AIP | Physics of Plasmas

Nonlinear dynamics of electromagnetic turbulence in a nonuniform magnetized plasma

P. K. Shukla, Arshad M. Mirza, and R. T. Faria Jr.

Citation: Physics of Plasmas (1994-present) **5**, 616 (1998); doi: 10.1063/1.872776
View online: http://dx.doi.org/10.1063/1.872776
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/5/3?ver=pdfcov
Published by the AIP Publishing

PHYSICS OF PLASMAS VOLUME 5, NUMBER 3 MARCH 1998

Nonlinear dynamics of electromagnetic turbulence in a nonuniform magnetized plasma

P. K. Shukla, Arshad M. Mirza,[a)] and R. T. Faria, Jr.[b)]
Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany

(Received 24 June 1997; accepted 17 November 1997)

By using the hydrodynamic electron response with fixed (kinetic) ions along with Poisson's equation as well as Ampère's law, a system of nonlinear equations for low-frequency (in comparison with the electron gyrofrequency) long-(short-) wavelength electromagnetic waves in a nonuniform resistive magnetoplasma has been derived. The plasma contains equilibrium density gradient and sheared equilibrium plasma flows. In the linear limit, local dispersion relations are obtained and analyzed. It is found that sheared equilibrium flows can cause instability of Alfvén-like electromagnetic waves even in the absence of a density gradient. Furthermore, it is shown that possible stationary solutions of the nonlinear equations without dissipation can be represented in the form of various types of vortices. On the other hand, the temporal behavior of our nonlinear dissipative systems without the equilibrium density inhomogeneity can be described by the generalized Lorenz equations which admit chaotic trajectories. The density inhomogeneity may lead to even qualitative changes in the chaotic dynamics. The results of our investigation should be useful in understanding the linear and nonlinear properties of nonthermal electromagnetic waves in space and laboratory plasmas. © *1998 American Institute of Physics.* [S1070-664X(98)00403-0]

I. INTRODUCTION

Recently, there has been a great deal of interest[1–3] in understanding the linear and nonlinear properties of finite amplitude low-frequency Alfvén-like waves in low-temperature space and laboratory plasmas. The linear theory[4–6] focuses on the derivation of the dispersion relation and identifying the unstable wave spectra, whereas the nonlinear analyses[1–3,6–8] compute the saturation level of fluctuations as well as develop mode coupling equations which are needed for predicting the salient features of fully developed electromagnetic turbulence in nonuniform magnetized plasmas.

Finite amplitude dispersive Alfvén-like electromagnetic waves accompany an electric field parallel to the external magnetic field lines. Intense electric fields may cause energization of electrons, which may produce interesting phenomena. The dispersive kinetic Alfvén waves[9] cannot be obtained within the framework of the ideal magnetohydrodynamic (MHD) equations and one has to invoke either a kinetic theory or a two–fluid model in order to include the dispersion caused by finite Larmor radius or electron inertial effects.

Linear theory dictates that nonthermal fluctuations can be generated provided that there exist free energy sources in the form of equilibrium anisotropic particle distributions, pressure and velocity gradients, etc. Under appropriate conditions, free energy of the system can be coupled to both the electrostatic and electromagnetic modes in nonuniform magnetoplasmas. Several authors[3–6] have investigated the instability of low-frequency (in comparison with the ion gyrofrequency), long wavelength (in comparison with the ion gyroradius) electrostatic convective cells and drift-acoustic waves[3] as well as of electromagnetic kinetic Alfvén-drift waves[6] in the presence of magnetic field-aligned plasma flow gradients. The nonlinear mode coupling equations excluding dissipative effects have also been derived and analyzed.[3,6] The results have been applied to understand the nonlinear structures in the auroral region of the Earth's ionosphere.

However, the experimental data from space[10] and laboratory[11] plasmas exhibit that the observed electromagnetic waves in nonuniform magnetoplasmas have a broad frequency and wavevector spectra. Specifically, the wave frequencies could be smaller or larger than the ion gyrofrequency, whereas wavelengths could be in the range between the collisionless electron skin depth and the ion gyroradius scale or even shorter. The waves accompany simultaneously finite density and sheared magnetic field perturbations. Thus, they can be categorized as sheared or kinetic Alfvén-like waves.[9]

In this paper, we focus on the linear and nonlinear properties of low-frequency (in comparison with the electron gyrofrequency) long as well as short wavelength electromagnetic Alfvén-like waves in a nonuniform magnetoplasma containing an equilibrium electron density gradient and sheared equilibrium plasma flows. For this purpose, we have employed the electron MHD equations consisting of the electron continuity and the parallel component of the electron momentum equations, supplemented by Ampère's law, in order to derive a set of nonlinear equations. For long

[a)]Permanent address: Department of Physics, Quaid-i-Azam University, Islamabad 45320, Pakistan.
[b)]Permanent address: Instituto de Física ''Gleb Wataghin,'' Universidade Estadual de Campinas, 13083-970, Campinas, SP, Brazil.

wavelength disturbances in an electron plasma with fixed ion background, we use Poisson's equation to eliminate the electron number density perturbation. On the other hand, for short wavelength waves, we use the Boltzmann response for the ion number density perturbation. In the linear limit, we obtain dispersion relations in the local approximation, whereas in the nonlinear case we discuss possible stationary and non-stationary solutions of the newly derived nonlinear equations.

The manuscript is organized as follows. In Sec. II, we present a derivation of the three-dimensional nonlinear mode coupling equations for low-frequency (in comparison with the electron gyrofrequency), long and short wavelengths sheared electromagnetic waves in a nonuniform dissipative magnetoplasma having an equilibrium density gradient and sheared equilibrium plasma flows. The linear dispersion relations are derived and analyzed in Sec. III. Section IV contains stationary solutions of the nonlinear equations when the dissipation is ignored. In Sec. V we study the non-stationary behavior of the nonlinear dynamical equations including dissipation. Finally, the main results of our investigation are summarized in Sec. VI.

II. GOVERNING NONLINEAR EQUATIONS

Let us consider the nonlinear propagation of low-frequency (in comparison with the electron gyrofrequency $\omega_{ce}=eB_0/m_ec$, where e is the magnitude of the electron charge, B_0 is the strength of the external magnetic field, m_e is the electron mass, and c is the speed of light) electromagnetic waves in a nonuniform magnetized plasma containing the equilibrium density gradient $\partial n_0/\partial x$ and the equilibrium velocity gradient $\partial v_{j0}/\partial x$, where n_0 is the equilibrium plasma number density, v_{j0} is the magnetic field-aligned unperturbed plasma flow velocity of the particle species j (j equals e for the electrons and i for the ions) in a direction transverse to the equilibrium magnetic field $B_0\hat{\mathbf{z}}$; $\hat{\mathbf{z}}$ being the unit vector along the z axis. We assume that the equilibrium currents produced by the difference between the electron and ion drift velocities lead to a negligible shear component of the equilibrium magnetic field. This is justified, for example, for local phenomena in the Earth's ionosphere and in several laboratory devices, where the main component of the magnetic field is thousand times larger than the sheared equilibrium magnetic field component. Furthermore, it is supposed that the equilibrium density gradient is maintained by external sources (e.g. external electric fields, gravitational forces, etc.), although no such sources are required for a non-zero gradient of $\mathbf{v}_{j0}$ to exist because $\mathbf{v}_{j0}\cdot\nabla\mathbf{v}_{j0}\equiv 0$ and $\mathbf{v}_{j0}\times\mathbf{B}_0\equiv 0$ when $\mathbf{v}_{j0}=\hat{\mathbf{z}}v_{j0}(x)$. Thus, non-continuous injection of charged particles along the external magnetic field lines establishes sheared plasma flows.

