Measuring vortex charge with a triangular aperture

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A triangular aperture illuminated with a vortex beam creates a truncated lattice diffraction pattern that identifies the charge of the vortex. In this Letter, we demonstrate the measurement of vortex charge via this approach for vortex beams up to charge ±7. We also demonstrate the use of this technique for measuring femtosecond vortices and noninteger vortices, comparing these results with numerical modeling. It is shown that this technique is simple and reliable, but care must be taken when interpreting the results for the noninteger case.

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Vortex beams have been an active area of research since their introduction by Nye and Berry [1]. Their intrinsic orbital angular momentum is of particular interest because it is quantized and can be either right-handed or left-handed [2]. Vortex beams are readily generated from diffractive plates [3] or programmable spatial light modulators (SLM) [4]. One can also generate femtosecond vortices [5] and noninteger vortices [6,7]. It has been demonstrated that vortex beams may be used to rotate and trap microparticles, create vortex states in Bose–Einstein condensates, and entangle orthogonal quantum states, among other applications [2,8].

Vortex beams can be characterized in terms of Laguerre–Gauss modes (LG_p^m) described by the topological charge m, the number of times the phase completes 2π on a closed loop around the propagation axis, and p, the number of radial nodes. For integer $m \neq 0$, the phase singularity on the beam axis results in an annular beam whose radius increases with m. The charge m of the vortex may be measured, both magnitude and sign, in several ways [2], including using a second SLM [9], Dammann phase masks [10], or, most surprisingly, with a triangular aperture [11].

Hickmann and colleagues [11] showed recently that when a vortex beam illuminates an equilateral triangular aperture, the far-field diffraction pattern consists of a truncated optical lattice. The lattice results from interference between waves diffracted from the aperture's edges shifted in reciprocal space by an amount proportional to the charge m. The number of bright spots in the triangular diffraction pattern is directly related to the vortex's charge m.

In this Letter, we demonstrate the measurement of vortex charge via the truncated lattice approach [11] for vortex beams from charge 0 to 7. We compare cw and femtosecond vortices, and demonstrate that the technique is capable of measuring the charge in either case. We also look at noninteger vortices both experimentally and numerically, including a systematic increase in charge from 0 to 3 in steps of 0.1.

We used a Ti:sapphire laser oscillator operating at 800 nm with a repetition rate of 80 MHz and an attenuated power of 50 mW. This laser was switched between cw and femtosecond modes for the various experiments. We generated our vortex beams with a programmable SLM (Hamamatsu X8267). We coupled the laser beam through a microstructured optical fiber (NKT, Femtowhite 800), which in our case merely acts as a spatial filter. The rapidly expanding beam that exited the fiber was collimated with a 70 cm focal length lens of 2 in. diameter (much of the power exiting the fiber was not collected by this lens). This setup provided a near plane-wave beam after the lens. This beam then impinged on our SLM. The SLM is computer controlled via a VGA interface and is easily programmed for a 0 to 2π phase modulation at the desired wavelength. The phase pattern used is the forked grating (spiral phase plus blazed grating) shown in [6]. The return beam from the SLM propagated back through the 70 cm lens and hit a pickoff mirror. This vortex beam came to a tight focus, and was then magnified with a 10 cm focal length lens, putting the beam waist at the triangular aperture, with a vortex beam size roughly matching the aperture.

It was a fairly simple matter to produce triangular apertures under a microscope. Pieces of metallic Scotch tape were aligned with the edges of equilateral triangles printed on computer paper. These pieces of tape were adhered to a Plexiglas housing with holes in it, so as to provide an aperture in air. Measurements of the aperture size were made under the microscope. The apertures are shown in Fig. 1(a). This Plexiglas holder was then affixed to a tip-tilt kinematic mount and attached to an xyz-translation stage. The xyz-translation stage proved critical for fine tuning of the optical alignment. A 15 cm focal length lens was placed immediately after the aperture, and at one focal length from this lens was placed a CCD camera

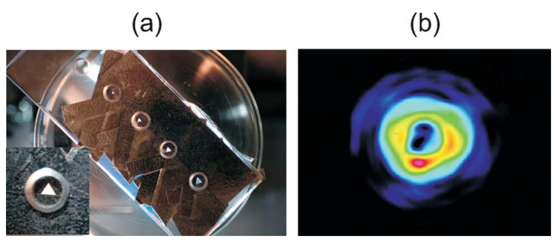

Fig. 1. (Color online) (a) Triangular apertures formed with metallic tape on a Plexiglas housing. The size of each triangle (length of one side in mm) is 0.73, 1.21, 1.76, and 2.44. The inset shows a close-up of the 1.76 mm triangle, centered on a hole in the Plexiglas. (b) Spatial profile of our charge 1 vortex beam at the aperture's position.

