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Hidden Spin-Current Conservation in 2D Fermi Liquids

Paulo F. Farinas
Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas 13083-970, Campinas, São Paulo, Brazil

Kevin S. Bedell
Department of Physics, Boston College, Chestnut Hill, Massachusetts 02167

Nelson Studart
Departamento de Física, Universidade Federal de São Carlos 13565-905, São Carlos, São Paulo, Brazil
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We report the existence of regimes of the two-dimensional Fermi liquid that show unusual conservation of the spin current and may be tuned by varying some parameter such as the density of fermions. We show that for reasonable models of the effective interaction the spin current may be conserved in general in 2D, not only for a particular regime. Low-temperature spin waves propagate distinctively in these regimes and entirely new "spin-acoustic" modes are predicted for scattering-dominated temperature ranges. These new high-temperature propagating spin waves provide a clear signature for the experimental search of such regimes. [S0031-9007(99)09124-3]

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Fermi-liquid theory (FLT) was first introduced by Landau [1] and further developed to include spin waves a long time ago [2]. The progress of FLT in 3D revealed that it is one of the broader theories in condensed matter physics, explaining the experimental results of a wide range of different systems. Recently, interest in FLT regained momentum, driven in part by the discovery of high-temperature superconductors and also by the refinement of experimental techniques in low-dimensional physics. While many results on the (ab)normal phases of the former allow one interpretation that casts doubts on the validity of FLT in 2D [3], the latter has been consolidating a source of examples of practical 2D systems that behave as predicted by 2D FLT, as can be appreciated in the experiments reported in ^{3}He films [4,5]. This also seems to be the case for doped semiconductors, where thickness and doping can be controlled. Experiments on these charged systems have directly observed expected 2D Fermi-liquid behavior in GaAs heterostructures [6]. This comes from extracting the quasiparticle lifetimes from the tunneling peaks in the current-voltage profile of two biased 2D doped semiconductor contacts with a quantum well between them. The result is the one predicted by 2D FLT [7].

Spin waves were observed in bulk alkali metals some three decades ago by conduction-electron spin resonance (CESR) techniques [8], confirming the predictions of FLT [9]. Later, using nuclear magnetic resonance (NMR) in bulk ^{3}He, Ref. [10] confirmed the existence of the Leggett-Rice effect formerly predicted [11]. The suppression of the Leggett-Rice effect was also indirectly observed for a particular region of the parameter space in bulk ^{3}He-^{4}He mixtures [12–14], which was pointed out in Ref. [15]. All of these results agree with the known fact that spin current is not conserved in 3D [16]. The majority of these experiments were not repeated for 2D systems up to this date.

The central purpose of this Letter is to present exact results indicating that spin current may be a conserved quantity in 2D Fermi liquids, at least for some regions of the parameter space. We will see that for microscopic models that assume short range potentials for the effective interaction spin-current conservation holds for the entire parameter space, so that such models validate the bold statement that spin current is conserved in 2D (not only at particular regimes). This is rather compelling since our results are for the spin channel, and the transition from short to long range in the charge channel should not essentially modify the physics presented here, provided one has no broken symmetries. We also calculate the dispersions of transverse spin waves under this "spin-space Galilean invariance" and find that an experimental observation of these new collective modes will show rather distinct features that make such regimes easy to identify. Before proceeding, we would like to give a precise meaning for 2D in the context of this article. Let us establish that the system is 2D whenever one of its three dimensions is comparable to or less than the quasiparticle's typical wavelength.