For low-frequency, long wavelength (in comparison with the electron gyroradius ρ_e) electromagnetic fields in an isothermal plasma, the electron fluid velocity perturbation is given by

$$\mathbf{v}_e\approx\mathbf{v}_{EB}+\mathbf{v}_{De}+\mathbf{v}_{pe}+(v_{e0}+v_{ez})\mathbf{B}_{1\perp}/B_0+\hat{\mathbf{z}}v_{ez}, \tag{1}$$

where $\mathbf{v}_{EB}=(c/B_0)\hat{\mathbf{z}}\times\nabla\phi$, $\mathbf{v}_{De}=-(cT_e/eB_0n_e)\hat{\mathbf{z}}\times\nabla n_e$, and $\mathbf{v}_{pe}=(c/B_0\omega_{ce})[\partial_t+v_{e0}\partial_z-\mu_e\nabla_\perp^2+(\mathbf{v}_{EB}+\mathbf{v}_{De})\cdot\nabla+v_{ez}\partial_z]\nabla_\perp\phi$ are the $\mathbf{E}\times\mathbf{B}_0$, the diamagnetic, and the polarization drift velocities, respectively, $\mathbf{E}=-\nabla\phi-c^{-1}\partial_tA_z\hat{\mathbf{z}}$ is the electric field vector, ϕ is the electrostatic potential, and A_z is the component of the vector potential along the z axis. Furthermore, n_e is the electron number density, T_e is the constant electron temperature, $\mu_e=0.51\nu_e\rho_e^2$ is the electron gyro-viscosity,[12] ν_e is the electron collision frequency, and $\mathbf{B}_{1\perp}=\nabla A_z\times\hat{\mathbf{z}}$ is the two-dimensional magnetic field perturbation. By adopting the low-β approximation, the compressional magnetic field perturbation along the $\hat{\mathbf{z}}$ direction has been neglected.

The z-component of the electron fluid velocity perturbation is obtained from the Ampère's law

$$v_{ez}\approx(c/4\pi n_ee)\nabla_\perp^2A_z. \tag{2}$$

To derive the appropriate nonlinear mode coupling equations for the electrons in the presence of the electromagnetic fields, we substitute Eq. (1) into the electron continuity equation and make use of Eq. (2), and radially obtain in the limit of $\rho_e^2\nabla^2\ll 1$

$$\begin{aligned}&\mathscr{L}_t\left(n_{e1}+\frac{cn_0}{B_0\omega_{ce}}\nabla_\perp^2\phi\right)-D_c\nabla_\perp^2n_{e1}-\frac{cn_0}{B_0\omega_{ce}}\mu_e\nabla_\perp^4\phi\\&-\frac{c}{B_0}\hat{\mathbf{z}}\times\nabla n_0\cdot\nabla\phi-\frac{1}{B_0e}\hat{\mathbf{z}}\times\nabla j_{e0}\cdot\nabla A_z\\&+\frac{c}{4\pi e}\mathscr{L}_z\nabla_\perp^2A_z=0,\end{aligned} \tag{3}$$

where $\mathscr{L}_t\equiv\partial_t+v_{e0}\,\partial_z+\mathbf{v}_{EB}\cdot\nabla+v_{ez}\,\partial_z$, $\mathscr{L}_z=\partial_z+B_0^{-1}\nabla A_z\times\hat{\mathbf{z}}\cdot\nabla$, $j_{e0}=-n_0ev_{e0}$, $D_c=\nu_e\rho_e^2$ is the coefficient of the electron diffusion, and $n_{e1}(=n_e-n_0\ll n_0)$ is the perturbed electron number density.

Similarly if we substitute the z-component of the electric field $E_z=-\partial_z\phi-c^{-1}\,\partial_tA_z$ into the z-component of the resistive electron momentum equation and use Eqs. (1) and (2), then we have

$$\begin{aligned}&(d_t+\mathbf{v}_{D0}\cdot\nabla)A_z-\lambda_e^2(\mathscr{L}_t+\nu_e)\nabla_\perp^2A_z+c(\partial_z+\mathbf{S}_{v0}\cdot\nabla)\phi\\&-(cT_e/en_0)\mathscr{L}_zn_{e1}=0,\end{aligned} \tag{4}$$

where $d_t=\partial_t+\mathbf{v}_{EB}\cdot\nabla$, $\mathbf{v}_{D0}=-(cT_e/eB_0n_0)\hat{\mathbf{z}}\times\nabla n_0$ is the equilibrium electron diamagnetic drift velocity, $\lambda_e=c/\omega_{pe}$ is the collisionless electron skin depth, $\omega_{pe}=(4\pi n_0e^2/m_e)^{1/2}$ is the electron plasma frequency, and $\mathbf{S}_{v0}=\hat{\mathbf{z}}\times\nabla v_{e0}/\omega_{ce}$. We note that the $\lambda_e^2\mathscr{L}_t\nabla_\perp^2A_z$ term in Eq. (4) is the contribution of the linear and nonlinear electron inertial forces.

In the following, we consider two types of plasma response. First, when the wave period is shorter than the ion plasma and ion gyroperiods, the ions can be regarded as a static charge neutralizing background and they do not have time to respond to electromagnetic disturbances. Thus, the linear and nonlinear phenomena occurs on scales much below the ion inertial length c/ω_{pi}, where the plasma dynamics is governed by electron flows and their self-consistent magnetic fields. The ion plasma frequency is denoted by

ω_{pi}. Here, we can approximate n_{e1} by $(1/4\pi e)\nabla^2\phi$ and write Eqs. (3) and (4) for long wavelength perturbations as

$$\mathscr{L}_t\left(\nabla^2+\frac{\omega_{pe}^2}{\omega_{ce}^2}\nabla_\perp^2\right)\phi-D_c\left(\nabla^2+0.51\frac{\omega_{pe}^2}{\omega_{ce}^2}\nabla_\perp^2\right)\nabla_\perp^2\phi$$
$$-\frac{\omega_{pe}^2}{n_0\omega_{ce}}\hat{\mathbf{z}}\times\nabla n_0\cdot\nabla\phi-\frac{4\pi}{B_0}\hat{\mathbf{z}}\times\nabla j_{e0}\cdot\nabla A_z$$
$$+c\mathscr{L}_z\nabla_\perp^2A_z=0 \tag{5}$$

and

$$(d_t+\mathbf{v}_{D0}\cdot\nabla)A_z-\lambda_e^2(\mathscr{L}_t+\nu_e)\nabla_\perp^2A_z+c(\partial_z+\mathbf{S}_{v0}\cdot\nabla)\phi$$
$$-c\lambda_{De}^2\mathscr{L}_z\nabla^2\phi=0. \tag{6}$$

Second, for short wavelength (in comparison with the ion gyroradius ρ_i), the ion number density is given by $n_i\approx n_0\exp(-e\phi/T_i)$, where T_i is the constant ion temperature. Hence, from Poisson's equation we have $n_e=(1/4\pi e)[\nabla^2\phi+(T_i\lambda_{Di}^{-2}/e)\exp(-e\phi/T_i)]$, where $\lambda_{Di}=(T_i/4\pi n_0e^2)^{1/2}$ is the ion Debye length. Accordingly, for this case Eqs. (3) and (4) can be cast in the form

$$\mathscr{L}_t(1-\rho_a^2\nabla_\perp^2)\phi-D_c(1-0.51\rho_a^2\nabla_\perp^2)\nabla_\perp^2\phi$$
$$+\rho_a^2\frac{\omega_{ce}}{n_0}(\hat{\mathbf{z}}\times\nabla n_0)\cdot\nabla\phi$$
$$+\frac{4\pi\lambda_{Di}^2}{B_0}(\hat{\mathbf{z}}\times\nabla j_{e0})\cdot\nabla A_z-c\lambda_{Di}^2\mathscr{L}_z\nabla_\perp^2A_z=0, \tag{7}$$

and

$$(d_t+\mathbf{v}_{D0}\cdot\nabla)A_z-\lambda_e^2(\mathscr{L}_t+\nu_e)\nabla_\perp^2A_z+c(\partial_z+\mathbf{S}_{v0}\cdot\nabla)\phi$$
$$+c\sigma\mathscr{L}_z\phi=0, \tag{8}$$

where $\rho_a=c_a/\omega_{ce}$, $c_a=(T_i/m_e)^{1/2}$ is the electron-acoustic velocity, and $\sigma=T_e/T_i$. In Eq. (7), we have assumed that $\nabla^2\lambda_{Di}^2\ll 1$ and $\partial_t\phi^2\ll\omega_{ce}\rho_a^4|\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla\nabla_\perp^2\phi|$.