(Dataray WinCamD). For a well-defined diffraction pattern, the edges of the triangular aperture ideally should be illuminated by the inner border of the incident annular beam [11]. Our setup, with multiple apertures, allows us to conveniently match the aperture size to the beam size for different values of the vortex's charge.

Results are shown in Fig. 2. As the charge of the vortex is increased from 0 to 5, the number of bright spots in the resultant triangle is seen to increase. As identified in [11], the number of spots N along one edge of the triangle represents a vortex charge of $m = N - 1$. Thus the charge 0 vortex makes one spot, a charge 1 vortex makes two spots (along one edge), and so on. There is another relationship one can derive between the charge of the vortex m and the total number of spots N_T in the triangle. This relationship is simply an arithmetic series $N_T = \sum_{i=0}^{m+1} i$, which is equivalent to $N_T = (m+1)(m+2)/2$. This equation gives a quadratic, which can be solved for m in terms of N_T, yielding $m = (\sqrt{1+8N_t} - 3)/2$. Therefore, counting the total number of spots in the diffraction pattern leads to a unique charge. This could be advantageous in the cases of higher charge where the sides of the triangle are not well defined; including information from the entire pattern might yield more accurate results. It is quite easy to resolve the number of spots along one side of the triangle (6, for example) to deduce the charge (5), or to use the total number of spots (21) to calculate the charge (5).

There is a noticeable nonuniform distribution of energy among the spots. This is due to two factors. First, the vortex beams did not have perfectly distributed energy; namely, the light intensity was not uniform around the ring as illustrated in Fig. 1(b). Second, the alignment of the vortex beam in the triangular aperture is critical. For each figure presented here, there were small tweaks to the alignment of the triangle, and it was found that small misalignments can lead to large fluctuations in the distribution of energy among the spots.

The patterns in Fig. 2 indicate this is a very robust measurement technique for measuring charge up to $m = 5$. With care, we were able to measure up to charge 8, but it became increasingly difficult to resolve the spots any higher than this. In all of the results presented herein, it should be noted that the orientation of the triangular aperture was vertical (its base is parallel to the optical table), as seen in the inset of Fig. 1(a). The diffracted

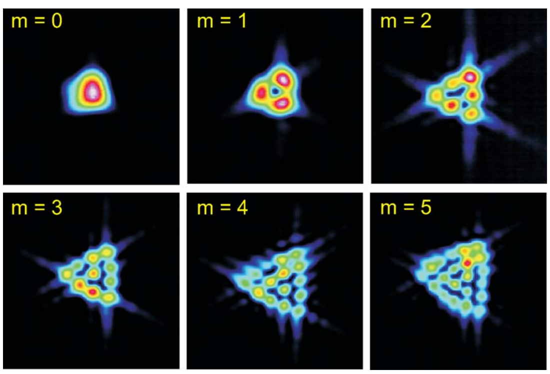

Fig. 2. (Color online) Measured diffraction patterns for vortices of integral charge. Images have been rescaled for clarity.

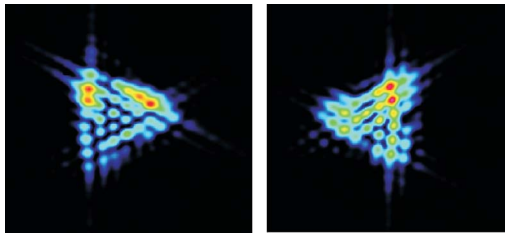

Fig. 3. (Color online) Diffraction patterns for vortex of charge −7 (left) and +7 (right) with a vertically oriented triangular aperture of size 1.21 mm.

triangular pattern is horizontal, however, which is characteristic of a vortex. It was shown theoretically in [11] that the orientation of the triangle identifies the sign of the vortex charge. A negatively charged vortex will generate a triangular lattice that is flipped 180° from its corresponding positively charged vortex. With a programmable SLM, it is straightforward to flip the sign of the vortex, and the change in orientation of the diffraction pattern between charge ±7 vortices is demonstrated in Fig. 3.

