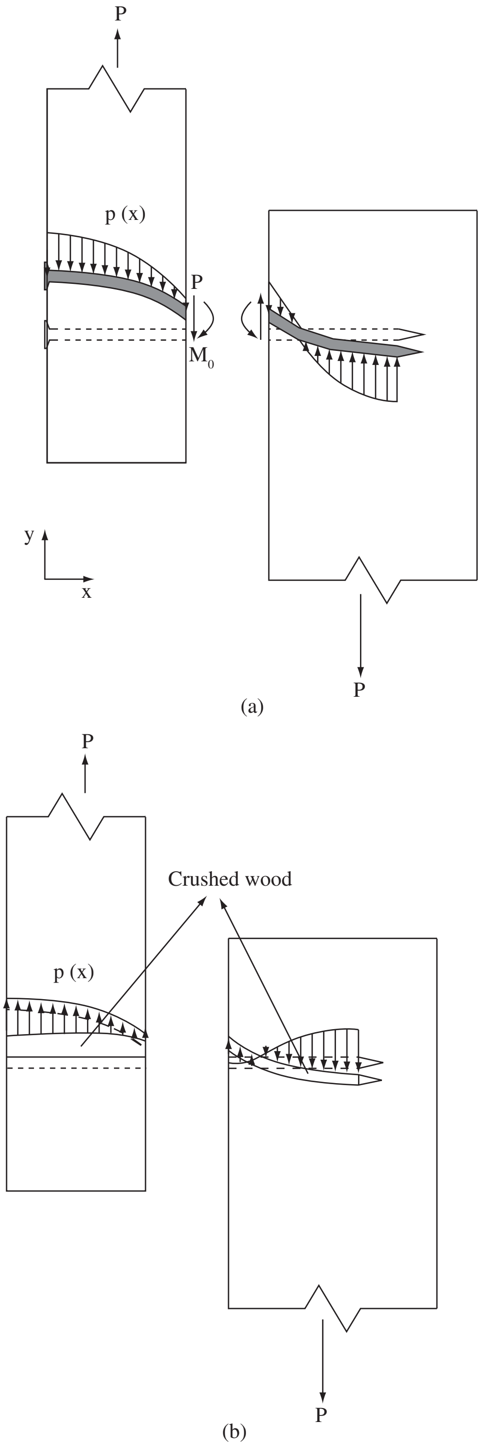
The analysis of a composite beam in general is developed from equilibrium equations for elements of infinitesimal length of the parts. Although the method of analysis of composite beams is well established, the concept of this "infinitesimal connection" has not been totally explored.

In this part we develop the concept of infinitesimal connection and define its "equivalent slip modulus". Foschi[14], in 1982, used this concept in the analysis of structures with continuous connections.

If two connections undergo the same slip, under the action of the same load, we can say that these two connections are equivalent as for the stiffness. Let us consider two equivalent connections: (a) a real one with a single nail, part of a built-up beam with nail spacing s, and (b) a fictitious one with infinitely small nails, distributed uniformly along of the length of the connection s, as shown in Figure 5.

The real connection with a single nail have slip modulus K, in such a way that its load-slip relationship is described by $P = K\Delta$. Be P the force transmitted by the real and the fictitious connection. We define p as a distributed force transmitted per unit length of the fictitious connection in such a way that $p = P/s$. In this case, the group effect does not apply.

Both connections undergo the same slip Δ, since they are defined to be equivalent. Because of the geometrical compatibility, each part of unit length of the fictitious connection undergoes the same slip.



(a)

(b)

Figure 4. Scheme of the forces acting in the elements of a connection according to the usual assumption a) reaction of the crushed wood over the nail and b) reaction of the nail over the wood.

Hence, if each part of unit length undergoes the same slip under a load that is s times lower than the real connection, then it has a slip modulus s times lower. For the fictitious connection the relationship $p = (K/s)\Delta$ is valid.