Equations (5) and (6) govern the nonlinear dynamics of long wavelength electromagnetic waves, whereas Eqs. (7) and (8) are for nonlinearly interacting short wavelength disturbances in a nonuniform resistive magnetized plasma with sheared plasma flows. We note that Eqs. (5)–(8) govern the mode coupling within the electromagnetic wave spectra. We thus have the possibility of energy cascading from short wavelength part of the spectrum to long wavelengths due to the nonlinear interactions of short wavelength oscillations. Thus, the physics of the modulation interaction[13] remains in tact within our formalism. For example, in an electron plasma with fixed ion background, magnetic electron drift modes could be nonlinearly excited by Alfvén-like electron convective cells. Thus, nonlinearly coupled magnetic electron drift modes and Alfvén-like electron convective cells can play a very important role in the electron magnetohydrodynamics (EMHD),[14] where the nonlinear phenomena occur on a short time (in comparison with the ion plasma and ion gyroperiods) scale over a typical scale size of the order of the collisionless electron skin depth λ_e.

III. LOCAL DISPERSION RELATIONS

In the following, we obtain the local linear dispersion relations for both the long and short wavelength electromagnetic modes. Accordingly, we neglect the nonlinear terms in Eqs. (5)–(8) and assume that the wavelengths of the perturbations are much smaller than the scalelengths of the equilibrium velocity and density gradients. The governing Eqs. (5)–(8) are then Fourier transformed by supposing that the perturbed quantities ϕ and A_z are proportional to $\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$, where $\mathbf{k}$ and ω are the wavevector and the frequency, respectively. Thus, Eqs. (5) and (6) give

$$(\Omega-\omega_{c*}+i\Gamma_c)\phi=\left(\frac{4\pi j_{e0}}{B_0}\mathbf{k}_J\cdot\mathbf{k}+ck_zk_\perp^2\right)k_0^{-2}A_z \tag{9}$$

and

$$\left(\omega-\frac{k_zv_{e0}b_e}{1+b_e}-\omega_{m*}+i\Gamma_m\right)A_z$$
$$=(1+b_e)^{-1}c(k_z+\mathbf{k}\cdot\mathbf{S}_{v0})\phi, \tag{10}$$

where $\Omega=\omega-k_zv_{e0}$, $\omega_{c*}=a\omega_{ce}\mathbf{k}_n\cdot\mathbf{k}/k_0^2$, $a=\omega_{pe}^2/\omega_{ce}^2$, $k_0^2=k^2+ak_\perp^2$, $\mathbf{k}_n=\hat{\mathbf{z}}\times\nabla\ln n_0$, $\mathbf{k}_J=\hat{\mathbf{z}}\times\nabla\ln j_{e0}$, $\Gamma_c=(k^2+0.51ak_\perp^2)D_ck_\perp^2/k_0^2$, $\omega_{m*}=\omega_{e*}/(1+b_e)$, $\omega_{e*}=\mathbf{k}\cdot\mathbf{v}_{D0}$, $b_e=k_\perp^2\lambda_e^2$, $\Gamma_m=b_e\nu_e/(1+b_e)$, and $k^2\lambda_{De}^2\ll 1$.

Combining Eqs. (9) and (10) we obtain the linear dispersion relation for long wavelength electromagnetic waves

$$(\Omega-\omega_{c*}+i\Gamma_c)[\omega+b_e\Omega-\omega_{e*}+ib_e\nu_e]$$
$$=\frac{c}{k_0^2}\left(\frac{4\pi j_{e0}}{B_0}\mathbf{k}_J\cdot\mathbf{k}+ck_zk_\perp^2\right)(k_z+\mathbf{k}\cdot\mathbf{S}_{v0}). \tag{11}$$

Equation (11) exhibits a linear coupling between the electron drift-convective cells (the first term in the parenthesis on the left-hand side) and the magnetostatic drift modes (the term in the square bracket on the left-hand side) due to finite k_z and the equilibrium sheared flow. In the absence of the latter, the two modes degenerate, and we obtain $\omega=\omega_{c*}-i\Gamma_c$ and $\omega=(\omega_{e*}-ib_e\nu_e)/(1+b_e)$, which are damped normal electromagnetic modes of the magnetized electron plasma containing an equilibrium density gradient.

On the other hand, for short wavelength electromagnetic modes, Eqs. (7) and (8) yield

$$(\Omega-\omega_{i*}+i\Gamma_s)\phi=\frac{\lambda_{Di}^2}{1+b_a}\left(\frac{4\pi j_{e0}}{B_0}\mathbf{k}_J\cdot\mathbf{k}+k_zck_\perp^2\right)A_z \tag{12}$$

and

$$\left(\omega-\frac{k_zv_{e0}b_e}{1+b_e}-\omega_{m*}+i\Gamma_m\right)A_z$$
$$=(1+b_e)^{-1}c[(1+\sigma)k_z+\mathbf{k}\cdot\mathbf{S}_{v0}]\phi, \tag{13}$$

where $\omega_{i*}=\rho_a^2\omega_{ce}\mathbf{k}\cdot\mathbf{k}_n/(1+b_a)$, $b_a=k_\perp^2\rho_a^2$, and $\Gamma_s=D_c(1+0.51b_a)k_\perp^2/(1+b_a)$.

Combining Eqs. (12) and (13), we obtain the linear dispersion equation for short wavelength electromagnetic waves

$$(\Omega-\omega_{i*}+i\Gamma_s)(\omega+b_e\Omega-\omega_{e*}+ib_e\nu_e)$$
$$=\frac{c\lambda_{Di}^2}{1+b_a}\left(\frac{4\pi j_{e0}}{B_0}\mathbf{k}_J\cdot\mathbf{k}+ck_zk_\perp^2\right)[(1+\sigma)k_z+\mathbf{k}\cdot\mathbf{S}_{v0}]. \tag{14}$$

Equation (14) shows that finite k_z and the equilibrium sheared plasma flow can cause a linear coupling between the ion-drift (the first term on the left-hand side in the parenthesis) and magnetostatic drift modes. In the absence of the equilibrium flow, the frequency of a flute-like damped ion-drift wave is $\omega=\omega_{i*}-i\Gamma_s$.

For flute perturbations (viz. $k_z=0$), Eqs. (11) and (14) become, respectively,

$$(\omega-\omega_{c*}+i\Gamma_c)(\omega-\omega_{m*}+i\Gamma_m)$$
$$=\frac{k_y^2\omega_{pe}^2}{k_\perp^2(1+a)(1+b_e)\omega_{ce}^2n_0e}\frac{\partial v_{e0}}{\partial x}\frac{\partial j_{e0}}{\partial x} \tag{15}$$

and

$$(\omega-\omega_{i*}+i\Gamma_s)(\omega-\omega_{m*}+i\Gamma_m)$$
$$=\frac{k_y^2\lambda_{Di}^2\omega_{pe}^2}{(1+b_a)(1+b_e)\omega_{ce}^2n_0e}\frac{\partial v_{e0}}{\partial x}\frac{\partial j_{e0}}{\partial x}. \tag{16}$$

In the absence of the equilibrium density gradient (viz. $\omega_{c*}=0$, ω_{i*} and $\omega_{m*}=0$) and dissipation, Eqs. (15) and (16) admit purely growing instabilities, provided that the velocity and current gradients oppose each other. The growth rates for long and short wavelength modes are, respectively,

$$\gamma_c\approx\frac{\omega_{pe}}{\omega_{ce}}\left[\frac{1}{(1+a)(1+b_e)n_0e}\right]^{1/2}\left|\frac{\partial v_{e0}}{\partial x}\frac{\partial j_{e0}}{\partial x}\right|^{1/2} \tag{17}$$

and

$$\gamma_s\approx k_y\lambda_{Di}\frac{\omega_{pe}}{\omega_{ce}}\left[\frac{1}{(1+b_a)(1+b_e)n_0e}\right]^{1/2}\left|\frac{\partial v_{e0}}{\partial x}\frac{\partial j_{e0}}{\partial x}\right|^{1/2}. \tag{18}$$

It is evident from Eqs. (17) and (18) that the growth rates are directly proportional to the square root of the product of the equilibrium velocity and current gradients. Thus, the equilibrium sheared plasma flow is responsible for the instability.