At low temperatures, the phase space available for scattering in 3D is a spherical shell and the incoming momenta are not, in general, coplanar with the outgoing pair. As a result, the values of the spin currents carried by two quasiparticles with antiparallel spins (singlet channel), before and after they collide, are not related, $\mathbf{j}_{\mathrm{in}} \equiv \sigma_1\mathbf{p}_1 + \sigma_2\mathbf{p}_2 = \mathbf{p}_1 - \mathbf{p}_2 \neq \mathbf{j}_{\mathrm{out}} = \mathbf{p}_3 - \mathbf{p}_4$, where $1, 2$ and $3, 4$ refer to incoming and outgoing momenta. If $\mathbf{q}$ is the total exchange of momentum in the collision, we can write $\mathbf{j}_{\mathrm{out}} = \mathbf{j}_{\mathrm{in}} + \mathbf{q}(\phi)$, where ϕ is the angle between the scattering planes. While the triplet channel conserves spin current trivially, the two scattering processes in the singlet channel are completely accounted for by fixing the spins on all momenta and varying ϕ. One can turn from small momentum exchange, near $\phi = 0$ (forward), continuously to large $q \sim 2k_F$, near $\phi = \pi$ (backward). Since $\mathbf{j}_{\rm out}$ points in a random direction relative to $\mathbf{j}_{\rm in}$, we can say that the spins "walk" randomly throughout the system and hence spin transport is diffusive. In 2D, however, due to the reduced phase space, one is left with only three possibilities, $\mathbf{j}_{\rm out} = \mathbf{j}_{\rm in}$, $\mathbf{j}_{\rm out} = -\mathbf{j}_{\rm in}$, and $\mathbf{j}_{\rm in}(\mathbf{k} = 0) \neq \mathbf{j}_{\rm out}(\mathbf{k} = 0)$, where $\mathbf{k}$ is the total momentum. This latter region of the phase space brings no contributions to the scattering integral to leading order in temperature [7,17,18]. In fact, as we see below, the scattering amplitude for $\mathbf{k} = 0$ processes is zero in a regime that conserves spin current, so that we are left with only two scattering processes: forward, which conserves spin current, and backward, which flips the direction of spin current. Hence, spin current is not conserved in 2D as long as the balance between two clearly distinguishable processes remains.

To study the circumstances which allow this balance to break in such a way as to favor spin-current conservation, we consider a planar Fermi liquid with a weak magnetization perpendicular to the plane and whose gradient is such that $\nabla M = |\nabla M|\hat{z}$, where $\hat{z}$ is an in-plane unit vector. The scattering integral for the Landau kinetic equation of FLT in 2D can, in general, be expanded in circular harmonics $\psi_l(x)$,

$$I[n_{\mathbf{p}\sigma}] = \sum_l I_l \psi_l(\mathbf{p} \cdot \hat{z}),$$

where the amplitudes depend on the quasiparticle energy. For spin-diffusion processes, while $I_0 = 0$ due to spin conservation in collisions, the symmetry of the distribution function implies that all but the first higher order angular contributions are small enough so that we can write $I_l = I_1 \delta_{l1}$. In 2D this dominant amplitude can be written explicitly in terms of the low-frequency four point vertex function Γ at $k = 0$ and $k = 2k_F$,

$$I_1 = C[u_1(x_{\mathbf{p}})S + u_2(x_{\mathbf{p}})|\Gamma^0_{\uparrow\downarrow\downarrow\uparrow}|^2], \tag{1}$$

where

$$C \propto \sigma|\nabla M|\left(\frac{T}{T_F}\right)^2 \left|\ln\left(\frac{T}{T_F}\right)\right|,$$

$$S = \sum_{\mathrm{spins},k=0,2k_F} |\Gamma^k_{\sigma_1\sigma_2\sigma_3\sigma_4}|^2 \delta_{\sigma_1+\sigma_2,\sigma_3+\sigma_4},$$

and u_i are functions of the normalized energy $x_{\mathbf{p}} = (\epsilon_{\mathbf{p}} - \mu)/k_B T$. Equation (1) yields

$$\sum_{\sigma\mathbf{p}} \sigma\mathbf{p} I[n_{\mathbf{p}\sigma}] \propto |\Gamma^0_{\uparrow\downarrow\downarrow\uparrow}|^2. \tag{2}$$