The "equivalent slip modulus", K_{eq}, can then be defined as given below:

$$K_{eq} = \frac{K}{s} \tag{10}$$

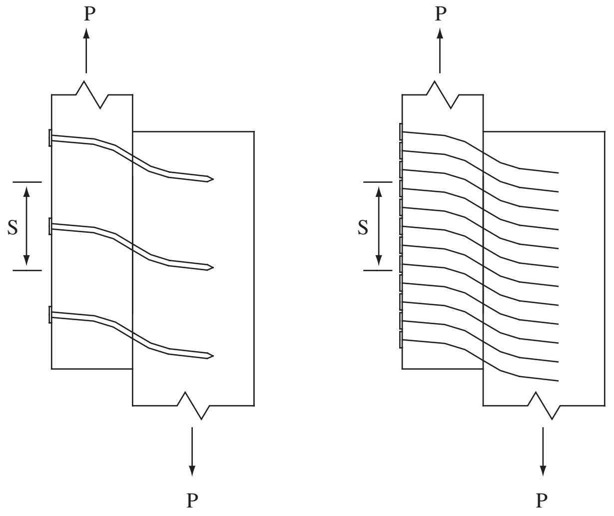


Figure 5. Two equivalent connections (real and fictitious connection).

It could be formally defined as “the slip modulus per unit length of a continuous connection” composed by single nail connections with slip modulus K and spacing s. Its dimension is force per square length, or better, force per length per length.

The same development could be done for the case of n parallel rows of nails, from which, assuming that the total load P is distributed equally among the n rows, we would arrive to:

$$K_{eq} = n\frac{K}{s} \tag{11}$$

that is the general form of the equivalent slip modulus.

As a more general result, the equivalent slip modulus is the relationship between the shear unit force at the interface and the relative displacement between the connected elements in a composite beam.

It is important to emphasize that when nails are disposed in a row, it is expected that nails receive different parts of load. However, in the case of a continuous connection in a beam, the nails are submitted to a particular distribution of load, following the distribution of shear forces between the parts connected. Each nail will receive a load that is the resultant of the shear forces distributed in the length equal to the spacing between nails. In this way, we may think in a stepped distribution of forces between nails.

The way in the load distribution of the load between nails depends on the way in which work in the connection. Riley and Gebremedhin[11] applied the concept of slip modulus in the study of the stiffness of metal toothed plates submitted to combined efforts. In this kind of connection, for instance, the distribution of load among connectors is very particular.

In this context, the problem of the distribution of load in a row of nails in connection in a composite beam can not be compared with the problem of distribution of load in a row of nails in a connection submitted to traction. It is clear that the effect of the alignment of the nails has some influence on the former problem also. But we may consider it of less importance if compared with the problem of modeling the distribution of shear forces in a partially composite beam, at least in the current level of knowledge about this issue.

It is important to emphasize that in this paper the determination of the equivalent slip modulus aims the fitting of the properties of the real discrete and repeated connection to the continuous approach used in the modeling of beams.

3. Experimental Results

In the experimental part, tests were performed in specimens obtained from models of nailed plywood timber beams with box section with plywood webs. The specimens were obtained by cutting the beam in the cross direction, in such a way that the specimens had the same cross section of the models. Each specimen has four connections with six nails.

The species employed were Angico Preto (*Anadenanthera macrocarpa* (Benth.) Brenae) for the timber parts and Bucuíba-branca plywood (*Virola* sp.). The diameter of the nail was 2.4 mm.

In the test, each specimen was loaded to compression in the longitudinal direction, and the slip between timber and plywood parts were measured. The schemes of the specimens and of the test are shown in Figures 6 and 7 respectively.

The test performed as described is not a standardized test, but its main advantage is that it reproduces the connection such it is in the real beam, with the only difference that in the real beam the shear stresses are due to bending and in the test the shear stresses are due to the transfer of compression forces between connected elements. Due to the configuration of the specimen (one of the ends flat and the other not), the load is transferred in a non-uniform way, as in the real beam.