In order to understand the effect of finite k_z and the density inhomogeneity, we rewrite Eqs. (11) and (14) by assuming that $\omega\gg k_zv_{e0},\Gamma_c,\Gamma_m$. Thus, in a nonuniform collisionless plasma, we have, respectively,

$$\omega^2-(\omega_{c*}+\omega_{m*})\omega+\omega_{c*}\omega_{m*}-S_l=0, \tag{19}$$

where $S_l=[c/k_0^2(1+b_e)][(4\pi k_y(\partial j_{e0}/\partial x)/B_0)+ck_zk_\perp^2][k_z+k_y(\partial v_{e0}/\partial x)/\omega_{ce}]$, and

$$\omega^2-(\omega_{i*}+\omega_{m*})\omega+\omega_{i*}\omega_{m*}-S_s=0, \tag{20}$$

where $S_s=[c\lambda_{Di}^2/(1+b_a)(1+b_e)][(4\pi k_y(\partial j_{e0}/\partial x)/B_0)+ck_zk_\perp^2][(1+\sigma)k_z+k_y(\partial v_{e0}/\partial x)/\omega_{ce}]$.

It turns out that both the long and short scale electromagnetic waves are stable when S_l and S_s are positive. However, for $S_l,S_s<0$ and $|S_l|>(\omega_{c*}-\omega_{m*})^2/4$, and $|S_s|>(\omega_{i*}-\omega_{m*})^2/4$, Eqs. (19) and (20) predict oscillatory current convective instabilities. Physically, the latter arise because in the presence of velocity gradient the parallel component of the electron velocity perturbation and the wave potential are out of phase, as long as the parallel wavelengths are extremely large.

Next, we note that there also exist resistive instabilities when $\omega_{m*},k_zv_{e0}\ll\omega\ll b_e\nu_e$. In this case, Eqs. (11) and (14) would lead to $\omega=\omega_{c*}-i\Gamma_c-iS_l/\nu_eb_e$ and $\omega=\omega_{i*}-i\Gamma_s-iS_s/\nu_eb_e$, respectively. Thus, for $S_l,S_s<0$ and $|S_l|>\nu_eb_e\Gamma_c$ and $|S_s|>\nu_eb_e\Gamma_s$, we have the excitation of long scale electron drift-convective cells and short scale ion drift waves in nonuniform collisional plasmas. Furthermore, the magnetic drift modes also become unstable if $|\omega-\omega_{c*}|\ll\Gamma_c$, $|\omega-\omega_{i*}|\ll\Gamma_s$, and $S_l,S_s<0$. The growth rates above threshold are $|S_l|/\Gamma_c$ and $|S_s|/\Gamma_s$, respectively.

IV. VORTEX SOLUTIONS

In the preceding section, we have seen that velocity gradient can cause the instability of electromagnetic waves which attain finite amplitude. The nonlinear interaction between finite amplitude modes can be responsible for the formation of either ordered structures or a chaotic state depending upon various plasma parameters. Although the general stationary and non-stationary solutions of Eqs. (5)–(8) cannot be found analytically, we discuss here some approximate solutions. First, we present vortex solutions of a non-dissipative inhomogeneous magnetized system, by assuming that $c\omega_{ce}|\nabla_\perp^2A_z\partial_z|\ll\omega_{pe}^2|\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla|$ and $\partial_z^2\ll\nabla_\perp^2$. Specifically, we seek traveling localized solutions of our governing equations by letting[2,15,16] $\xi=y+\alpha z-ut$, where α and u are constants, and assume that ϕ and A_z are functions of x and ξ only. Conditions for the formation of different types of vortices as well as their structures are presented below. The introduction of the new reference frame ξ with constant α and u for an inhomogeneous medium is a well established fact for cases involving Rossby and gravity dipolar vortices in fluids[2,15] as well as drift-acoustic[16] and drift-Alfvén[2,7,8] vortices in nonuniform magnetized plasmas. Therefore, it does not make sense to allow α and u to be a function of x, as ξ is an independent variable (characteristic line) which depends only on y and z. The physical field variables are certainly functions of x and ξ, as indicated above.

A. Long scale vortices

In the stationary frame, Eqs. (5) and (6) may be written as

$$\mathscr{L}_\phi(\nabla_\perp^2\phi+u_c\phi)+u_J\partial_\xi A_z-\frac{c\alpha}{u_\alpha(1+a)}\mathscr{L}_A\nabla_\perp^2A_z=0 \tag{21}$$

and

$$\mathscr{L}_\phi(A_z-\lambda_e^2\nabla_\perp^2A_z)+\frac{V_*+\alpha v_{e0}}{u_\alpha}\partial_\xi A_z-\frac{c\alpha_0}{u_\alpha}\partial_\xi\phi=0, \tag{22}$$

where $\mathscr{L}_\phi=\partial_\xi-(c/u_\alpha B_0)(\partial_x\phi\partial_\xi-\partial_\xi\phi\partial_x)$, $\mathscr{L}_A=\partial_\xi-(1/\alpha B_0)(\partial_xA_z\partial_\xi-\partial_\xi A_z\partial_x)$, $\nabla_\perp^2=\partial^2/\partial x^2+\partial^2/\partial\xi^2$, $u_c=a\omega_{ce}/L_n(1+a)u_\alpha$, $u_\alpha=u-\alpha v_{e0}$, $u_J=4\pi j_{e0}/L_JB_0(1$

$+a)u_\alpha$, $L_n=n_0/(\partial n_0/\partial x)$, $L_J=j_{e0}/(\partial j_{e0}/\partial x)$, $V_*=cT_e/eB_0L_n$, and $\alpha_0=\alpha+(\partial v_{e0}/\partial x)/\omega_{ce}\equiv\alpha+v'_{e0}/\omega_{ce}$.

It is somewhat difficult to find the general solutions of Eqs. (21) and (22). Thus, we consider two limiting cases. First, we consider $\lambda_e^2\nabla_\perp^2 A_z\ll A_z$ so that the scale sizes of the vortices are much smaller than the collisionless electron skin depth. Here, Eq. (22) gives

$$\mathscr{L}_{\phi u_*}\left[A_z-\frac{c\alpha_0}{u_*}\phi\right]=0, \tag{23}$$

where $\mathscr{L}_{\phi u_*}=\partial_\xi-(c/u_*B_0)(\partial_x\phi\partial_\xi-\partial_\xi\phi\partial_x)$, and $u_*=u+V_*$.

Second, we take $u\gg V_*$ *and* αv_{e0} so that Eq. (22) simplifies as

$$\mathscr{L}_{\phi u}\left[(1-\lambda_e^2\nabla_\perp^2)A_z-\frac{c\alpha_0}{u}\phi\right]=0, \tag{24}$$

where $\mathscr{L}_{\phi u}\equiv\partial_\xi-(c/uB_0)(\partial_x\phi\partial_\xi-\partial_\xi\phi\partial_x)$.

A typical localized solution of Eq. (23) is

$$A_z=\frac{c\alpha_0}{u_*}\phi, \tag{25}$$

whereas Eq. (24) is satisfied by the ansatz

$$\nabla_\perp^2 A_z=\frac{1}{\lambda_e^2}\left(A_z-\frac{c\alpha_0}{u}\phi\right). \tag{26}$$

Inserting Eq. (25) into Eq. (21), we obtain

$$\left[1-\frac{\alpha\alpha_0c^2}{u_\alpha u_*(1+a)}\right]\partial_\xi\nabla_\perp^2\phi+\frac{a\omega_{ce}}{u_\alpha(1+a)}\left(\frac{1}{L_n}-\frac{\alpha_0v_{e0}}{u_*L_J}\right)\partial_\xi\phi$$
$$-\frac{c}{u_\alpha B_0}\left[1-\frac{\alpha_0^2c^2}{u_*^2(1+a)}\right]J(\phi,\nabla_\perp^2\phi)=0, \tag{27}$$

where $J(\phi,\nabla_\perp^2\phi)=\partial_x\phi\partial_\xi\nabla_\perp^2\phi-\partial_\xi\phi\partial_x\nabla_\perp^2\phi$.