This result comes from the vanishing of the term proportional to S in Eq. (1) under energy integration. Details will be presented elsewhere but they follow from a similar analysis of the scattering integral in 2D found in Refs. [7] and [18]. The fact that in 2D one can write exact expressions in terms of the Landau parameters for the vertex part [19] yields closed expressions in terms of this function at particular points of the phase space. It is clear from Eq. (2) that spin current is conserved *exactly* if $\Gamma^0_{\uparrow\downarrow\downarrow\uparrow} = 0$. In the same spirit, the diffusion coefficient may also be expressed in terms of the vertex part [18] for a weakly polarized Fermi liquid, and when $\Gamma^0_{\uparrow\downarrow\downarrow\uparrow} = 0$ it diverges, as it should if spin current is conserved.

To see that $\Gamma^0_{\uparrow\downarrow\downarrow\uparrow}$ may in fact be zero, we start with a dilute-gas result [20] that gives $\Gamma^{k=0}_{t\,\mathrm{matrix}} = 0$. This result comes from resumming the logarithm divergences for an arbitrary spin independent short range potential, similar to what is done in 3D [21]. This is a physically compelling starting point, since, as mentioned earlier, one does not expect the physics in the spin channel to radically change in the absence of broken symmetries for potentials with longer tails in the charge channel. We plug the t matrix result for an arbitrary $\mathbf{k}$ in the Bethe-Salpeter equation [19] for the vertex function and obtain $\Gamma^0_{\uparrow\downarrow\downarrow\uparrow} = 0$. This is equivalent to including contributions from particle-hole correlations to all orders in the coupling constant (g) in addition to particle-particle ladder bubbles to all orders and particle-hole ladder bubbles up to second order that are included in $\Gamma^k_{t\,\mathrm{matrix}}$. Contributions from other diagrams, if not vanishing at the particular point $k = 0$, will be small, leading to a very long diffusion relaxation time.

For this result the spin-diffusion relaxation time due to I_1 is infinite so that the symmetry due to higher order amplitudes becomes dominant. We stress that even in a more general scenario where $\Gamma^0_{\uparrow\downarrow\downarrow\uparrow}$ remains small but finite it suffices that it is small enough to render the relaxation time associated with I_1 long compared to the scales arising from higher angular terms. Under such a condition spin current will be conserved in the relevant finite time scales; the system will relax due to a higher order process that we call "spin viscosity" in analogy to what happens with sound.

We discussed the possibility that a new conservation exists in 2D Fermi liquids in general. This is not the only possibility. We now turn to a more phenomenological analysis of the particular regimes that conserve spin current in 2D. This is based on the fact that the condition for $\Gamma^0_{\uparrow\downarrow\downarrow\uparrow} = 0$, if not valid in general in 2D, can be achieved *by tuning* the appropriate values of the Landau parameters. To be more specific, let a be the Greens' function quasiparticle residue and let $N(0)$ be the density of states at the Fermi surface. For a 2D Fermi liquid we can write [18,19]

$$N^2(0)a^2|\Gamma^0_{\uparrow\downarrow\downarrow\uparrow}|^2 = \left[\sum_{l=-\infty}^{+\infty} (-1)^l c_l A^a_l\right]^2, \tag{3}$$