At the upper end (non-flat end) of the specimen the timber parts receive theoretically 100% of the load. At the lower end (flat end) of the specimen, the load is distributed among all the timber and plywood parts, according to the axial stiffnesses of each of them.

In the case of our specimen, knowing the axial stiffness of timber and plywood parts, and using the condition of compatibility of displacements, we calculated that timber parts receive 80% of the load, and plywood parts, 20% (see Table 1). Hence, we know the load that is transmitted by each of the four connections of the specimen, that is, 5% of the total applied load.

Figure 8 shows the diagrams of load transmitted by each connection vs. slip obtained for three specimens. For each diagram, a straight line was adjusted in the linear-elastic range. With the slope of this line we obtained the slip modulus of the connection with six nails (K_6), which is the same for each of the four connections. Dividing this value by six times the spacing, we obtain the equivalent slip modulus of the connection (K). We know that the load is not uniformly distributed among all the nails of a row, but the proportionality between load and slip is the same. Thus, we can calculate the equivalent slip modulus for all the six nails and obtain a mean equivalent slip modulus.

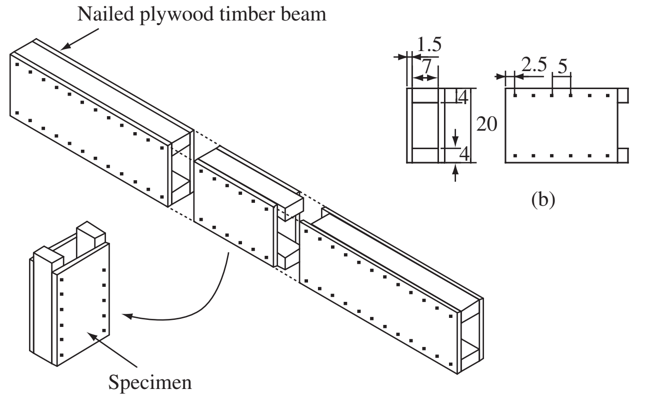
Table 2 exhibits the obtained results. The average value of the equivalent slip modulus was 5,34 kN/cm^2.

In order to calculate the theoretical result, the value of the elastic capacity constant, K_o, of both timber and plywood is required. These values are not available for Brazilian species, what justifies a work on the characterization of this property.

Then, some theoretical values were calculated with Equations 7 (with Equations 5a to 5f and 6a and 6b), 8 and 10. By adopting values for the foundation depth sucessively, we found that a foundation depth of about zf = 2,4 D leads to the slip modulus equal to 53,4 N/mm^2. With D = 2.4 mm, the foundation depth is 5.76 mm, or 0,23 inch. However, there is no way to confirm this value theoretically.

This fact indicates that Equation 8 is not a good way for the determination of the foundation modulus (although it is very useful to clarify the behavior of the elastic foundation). A more simple and direct way for the determination of the foundation modulus is Equation 9 with the value of the elastic bearing constant for wood known.

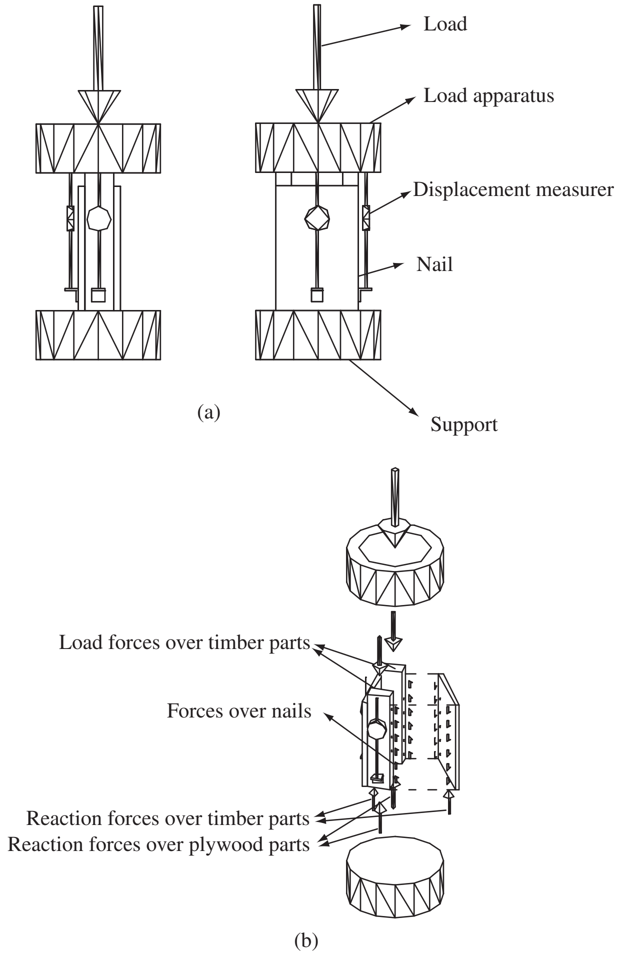
As already commented, from the embedding test, it is possible to obtain a curve of the distributed compressive force along the axis