Equation (27) is satisfied by the ansatz

$$\nabla_\perp^2\phi=F_1\phi+F_2x, \tag{28}$$

where the constants F_1 and F_2 are related by $\delta F_1+(c/u_\alpha B_0)[1-\alpha_0^2c^2/u_*^2(1+a)]F_2+[a\omega_{ce}/u_\alpha(1+a)]\times(L_n^{-1}-\alpha_0v_{e0}/u_*L_J)=0$, and where $\delta=1-\alpha\alpha_0c^2/u_\alpha u_*(1+a)$.

It can be readily shown[15–18] that Eq. (28) admits a dipolar vortex solution, the form of which is

$$\phi_{\text{out}}=\phi_0K_1(k_1r)\cos\theta, \tag{29}$$

in the outer region defined by $r=(x^2+\xi^2)^{1/2}>R$, where R is the vortex radius, ϕ_0 is a constant, K_1 is the modified Bessel function, $k_1^2=[a\omega_{ce}/u_\alpha(1+a)][(\alpha_0v_{e0}/u_*eu_J\delta)\ -1/L_n]>0$, and $\cos\theta=x/r$. In the inner region ($r<R$), the solution reads

$$\phi_{\text{in}}=\left(\phi_iJ_1(k_2r)+\frac{F_{2i}}{k_2^2}r\right)\cos\theta, \tag{30}$$

where ϕ_i is a constant, J_1 is the Bessel function of the first order, and $[1-\alpha_0^2c^2/u^{*2}(1+a)]F_{2i}=\delta(k_1^2+k_2^2)u_\alpha B_0/c$.

The constants ϕ_0, ϕ_i and F_{2i} are determined from the equations that come from the matching conditions of the inner and outer ϕ, $\nabla_\perp^2\phi$ and $\nabla_\perp\phi$ at $r=R$. One finds that $\phi_0=RF_{2i}/(k_1^2+k_2^2)K_1(k_1R)$, $\phi_i=-k_1^2RF_{2i}/k_2^2(k_1^2+k_2^2)J_1(k_2R)$, and

$$K_2(k_1R)/k_1K_1(k_1R)=-J_2(k_2R)/k_2J_1(k_2R), \tag{31}$$

where J_2 and K_2 are the Bessel function and the modified Bessel function of the second order, respectively. For a given value of k_1, Eq. (31) determines k_2.

On the other hand, when $L_n\approx L_Ju_*/\alpha_0v_{e0}$, then Eq. (27) takes the form of a stationary Navier-Stokes equation, namely,

$$\partial_\xi\nabla_\perp^2\phi-\frac{\mu_lc}{u_\alpha B_0}J(\phi,\nabla_\perp^2\phi)=0, \tag{32}$$

where $\mu_l=[1-\alpha_0^2c^2/u_*^2(1+a)]/\delta>0$. Equation (32) is satisfied by

$$\nabla_\perp^2\phi=\frac{4\phi_lK_l^2}{a_l^2}\exp\left[-\frac{2}{\phi_l}\left(\phi-\frac{u_\alpha B_0}{\mu_lc}x\right)\right], \tag{33}$$

where ϕ_l, K_l and a_l are arbitrary constants. The solution of Eq. (33) is given by[3]

$$\phi=\frac{u_\alpha B_0}{\mu_lc}x+\phi_l\ \ln\left[2\cosh(K_lx)+2\left(1-\frac{1}{a_l^2}\right)\cos(K_l\xi)\right]. \tag{34}$$

For $a_l^2>1$ the vortex profile given by Eq. (34) resembles the Kelvin-Stuart ''cat's eyes'' that are chains of vortices.

Next, we substituting for $\nabla_\perp^2A_z$ from Eq. (26) into Eq. (21), we have

$$\mathscr{L}_\phi(\nabla_\perp^2\phi+\beta_1\phi-\beta_2A_z)=0, \tag{35}$$

provided that $\alpha=\alpha_0+(4\pi j_{e0}\lambda_e^2/B_0cL_J)$. Here, the vortex size is of the order of λ_e. A possible solution of Eq. (35) is

$$\nabla_\perp^2\phi+\beta_1\phi-\beta_2A_z=F_3\left(\phi-\frac{uB_0}{\mu_lc}x\right), \tag{36}$$

where $\beta_1=[c^2\alpha\alpha_0/u^2(1+a)\lambda_e^2]+a\omega_{ce}/u(1+a)L_n$, $\beta_2=\alpha_0c/u(1+a)\lambda_e^2$, and F_3 is an arbitrary constant of integration.

Combining Eqs. (26) and (36), we obtain a fourth order differential equation

$$\nabla_\perp^4\phi+C_1\nabla_\perp^2\phi+C_2\phi-F_3\frac{uB_0}{\lambda_e^2\mu_lc}x=0, \tag{37}$$

where $C_1=\beta_1+F_3-1/\lambda_e^2$ and $C_2=[(F_3-\beta_1)+c\beta_2\alpha_0/u]/\lambda_e^2$. Equation (37) admits spatially-bounded dipolar vortex solutions. In the outer region ($r>R$), we set $F_3=0$ and write the solution of Eq. (37) as

$$\phi=[Q_1K_1(s_1r)+Q_2K_1(s_2r)]\cos\theta, \tag{38}$$

where Q_1 and Q_2 are constants and $s_{1,2}^2=[-\alpha_1\pm(\alpha_1^2-4\alpha_2)^{1/2}]/2$ for $\alpha_1<0$ and $\alpha_1^2>4\alpha_2>0$. Here, $\alpha_1=\beta_1-\lambda_e^{-2}$ and $\alpha_2=-(\beta_1/\lambda_e^2)+c\beta_2\alpha_0/u\lambda_e^2$. Thus, $u^2\gg\alpha\alpha_0$ is required for the localization of the outer solution. In the inner region ($r<R$), the solution reads

$$\phi=\left[Q_3J_1(s_3r)+Q_4I_1(s_4r)-\frac{F_3}{\lambda_e^2}\frac{uB_0}{\mu_l cC_2}r\right]\cos\theta, \quad (39)$$

where Q_3 and Q_4 are constants.[3] We have defined $s_{3,4}=[(C_1^2-4C_2)^{1/2}\pm C_1]/2$ for $C_2<0$. Evidently, the outer and inner region profiles of inertial electromagnetic vortices are different from those of non-inertial vortices. It is worthwhile to mention here that the sheared equilibrium electron flow is responsible for complete localization of the dipolar vortex in the outer region. Without the sheared plasma flow, we have $\alpha_0=\alpha$ and $C_2=0$ in the outer region and the solution of Eq. (37) has a long tail.[19]

The constants Q_1, Q_2, Q_3, Q_4 and F_3 can be determined by matching the inner and outer solutions of ϕ and A_z and the higher derivatives $\nabla\phi$, $\nabla_\perp^2\phi$, $\nabla_\perp A_z$ and $\nabla_\perp^2 A_z$ at the vortex interface $r=R$. This exercise has been carried out by Mikhailovskii *et al.*[7] and Liu and Horton,[7] and explicit expressions for the various constants had been found.

B. Short scale vortices

Here, we discuss the vortex solutions of Eqs. (7) and (8) by ignoring dissipation. Thus, in the stationary frame, we rewrite Eqs. (7) and (8) as

$$\mathscr{L}_\phi\left[1-\rho_a^2\nabla_\perp^2-\frac{c_a}{u_\alpha L_n}\right]\phi-\frac{4\pi j_{e0}\lambda_{Di}^2}{B_0L_Ju_\alpha}\partial_\xi A_z+\frac{c\alpha\lambda_{Di}^2}{u_\alpha}\mathscr{L}_A\nabla_\perp^2A_z=0 \quad (40)$$

and

$$\mathscr{L}_\phi(1-\lambda_e^2\nabla_\perp^2)A_z+\frac{(V_*+\alpha v_{e0})}{u_\alpha}\partial_\xi A_z-\frac{c\alpha_0}{u_\alpha}\partial_\xi\phi-\frac{c\alpha\sigma}{u_\alpha}\mathscr{L}_A\phi=0, \quad (41)$$

where $c_a=\rho_a^2\omega_{ce}$.