where A^a_l are the antisymmetric scattering amplitudes that relate to the Landau coefficients through $A^a_l = F^a_l/(1 + c_l F^a_l)$ and $c_l = 1/(2|l| + 1)$. A quick technical remark about our choice of the 2D basis will avoid confusion. We choose $\psi_l(\mathbf{p}\cdot\hat{z}) = c_l e^{il\theta}$. This shifted basis leads mostly to formulas that look identical to their 3D counterparts, and is only a matter of convenience. We see from Eq. (3) that regions of the parameter space which conserve spin current *exactly* are tuned when $c_l A_l^a = 2^{\delta_{l0}} c_{l+1} A_{l+1}^a$, for $l = 0, 2, 4, \ldots$, $c_l A_l^a = 2^{\delta_{l0}} c_{l+3} A_{l+3}^a$, for $l = 0, 1, 2, \ldots$, and for an infinite number of other possibilities. One can also keep only the first few scattering amplitudes following another dilute-gas calculation for which the nth Landau coefficient is proportional to g^n [22]. One then finds that $\Gamma^0_{\uparrow\downarrow\downarrow\uparrow} = 0$ for specific combinations of the Landau parameters. For instance, when the first two Landau parameters are such that $F_0^a = 2F_1^a/3(1 - F_1^a/3)$, the contribution from the first two terms in the sum vanishes and one is left only with terms that are of order g^2 or higher. Throughout this article, the 2D coupling constant, $g \equiv -1/2\ln(k_F a_s)$, arises from microscopic models based on short-range potentials with characteristic length a_s. One can easily verify that, for the majority of choices made to give $\Gamma^0_{\uparrow\downarrow\downarrow\uparrow} = 0$, one always finds a very reasonable value for g (between 0.1 and 0.4). It is clear that these regimes may be tuned by externally varying some parameter such as the density. This particular range of the coupling constant corresponds to second-layer coverage densities between 5 and 18 $\times 10^{13}$ cm^{-2} in the data from ^{3}He films on Grafoil experiments [23].

Given the infinite number of combinations that lead to $\Gamma^0_{\uparrow\downarrow\downarrow\uparrow} = 0$, we feel compelled to investigate the signatures that should indicate the experimental tuning of such regimes in the spin-wave modes. The dispersion relations for transverse spin waves are the simpler and broader objects that we can think of for this purpose. They may be used to figure out the form of the effective "diffusion" (in this case we should say effective "viscosity") equivalent to the Platzman and Wolff result for charged systems so that CESR experiments can be done in electronic planes. We will hence leave the consequences of more sophisticated phenomena, such as the Leggett-Rice effect [11], for future publications. Also, the detailed analysis of the dispersion relations will be published in a longer article; here we show only the results and outline the derivation.

Spin-current conservation equalizes the spin and charge channels regarding the number of conserved quantities. As a consequence, we expect propagation of spin waves to become analogous to sound propagation. This is partly true; the presence of an additional scale set by the external magnetic field keeps the propagation of spin waves distinct from sound, but they present various new common features. We project out the Landau kinetic equation [16] on the 2D basis, and solve for the Fermi-surface distortions associated with transverse spin waves in a relaxation time scheme. For $\Gamma^0_{\uparrow\downarrow\downarrow\uparrow} = 0$, the relaxation time approximation is written as $I_l = (1 - \delta_{l0} - \delta_{l1})\omega_{\sigma\eta}$, where we have introduced the spin-viscous relaxation time $\omega_{\sigma\eta}^{-1}$. The dispersion relations are calculated for a weak but finite magnetic field such that in the long wavelength limit $qv_F \ll \omega_L$ (the Larmor frequency). For bulk ^{3}He this corresponds to magnetic fields between 0.25 and 1 T. The results are

$$\Delta\omega_{l=0} = -\frac{\alpha_0}{2\lambda\omega_L}(qv_F)^2 + \left(\frac{1}{\alpha_1} - \frac{\lambda\lambda_2}{2z}\right) \times \frac{\alpha_0^2}{4\lambda^3\omega_L^3}(qv_F)^4,$$

$$\Delta\omega_{l=1} = \omega_L\lambda\alpha_1 + \left[1 + \frac{(1 + \lambda_2\alpha_2)}{2(1 - z\alpha_2/\lambda\alpha_1)}\right] \times \frac{\alpha_0}{2\lambda\omega_L}(qv_F)^2,$$

and

$$\Delta\omega_{l=2} = \omega_L z\alpha_2 - \frac{(1 + \lambda_2\alpha_2)}{2(1 - z\alpha_2/\lambda\alpha_1)}\frac{\alpha_0}{2\lambda\omega_L}(qv_F)^2,$$