(b)

(a)

Figure 6. Scheme of the specimen a) obtaining the specimen from the beam and b) specimen dimensions.



(a)

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Figure 7. Scheme of the test. a) scheme of test setup; and b) scheme of forces.

of the nail of diameter D versus the foundation displacement. From the slope of this curve, we obtain the foundation modulus. Santana[6] and Mascia and Santana[15] presented some of these values for some Brazilian tropical spec ies of wood and plywood.

4. Numerical Simulation

To illustrate the theoretical and experiments aspects of this paper, an example calculation of the equivalent slip modulus for a plywood timber beam with box section is presented. In this example, consider that both plywood webs have width equal to 15 mm, the timber flanges have width equal to 50 mm, and the nails which make the connection between webs and flanges have length equal to 41 mm, in such a way that the nails penetrates 26 mm inside the timber flanges at each side. In this way we have $t_1 = 15$ mm and $t_2 = 26$ mm.

The moduli of elasticity of the plywood and wood are respectively 2,826 GPa and 12,219 GPa, in the direction parallel to the axis of the beam. The steel of the nail have modulus of elasticity equal to 210 GPa. The diameter of the nail is 2.4 mm. The spacing between nails is 50 mm and each connection has one row of nails.

The foundation moduli of the plywood and wood were calculated with Equation 8 with foundation depth adopted as twice the diameter of the nail. We did not use Equation 9, since we do not have the values of the elastic bearing constant for Brazilian species. This value for the ratio foundation depth/diameter of the nail was estimated arbitrarily,

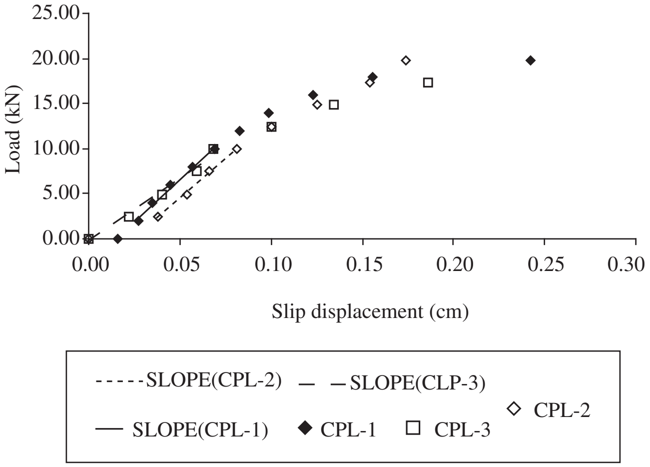

Figure 8. Experimental longitudinal load vs. measured displacement diagram, for specimens CPL-1, CPL-2 and CPL-3.

Table 2. Determination of the equivalent slip modulus from the test results.