When the perpendicular scale size of the nonlinear structure is smaller than λ_e, Eq. (41) is then approximated by

$$\mathscr{L}_{\phi u_*}\left[A_z-\frac{c(\alpha_0+\sigma\alpha)}{u_*}\phi\right]=0, \quad (42)$$

A possible localized solution of Eq. (42) is

$$A_z=\frac{c(\alpha_0+\sigma\alpha)}{u_*}\phi\equiv\frac{c\alpha_0^*}{u_*}\phi. \quad (43)$$

If we insert Eq. (43) into Eq. (40), we obtain

$$\left(\rho_a^2-\frac{\alpha\alpha_0^*c^2\lambda_{Di}^2}{u_\alpha u_*}\right)\partial_\xi\nabla_\perp^2\phi-\left(1-\frac{c_a}{u_\alpha L_n}-\frac{4\pi c\alpha_0^*\lambda_{Di}^2j_{e0}}{B_0u_\alpha u_*L_J}\right)\times\partial_\xi\phi-\frac{c\rho_a^2}{u_\alpha B_0}\left(1-\frac{\alpha_0^{*2}c^2\lambda_{Di}^2}{u_*^2\rho_a^2}\right)J(\phi,\nabla_\perp^2\phi)=0. \quad (44)$$

Equation (44) is satisfied by the ansatz

$$\nabla_\perp^2\phi=F_1^*\phi+F_2^*x. \quad (45)$$

Here the constants F_1^* and F_2^* are related by $\delta^*F_1^*-(c\rho_a^2/u_\alpha B_0)(1-\alpha_0^{*2}c^2\lambda_{Di}^2/u_*^2\rho_a^2)F_2^*-[1-(c_a/u_\alpha L_n)-4\pi c\alpha_0^*\lambda_{Di}^2j_{e0}/B_0u_\alpha u_*L_J]=0$, where $\delta^*=\rho_a^2-\alpha\alpha_0^*\times c^2\lambda_{Di}^2/u_\alpha u_*$. Clearly, for $[1-(c_a/u_\alpha L_n)-4\pi c\alpha_0^*\times\lambda_{Di}^2j_{e0}/B_0u_\alpha u_*L_J]/\delta^*\equiv P/\delta^*>0$, the dipolar vortex solution of Eq. (45) is also similar to Eqs. (29) and (30).

On the other hand, when $P=0$ (where the double vortex solutions are forbidden), Eq. (45) takes the form

$$\partial_\xi\nabla_\perp^2\phi-\frac{\mu^*c}{u_\alpha B_0}J(\phi,\nabla_\perp^2\phi)=0, \quad (46)$$

where $\mu^*=(\rho_a^2-\alpha_0^{*2}c^2\lambda_{Di}^2/u_*^2)/(\rho_a^2-\alpha\alpha_0^*c^2\lambda_{Di}^2/u_\alpha u_*)>0$. Equation (46) is again satisfied by

$$\nabla_\perp^2\phi=\frac{4\phi_s^*K^{*2}}{a^{*2}}\exp\left[-\frac{2}{\phi_s^*}\left(\phi-\frac{uB_0}{\mu^*c}x\right)\right], \quad (47)$$

where ϕ_s^*, K^* and a^* are arbitrary constants. The solution of Eq. (47) is again similar to Eq. (34), but the condition under which the short scale vortex street arises is completely different.

Finally, we present the vortex solutions of Eqs. (40) and (41) by assuming that $u\gg V_*$, αv_{e0} and $\sigma\ll1$. Here, Eq. (41) is also satisfied by Eq. (26), so that Eq. (40) can be cast in the form

$$\mathscr{L}_\phi(\nabla_\perp^2\phi+\beta_1^*\phi-\beta_2^*A_z)=0, \quad (48)$$

provided that $v_{e0}\rho_ac_a/c^2L_J=(\alpha\lambda_{Di}^2/\lambda_e^2)-\alpha_0$. A possible solution of Eq. (48) is

$$\nabla_\perp^2\phi+\beta_1^*\phi-\beta_2^*A_z=F_3^*\left(\phi-\frac{uB_0}{\mu^*c}x\right), \quad (49)$$

where $\beta_1^*=(c^2\alpha\alpha_0\lambda_{Di}^2/u^2\rho_a^2\lambda_e^2)+(c_a/u\rho_a^2L_n)-1/\rho_a^2$, $\beta_2^*=\alpha_0c\lambda_{Di}^2/u\rho_a^2\lambda_e^2$, $\mu^*=(1-c^2\alpha_0^{*2}\lambda_{Di}^2/\rho_a^2u_*^2)/(1-c^2\alpha\alpha_0^*\times\lambda_{Di}^2/u_*u_\alpha)$, and F_3^* is an arbitrary constant of integration.

Combining Eqs. (26) and (49), we obtain a fourth order differential equation

$$\nabla_\perp^4 \phi + C_1^* \nabla_\perp^2 \phi + C_2^* \phi - \frac{F_3^*}{\lambda_e^2} \frac{u B_0}{\mu^* c} x = 0, \tag{50}$$

where $C_1^* = \beta_1^* - F_3^* - 1/\lambda_e^2$ and $C_2^* = [(F_3^* - \beta_1^*) + c\beta_2^* \alpha_0/u]/\lambda_e^2$. Equation (50) also admits spatially-bounded dipolar vortex solutions,[7,8] which are similar to the long wavelength case, as discussed in Sec. IV B. An examination of the vortex analyses reveals that the constants u and α are related in terms of the density and velocity gradients as well as other plasma parameters. Thus, for given values of constant density and velocity gradient scalelengths we find that u is completely determined by a specific choice of α and prescribed values for the unperturbed plasma number density, the plasma temperature and the external magnetic field strength. The scale sizes of the vortices, as found here, are typically of the order of λ_e and ρ_a, which are smaller than the scale length of the equilibrium density and velocity gradients. Such scenarios are common in space[10] and laboratory plasmas.[11] Finally, we note that the present methods of solutions do not allow to construct vortices whose scale sizes are of the order of the inhomogeneity scale lengths. However, we anticipate that in such a situation, the governing Eqs. (5)–(8) may admit global vortex patterns[8] provided that the profiles of the density and velocity inhomogeneities are known. A detailed investigation of this problem is beyond the scope of the present paper.

V. CHAOTIC BEHAVIOR OF ELECTROMAGNETIC TURBULENCE

In the following, we follow Lorenz[20] and Stenflo[21,22] and derive a set of equations which are appropriate for studying the temporal behavior of chaotic motion involving two-dimensional low-frequency nonlinearly interacting electromagnetic waves in a dissipative magnetoplasma without the density gradient. Accordingly, we introduce the Ansatz

$$\phi = \phi_1(t)\sin(K_x x)\sin(K_y y) \tag{51}$$

and

$$A_z = A_1(t)\sin(K_x x)\cos(K_y y) - A_2(t)\sin(2K_x x), \tag{52}$$

where K_x and K_y are constant parameters, and ϕ_1, A_1 and A_2 are amplitudes which are only functions of time.