where $\alpha_l = 1 + c_l F_l^a$. In addition to the usual strength $\lambda \equiv \alpha_0^{-1} - \alpha_1^{-1}$, we defined $\lambda_2 \equiv \alpha_0^{-1} - \alpha_2^{-1}$. Here $\Delta\omega = \omega - \omega_L$ and $z \equiv \lambda_2 - i\omega_{\sigma\eta}/\omega_L$. We write the lowest distortions to higher order in q since these modes attenuate only to relative order q^2 under the new condition of spin-current conservation. A third dispersion branch emerges as a result of the higher order "spin-viscous" attenuation. We see that the quadrupolar fluctuations show similar behavior to the lower order distortions in a nonconserving regime: It propagates at low T and is purely damped at high T. The dipole modes, however, propagate almost undamped both at high and low T's, showing more attenuation for intermediate temperatures. The density fluctuation propagates almost undamped and with the same dispersion for any T to leading order in q, and is weakly damped to relative order q^2 at intermediate temperatures. Hence the $l = 0$ and $l = 1$ modes propagate both in the collisionless and hydrodynamic regimes. This rather unusual behavior in the propagation of the two first spin-wave modes is a direct consequence of spin-current conservation and it is analogous to what happens in the propagation of sound. The ratio $\omega_{\sigma\eta}/\omega_L$ governs the interplay between the collisionless and hydrodynamic regimes, and a peak in the attenuation occurs when $\omega_{\sigma\eta}/\omega_L = \lambda_2$ for both modes.

For completeness, we address some regimes of the parameter space that are of particular interest if they coexist with the conservation of spin current. We do not wish to imply here that these additional regimes exist, only to present the changes that should be expected if they do. We look at the regime for which $\lambda = 0$. This regime where the interaction strength changes sign was observed in bulk ^{3}He-^{4}He mixtures [12–14], and it is known to show a remarkable suppression of the Leggett-Rice effect [11,15]. For $\lambda = 0$, the relevant changes occur in the lowest two branches which collapse into two physically equivalent branches corresponding to soundlike spin waves propagating in opposite directions with velocity $\alpha_0 v_F/\sqrt{2}$. These modes propagate at any temperature to leading order in q. The attenuation in this case is of relative order q and also presents a peak at $\omega_{\sigma\eta}/\omega_L = \lambda_2$. These modes are thus analogous to sound in the sense that besides having a linear dispersion they undergo a transition from a low-temperature zero-sound-like regime into a hydrodynamic regime as one raises the temperature.

The region of the parameter space for which $\lambda_2 = 0$ is readily obtained and brings no additional physics if $F_0^a \neq F_1^a/3$. However, for $\lambda = \lambda_2 = 0$ we have the interesting new feature that the attenuation of the two lowest order distortions decrease as T increases. This may be understood by recalling that the strength of the quasiparticles' interactions is measured by λ and λ_2. If both parameters are zero then no zero-sound-like modes are expected to propagate at low T. However, as scattering increases with T, spin currents are favored in a conserving regime.

In conclusion, we presented the possibility that spin current is conserved in 2D Fermi liquids, if not in general, for some particular regimes. This is basically due to the restricted geometry combined with the degeneracy of the Fermi surface. We showed some consequences this conservation brings to the propagation of transverse spin waves in such regimes, and predicted that spin waves that are known to occur only for very low T will also propagate in scattering-dominated regimes. This is the most remarkable property of such regimes and should serve to clearly distinguish what we call spin-viscous relaxation processes associated with such regimes from ordinary spin-diffusion relaxation. Nuclear magnetic resonance experiments probing the spin relaxation will show sharper absorption peaks for the two lowest modes due to the weak attenuation. As one scans temperatures within the Fermi-liquid regime, NMR peaks are expected to widen up to a maximum width and then to become sharp again, indicating the presence of a maximum value for the attenuation. The sole presence of a peak at higher temperatures will provide evidence for this conservation in 2D. The immediate candidate systems for such experiments are helium films, such as the ones pointed out here [5,23]. The existence of further consequences both in helium layers and in 2D Fermi systems in general, is an open question.

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