Slip modulus	Specimen CPL-1	Specimen CPL-2	Specimen CPL-3	Average
K_6* (N/m)	18611	16309	13103	16008
$\overline{K}$** (N/mm^2)	62,0	54,4	43,7	53,4

*Slip modulus of the real connection with 6 nails; and **The equivalent slip modulus for the equivalent fictitious connection.

Table 1. Distribution of load among timber and plywood parts at the low end of the specimen.

	Elasticity Modulus (MPa)	Area (mm^2)	Axial Stiffness (kN)	Load (%)
Timber parts	12220	2 x 2800 = 5600	68432	80
Plywood parts	2830	2 x 3000 = 6000	16980	20
Total			85412	100

based on the assumption that the foundation depth depends on the diameter and that for hard wood species this value is expected to be few times the diameter of the nail.

With Equation 8 applied to both wood and plywood, and Equations 6a and 6b, we calculated k_1 = 6,1095 GPa, k_2 = 1,4132 GPa, $\lambda_1 = 0.25851$ mm^{-1} and $\lambda_2 = 0.17928$ mm^{-1}.

We also have $J_1 = 0.0110 \times 10^{-2}$ mmN^{-1}, $J_2 = 0.0228 \times 10^{-2}$ mmN^{-1}, $K_1 = 0.0283 \times 10^{-4}$mm^{-1}N^{-1}, $K_2 = 0.0408 \times 10^{-4}$ mm^{-1}N^{-1}, $L_1 = 0.0042 \times 10^{-3}$ N^{-1} and $L_2 = 0.0127 \times 10^{-3}$ N^{-1}.

With Equation 7, we have that the slip modulus for each nail is K = 3141 N/mm.

Finally, with Equation 10 we obtain that the equivalent slip modulus of the connection is K_{eq} = 62,8 N/mm^2.

Observe that the data of the beam in the experimental investigation are the same as in the numerical example presented. The result obtained experimentally is about 15% lower than the theoretical value. This relatively high difference happened mainly due to the uncertainties involved in the determination of the material properties that affects directly the wood foundation properties. In the numerical example, we observe that the result is very sensible for the foundation depth. For example, if we adopt for the foundation depth a value of once and three times the diameter of the nail, we obtain respectively 105,5 N/mm^2 and 45,5 N/mm^2 for the equivalent slip modulus. Since the foundation modulus is an unknown property, estimated with a large uncertainty, we can conclude that the foundation properties of the wood need to be characterized as a design property.

5. Conclusions

In this work we focused the stiffness of nailed connections and presented the basis to its analytical determination, aiming the application of that determination in the analysis of nailed plywood timber beam. The theoretical development may be applied to pin-shaped connections in general.

The stiffness of a nailed connection, represented by the slip modulus, is a property whose correct characterization is essential in analysis including the behavior of the connection, in the elastic-linear range. The structure behavior is very sensitive to its value.

The model of elastic foundation, widely described in the literature, clarifies the behavior of the nailed and other pin-shaped connections. As can be observed from the theoretical model, the value of the slip modulus depends on geometrical properties of the connection and properties of the materials.

Finally, in the theoretical development, we presented the concept of the equivalent slip modulus, showing how the slip modulus, usually determined for a connection with a single nail, may be correctly inserted in the analysis of a nailed plywood timber beam.

In the experimental part we presented a proposal of a test to the determination of the equivalent slip modulus, in order to evaluate the analytical model. We presented an obtained result, but the theoretical value can not be compared with it, because the values for the elastic bearing constant, required for the determination of the theoretical value, are not available for Brazilian species. Any way, as a first effort to validate the theoretical model presented in the work, this test showed very suitable for obtaining experimental values of the equivalent modulus, because it reproduces the connection of the beam with more fidelity and also is simple to be assembled.

We conclude that the model of beam on elastic foundation is suitable for the development of design procedures for the slip modulus, provided that the material properties be correctly and systematically determined, specially the wood foundation modulus. We presented a proposal and the concepts for the systematization of the experimental verification of this property, through the embeddment test.

In general, we conclude that this work contributes to the knowledge of a mechanical property of connection, which is widely used, but is not well known.

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