As an illustration, we consider in detail the chaotic behavior of nonlinearly interacting flute-like short wavelength electromagnetic waves. Thus, by substituting Eqs. (51) and (52) into Eqs. (7) and (8), we readily obtain

$$(1+K^2\rho_a^2)\dot{\phi}_1 = -\mu_1 K^4 \phi_1 + \delta_1 K_y A_1 - \delta_2 (K^2 - 4K_x^2) \times K_x K_y A_1 A_2 - \delta_3 K^2 \phi_1, \tag{53}$$

$$(1+K^2\lambda_e^2)\dot{A}_1 = -\eta K^2 A_1 - \sigma_1 K_y \phi_1 + \frac{c}{B_0}[1+K^2\lambda_e^2 - 6K_x^2\lambda_e^2] K_x K_y A_2 \phi_1 + \sigma_2 K_x K_y \phi_1 A_2, \tag{54}$$

and

$$(1+4K_x^2\lambda_e^2)\dot{A}_2 = -\frac{c}{2B_0}(1+4K_x^2\lambda_e^2) K_x K_y \phi_1 A_1 - 4\eta K_x^2 A_2 - \sigma_2 K_x K_y \phi_1 A_1/2, \tag{55}$$

where $\mu_1 = \mu_e \rho_a^2$, $\delta_1 = 4\pi\lambda_{Di}^2(\partial j_{e0}/\partial x)/B_0$, $\delta_2 = c\lambda_{Di}^2/B_0$, $\delta_3 = \nu_e \rho_e^2$, $\eta = \nu_e \lambda_e^2$, is the plasma resistivity, $\sigma_1 = c(\partial v_{e0}/\partial x)/\omega_{ce}$ and $\sigma_2 = c\sigma/B_0$. The time derivative is defined by a dot on ϕ_1, A_1 and A_2. We note that the terms proportional to $\sin(3K_x x)$ have been dropped in the derivation of Eqs. (53)–(55). This approximation, which is often employed by many authors for deriving the relevant Lorenz-like equations in many branches of physics, can easily be generalized[21] to describe more realistic space dependence solutions. Furthermore, we note that in deriving Eqs. (53) and (54) we have assumed that $\partial_t >> V_* \partial_y$, $\rho_a^2 \omega_{ce} K_n \partial_y$, where $V_* = -(cT_e/eB_0)K_n$ and $K_n = n_0^{-1}\partial n_0/\partial x$, which justify the neglection of the density gradient from Eqs. (7) and (8).

Equations (53)–(55) can be appropriately normalized so that they can be put in a form which is similar to that of Lorenz and Stenflo. We have

$$\begin{pmatrix} d_\tau X \\ d_\tau Y \\ d_\tau Z \end{pmatrix} = \begin{pmatrix} -\sigma & \sigma + sZ & 0 \\ r - Z & -1 & 0 \\ Y & 0 & -b \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \tag{56}$$

which describes the nonlinear coupling between various amplitudes. Here, $\sigma = (\mu_1 K^2 + \delta_3)\ b_e^*/\eta(1+K^2\rho_a^2)$, $r = -\delta_1\sigma_1 K_y^2/\eta K^4(\mu_1 K^2 + \delta_3)$, $b = 4K_x^2 b_e^*/(1+4K_x^2\lambda_e^2)K^2$ and the new parameter $s = -\delta_2 a_2 a_3 (K^2 - 4K_x^2) b_e^* K_x K_y / a_1 \eta (1+K^2\rho_a^2)$, with $K^2 = K_x^2 + K_y^2$ and $\tau = t/t_0$; where $t_0 = \eta K^2/b_e^*$ and $b_e^* = 1 + K^2\lambda_e^2$.

A comment is in order. If we set $s=0$, which happens for $K_y^2 = 4K_x^2$, Eq. (56) then reduce to the Stenflo type equations. However, the normalizations used here are

$$\phi_1 = a_1 X = \frac{\sqrt{2}\,\eta K^2 B_0}{K_x K_y \sqrt{b_e^*(b_e^* - 6K_x^2\lambda_e^2 + B_0\sigma_2/c)[1+\sigma_2 B_0/c(1+4K_x^2\lambda_e^2)]}} X,$$

$$A_1 = a_2 Y = \frac{\sqrt{2}\,\eta K^4 B_0(\mu_1 K^2 + \delta_3)}{c\,\delta_1 K_x K_y^2 \sqrt{b_e^*(b_e^* - 6K_x^2\lambda_e^2 + B_0\sigma_2/c)[1+\sigma_2 B_0/c(1+4K_x^2\lambda_e^2)]}} Y,$$

and

$$A_2 = a_3 Z = \frac{-\eta K^4 B_0(\mu_1 K^2 + \delta_3)}{[c\delta_1 K_x K_y^2 (b_e^* - 6K_x^2 \lambda_e^2 + B_0 \sigma_2 / c)]} Z.$$

Equations (56) are the generalized Lorenz equations, whose properties can be studied both analytically as well as numerically by means of standard techniques.[23] We observe that the equilibrium points of Eq. (56) are

$$X_0 = \pm [b((r-2+sr^2/\sigma) + \sqrt{(r-2+sr^2/\sigma)^2 + 4(r-1)})/2]^{1/2}, \quad (57)$$

$$Y_0 = \frac{rbX_0}{(b+X_0^2)}, \quad (58)$$

and

$$Z_0 = \frac{X_0 Y_0}{b}. \quad (59)$$

In the absence of the s-term, we note that for $|r|>1$, the equilibrium fixed points $[X_0 = Y_0 = \pm\sqrt{b}(|r|-1)^{1/2}$, and $Z_0 = |r|-1]$ are unstable resulting in convective cell motions. Thus, the linear instability should saturate by attracting to one of these new fixed states. Furthermore, it is worth mentioning that a detailed behavior of chaotic motion for $K_y \neq \sqrt{3}K_x$ can be studied by numerically solving Eqs. (53)–(55). However, this investigation is beyond the scope of this paper.

The stability of the stationary states can be studied by a simple linear analysis. Letting $X = X_s + X_1$, $Y = Y_s + Y_1$ and $Z = Z_s + Z_1$, the linearized system is

$$\begin{pmatrix} d_\tau X_1 \\ d_\tau Y_1 \\ d_\tau Z_1 \end{pmatrix} = \begin{pmatrix} -\sigma & \sigma & 0 \\ r - Z_s & -1 & -X_s \\ Y_s & X_s & -b \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}, \quad (60)$$

where $X_1 \ll X_s$, $Y_1 \ll Y_s$ and $Z_1 \ll Z_s$ and (X_s, Y_s, Z_s) represents a stationary state. The corresponding characteristic equation is thus

$$(\lambda + b)[(\lambda^2 + (1+\sigma)\lambda + (1-r)\sigma)] = 0, \quad (61)$$

which governs the linear stability of the stationary state. For example, if we take $r<1$, the origin is a hyperbolic sink and is thus stable. On the other hand, for $r=1$, the eigenvalues are $\lambda = -b$ and $\lambda = -(1+\sigma)$, which are always negative. Finally, for $r>1$, the nontrivial stationary points are $X_s^\pm = Y_s^\pm = \pm\sqrt{b(r-1)}$ and $Z_s = r-1$. The eigenvalues of Eq. (61) are $\lambda = -(\sigma+b+1)$ and $\pm i\sqrt{2\sigma(\sigma+1)/(\sigma-b-1)}$, so that the stationary states $(X_s^\pm, Y_s^\pm, Z_s)$ are sinks for $r \in (1, r_H)$, where $r_H \equiv \sigma(\sigma+b+3)/(\sigma-b-1)$. A Hopf bifurcation occurs at r_H. For $\sigma > 1+b$, imaginary roots are possible and that for $r>r_H$ the nontrivial fixed points are saddles with two-dimensional unstable manifolds. Thus, for $r>r_H$ all the three fixed points are unstable but the attractor set still exists.[20] Further bifurcations at larger r values eventually lead to chaotic behavior.[20] The inclusion of the equilibrium density inhomogeneity may lead to even qualitative changes in the chaotic dynamics of electromagnetic turbulence, as discussed here.