We investigated, theoretically and experimentally, the measurement of partial vortices, namely those vortex beams with noninteger charge. These vortices were generated as described in [6]. The results are shown in Fig. 4, which illustrates the diffraction patterns for charge 0, 0.2, 0.4, 0.6, 0.8, and 1.0. The theoretical diffraction patterns were obtained by numerically calculating a two-dimensional Fourier transform of the product of the aperture's transmission function and a linear superposition of an LG_0^0 mode and an LG_0^1 mode as per [7]. The numerical and experimental results are displayed and show good agreement. It is seen that the individual spot from charge 0 slowly dissipates and generates two new spots for a total of three, which corresponds to a charge 1 vortex. This result indicates that it is difficult to be certain about the fractional vortex charge based only on counting the number of spots, either along the edge of the triangle or the total number inside it. Indeed, one must take into account the distribution of energy within those spots and their corresponding spacing on the lattice. In order to visualize this transformation, we composed a movie (Media 1) that consists of changing the vortex charge from 0 to 3 in steps of 0.1. This movie is not real-time; rather, it is a composite of still images obtained experimentally, where each image was optimized. It gives a good feel for how the energy redistributes itself as the charge increases.

And finally, we also measured the diffraction pattern for a cw beam versus a femtosecond beam (≈40 nm bandwidth) for a charge 3 vortex, as shown in Fig. 5. These results are encouraging for lower-order vortices, indicating that this approach does work for femtosecond vortices. But the pattern quickly becomes unrecognizable at higher charges, where the smearing of the individual spots near the edges of the pattern makes it impossible to count the number of spots. We numerically simulated diffraction of a femtosecond vortex beam by following [12]. The simulation includes a spatial chirp introduced by the diffraction grating used to extract the vortex beam [5] and shows a smearing of the spots similar to that observed in the experiment. Because of the

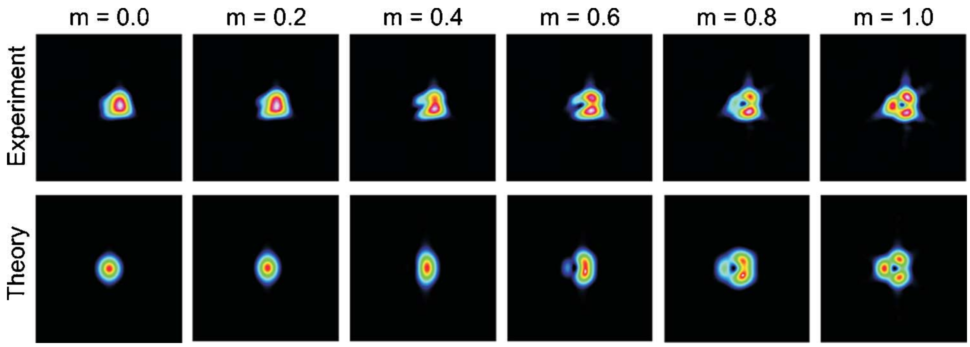

Fig. 4. (Color online) Diffraction patterns of fractional-charge vortices with aperture 1.21 mm. Movie online (Media 1) shows the change from charge 0 to 3 in steps of 0.1.

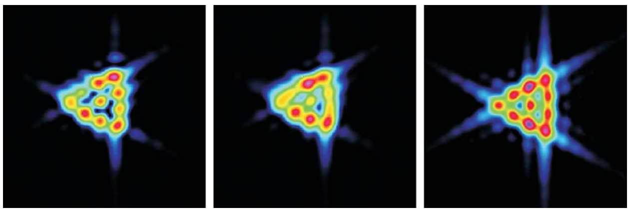

Fig. 5. (Color online) Measured diffraction patterns for a charge 3 vortex in cw mode (left) and femtosecond mode (center). Numerical simulation of femtosecond vortex diffraction with spatial chirp (right).

spatial chirp, the different colors in the beam will be slightly spatially misaligned from each other (but still mostly overlapping) when they impinge on the triangular aperture, smearing the spots. Without spatial chirp, a numerical simulation of the femtosecond diffraction pattern shows no significant difference from a simulated cw pattern.

In conclusion, we investigated the diffraction of vortex beams on a triangular aperture. We demonstrated experimentally that the technique is useful to determine integral vortex charges up to about ± 7 by simply counting the number of diffraction spots along the triangular lattice. However, the procedure is not so straightforward for noninteger vortices. The technique can be used with both cw and femtosecond lasers, at least for small-order vortices.

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