VI. DISCUSSION AND CONCLUSIONS

In this paper, we have investigated the linear as well as nonlinear properties of low-frequency electromagnetic waves in a nonuniform dissipative magnetized plasmas. For this purpose, we have employed the electron MHD equations supplemented by Ampere's law as well as the electron (ion) response given by Poisson's equation (the Boltzmann distribution) and have derived a set of nonlinear mode coupling equations. In the linear regime, our analyses show that electromagnetic disturbances of various scale sizes can be driven on account of the free energy stored in the sheared equilibrium plasma flows. The physical mechanism of the present instabilities is similar to the current convective electrostatic instability.[4–6] Furthermore, we have shown that the nonlinear mode coupling of finite amplitude electromagnetic waves in nonuniform magnetoplasmas with sheared plasma flows can lead to self-organization in the form of various types of vortex patterns. Explicit conditions for the existence of different types of vortices are obtained. For example, we have found that the electron magnetohydrodynamic (EMHD) equations in an electron plasma with stationary ions admit vortices whose characteristic transverse (to $\hat{\mathbf{z}}$) scale length $\lambda_\perp$ satisfies the inequality $\lambda_e \ll \lambda_\perp \ll \lambda_i$, where $\lambda_i = c/\omega_{pi}$ is the collisionless ion skin depth. Inclusion of the Boltzmann ion distribution allows shorter scale (in comparison with the ion gyroradius) dipolar vortices. Since the vortex solutions exist locally, our theory requires that the vortex sizes are much smaller than the scalelengths of the equilibrium density and velocity gradients. Furthermore, weakly interacting flute-like electromagnetic waves in a dissipative system without the density gradient are shown to obey the generalized Lorenz-Stenflo equations, which admit a chaotic state. The parameter regimes for the onset of chaos have been identified. We have thus pointed out the possibility of different classes of solutions including ordered structures as well as a chaos in a fully developed electromagnetic turbulence in nonuniform magnetoplasmas. Unfortunately, we are unsuccessful writing our complete set of Eqs. (5)–(8) in terms of the generalized Lorenz-Stenflo equations in the presence of the density gradient and the magnetic field-aligned variation terms. Therefore, the role of the latter on the chaotic motion could not be rigorously identified. It may well turn out that inclusion of the equilibrium density inhomogeneity may lead to even qualitative changes in the chaotic dynamics of electromagnetic turbulence, in contrast to what has been described here.

The present paper neither includes the sheared magnetic fields nor addresses the issue of the vortex stability. In the presence of equilibrium magnetic shear, the parallel wavevector is a function of position and one encounters an eigenvalue problem for the linear electromagnetic waves, which may also have a discrete spectrum. Furthermore, a critical evaluation of the literature[24] reveals that long-lived vortices can indeed exist around the mode rational surfaces when the magnetic shear is incorporated. On the other hand, a number of investigations[25–27] has been carried out in order to answer the question of the stability of dipolar vortices and the vortex chain, which are stationary solutions of the nonlinear partial differential equations. The latter are somewhat different from ours. Clearly, the procedure of those investigations can be utilized to examine the stability of long and short scale electromagnetic vortices. We anticipate that these nonlinear structures should remain stable, because the structures of our nonlinear equations are similar in form to those of earlier investigations.[25–27] A complete stability analysis of the vortices would lead us far beyond the scope of this paper.

In closing, we stress that the results of the present investigation, which is complementary to Ref. 3, shall provide a complete and better view of the linear and nonlinear features of long and short wavelength electromagnetic turbulence in magnetized plasmas with equilibrium density and velocity gradients, which are common in space and laboratory plasmas. Specifically, the measurements of broadband electric and magnetic fluctuations peaked in the current layer[10] do support the current as the source of the fluctuations.

ACKNOWLEDGMENTS

This research was partially supported by the Deutsche Forschungsgemeinschaft through the Sonderforschungsbereich 191 ‘‘ Physikalische Grundlagen der Niedertemperaturplasmen,’’ as well as by the Alexander von Humboldt (AvH) Foundation, the Deutscher Akademischer Austauschdienst (DAAD) and Fundação Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES).

[1]V. M. Chmyrev, O. A. Pokhotelov, V. A. Marchenko, V. I. Lazarev, A. V. Streltsov, and L. Stenflo, Phys. Scr. **38**, 841 (1988).
[2]V. I. Petviashvili and O. A. Pokhotelov, *Solitary Waves in Plasmas and in the Atmosphere* (Gordon and Breach, Reading, Birkshire, 1992).
[3]P. K. Shukla, G. T. Birk, J. Dreher, and L. Stenflo, Plasma Phys. Rep. **22**, 818 (1996).
[4]A. B. Mikhailovskii, *Theory of Plasma Instabilities* (New York, Consultants Bureau, 1974), Vol. 2, p. 63.
[5]A. B. Mikhailovskii, *Electromagnetic Instabilities in an Inhomogeneous Plasma* (Adam Hilger, Institute of Physics Publishing, Bristol, UK, 1992).
[6]P. K. Shukla, J. Srinivas, G. Murtaza, and H. Saleem, Phys. Plasmas **1**, 3505 (1994); P. K. Shukla, G. T. Birk, and R. Bingham, Geophys. Res. Lett. **22**, 671 (1995).
[7]A. B. Mikhailovskii, V. P. Lakhin, G. D. Aburdzhaniya, L. A. Mikahailovskaya, O. G. Onishenko, and A. I. Smolyakov, Plasma Phys. Controlled Fusion **29**, 1 (1984); J. Liu and W. Horton, J. Plasma Phys. **36**, 1 (1986).
[8]P. K. Shukla, in *A Variety of Plasmas; Proceedings of the Invited Talk of the 1989 International Conference on Plasma Physics, New Delhi*, edited by A. Sen and P. K. Kaw (Indian Academy of Sciences, Bangalore, 1991), pp. 297–314.
[9]A. Hasegawa and C. Uberoi, *The Alfvén Wave* (National Technical Information Service, Springfield, VA, 1982).
[10]B. T. Tsurutani, A. L. Brinca, E. J. Smith, R. T. Okida, R. R. Anderson, and T. E. Eastman, J. Geophys. Res. **94**, 1270 (1989); P. Louran, J. E. Wahlund, T. Chust, H. de Feraudy, A. Roux, B. Holback, P. O. Dover, A. I. Eriksson, and G. Holmgren, Geophys. Res. Lett. **21**, 1847 (1994).
[11]E. J. Hains, I. H. Tan, and S. C. Prager, Phys. Plasmas **2**, 1521 (1995); H. Saleem and P. K. Shukla, Phys. Fluids B **4**, 86 (1992); R. L. Stenzel, J. M. Urruita, and C. L. Rousculp, Phys. Rev. Lett. **74**, 702 (1995).
[12]H. Okuda and J. M. Dawson, Phys. Fluids **16**, 408 (1973).
[13]P. K. Shukla, M. Y. Yu, H. U. Rahman, and K. H. Spatschek, Phys. Rep. **105**, 227 (1984).
[14]A. S. Kingsep, K. V. Chukbar, and V. V. Yan'kov, in *Reviews of Plasma Physics*, Vol. 16, edited by B. B. Kadomtsev (Consultants Bureau, New York, 1990); A. V. Gordeev, A. S. Kingsep, and L. I. Rudakov, Phys. Rep. **243**, 215 (1994).
[15]L. Stenflo, Phys. Fluids **30**, 3297 (1987).
[16]J. D. Meiss and W. Horton, Phys. Fluids **26**, 990 (1983).
[17]L. Stenflo, Phys. Lett. A **186**, 133 (1994).
[18]L. Stenflo, Phys. Lett. A **222**, 378 (1996).
[19]M. Y. Yu, P. K. Shukla, and L. Stenflo, Astrophys. J. **309**, L-63 (1986).
[20]E. N. Lorenz, J. Atmos. Sci. **20**, 130 (1964).
[21]L. Stenflo, Phys. Lett. A **212**, 224 (1996).
[22]L. Stenflo, Phys. Scr. **53**, 83 (1996).
[23]C. Sparrow, *The Lorenz Equations: Bifurcations, Chaos and Strange Attractors* (Springer, New York, 1982).
[24]I. O. Pogutse, Fiz. Plazmy **17**, 874 (1991) [Sov. J. Plasma Phys. **17**, 511 (1991)]; I. O. Pogutse, *ibid.* **19**, 1479 (1993) [Plasma Phys. Rep. **19**, 779 (1993)]; D. J. Jovanovic and J. Vranjes, Phys. Scr. **T63**, 234 (1996).
[25]T. Dauxois, Phys. Fluids **6**, 1625 (1994).
[26]R. Benzi, S. Pierini, A. Vulpiani, and E. Salusti, Geophys. Astrophys. Fluid Dyn. **20**, 293 (1982).
[27]C. F. Carnevale and G. K. Vallis, J. Fluid Mech. **213**, 549 (1990).