Quasi-Fermi-levels in quantum-well photoluminescence

G. D. Mahan
Department of Physics and Astronomy, University of Tenneessee, Knoxville, Tennessee 37996-1200
and Solid State Division, Oak Ridge National Laboratory, P.O. Box 2008, Oak Ridge, Tennessee 37831-6030

L. E. Oliveira
Instituto de Fisica, Universidade Estadual de Campinas, Caixa Postal 6165, 13081 Campinas, São Paulo, Brazil
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The nonequilibrium quasi-Fermi-levels of electrons and holes in quantum wells are calculated during photoluminescence. It is assumed the electrons and holes are created by continuous laser excitation. Various recombination processes are included: electron radiative recombination with holes bound at neutral acceptors, electron radiative recombination with free holes, hole trapping at ionized acceptors, and Auger decay. A numerical example is presented for acceptors in $GaAs/Ga_{1-x}Al_xAs$ quantum wells.

I. INTRODUCTION

Quantum wells are one of the active areas of semiconductors physics. A typical geometry is the two-dimensional layer. The electrons and holes move freely in two dimensions. Motion in the third dimension is limited to a well of a thickness which can be varied experimentally, and is usually of the order of $L = 100$ Å. The wells are created by alternate layers of different semiconductors. Their lattice constants are usually matched to reduce strains. A popular system is $GaAs/Ga_{1-x}Al_xAs$, where x can be chosen by the experimentalist.

The semiconductors have impurities that cause donor and acceptor states. Bastard[1] was the first to consider impurity binding in quantum wells. Other calculations were done by Mailhoit *et al.*,[2] by Greene and Bajaj,[3] and others.[4-9] They consider the ground state plus low-energy excited states. They usually considered one quantum well with potential boundaries of a finite height. The matching at the boundaries of the wells included changes in the dielectric function and effective mass. Oliveira[7] also included r-dependent screening due to central-cell corrections.

An important parameter in these calculations is the position z_i of the impurity in the quantum well. Taking the center of the well as $z=0$, popular choices are putting the impurity at the center ($z_i=0$), at the edges ($z_i=\pm L/2$), or uniformly distributed throughout the well ($-L/2<z_i<L/2$).

Photoluminescence has been used as an experimental probe of acceptor states in quantum wells.[10-18] The usual experiment is to excite electrons and holes using continuous laser excitation. The luminescence shows a spectrum that is continuous in frequency. However, the spectrum has peaks which are associated with critical points due to placing the acceptors at the center or edge of the quantum wells. The interpretation of these spectra has been a subject of lively discussion in the literature.[19-21] Transient photoluminescence experiments have also been done.[22,23] A key parameter in the interpretation of the photoluminescence is the value of the quasi-Fermi-levels of the electrons and holes. For example, very different spectra are obtained if the distributions follow Maxwell-Boltzmann statistics compared to Fermi-Dirac statistics. The semiconductors are insulating without the laser excitations. The excitation creates the electrons and holes. The density of particles depends upon the laser intensity. A typical experiment may have an illumination intensity of $I=1$ W/cm^2. What is the density of electrons and holes at this or other intensities? The answer depends upon the value of numerous rate constants for different relaxation processes.

Here we try to calculate the quasi-Fermi-levels of electrons and holes as a function of the laser intensity and of material parameters. We hope this calculation will provide a quantitative interpretation of the photoluminescence experiments.

The present calculation considers the following processes which determine the quasi-Fermi-levels: (1) interband absorption, (2) electron radiative recombination with free holes, (3) electron nonradiative recombination (Auger) with free holes, (4) electron radiative recombination with bound holes, and (5) trapping of holes at acceptors. Numerical examples are given for a $GaAs/Ga_{1-x}Al_xAs$ quantum well.

II. THE MODEL

Many different processes can occur when photons are absorbed in a semiconductor. Only a small subset of these processes has been selected for consideration here. This choice was made with one eye on the experiments. We assume that the prominent impurities are acceptors. Their density is small—typically[24] around $N_A \simeq 10^{14}$ cm^{-3}—although it can be increased by doping. It is assumed that the temperature is low, for example, $T=1$ K. The acceptors are all neutral in equilibrium, so the chemical potential is initially between the acceptor level and the top of the valence band.

A. Interband absorption

Electrons and holes are created by interband absorption of photons from a laser whose frequency exceeds the energy gap of the semiconductor. Since the acceptors are neutral, the holes are created in the valence band. Call w the rate per unit area of creating electrons and holes. It is proportional to the intensity I of the laser,[25]

$$w = w_I I \ , \tag{1}$$

$$w_I = 4\pi\alpha_f \frac{\mu(\hat{\xi}\cdot\mathbf{p}_{cv})^2}{m_e^2(\hbar\omega)^2} G(\hbar\omega - \varepsilon_g) \ , \tag{2}$$

$$G(E) = \frac{2}{1+e^{-(\pi/2)\sqrt{E_x E}}} \ . \tag{3}$$

The fine-structure constant is denoted by α_f, the electron mass is m_e, the effective reduced mass of the electron-hole systems is μ, the light polarization is $\hat{\xi}$, and the interband matrix element is $\mathbf{p}_{cv}$. The factor $G(E)$ includes exciton effects due to the Coulomb attraction of the electron and hole. For empty bands in two dimensions, excitons of binding energy E_x increase w_I by a factor of 2 at the bottom of the band, i.e., for photon energies just above the energy gap. This factor of 2 decreases slowly to unity for photon energies which are increased above the band minimum—the energy scale is the exciton binding energy. For heavily doped bands, the Mahan exciton increases the interband absorption at threshold. The energy gap for three-dimensional bulk semiconductors is E_g. In quantum wells, the minimum energies of electron and hole are raised due to the quantization in the z direction. The effective band gap ε_g of the quantum well is larger than E_g.

B. Quasi-Fermi-levels

The electrons and holes recombine by various processes. Under cw excitation, these processes create a nonequilibrium but steady-state distribution of electrons (n_e) and holes (n_h). The Coulomb scattering between particles is very efficient at equilibrating the electrons and holes into Fermi-Dirac distributions with an effective temperature $T_p > T$. The steady-state distribution can be represented by a quasi-Fermi-level, or quasichemical potential, for the electrons and holes. An important consideration is the rate at which the particle temperature T_p relaxes back to the phonon temperature T. This relaxation is done through the electron-phonon and hole-phonon interactions. This issue has been discussed in the "hot-electron" literature. Here we assume that such relaxation occurs, and that the particle temperature remains small. That is, it is small enough that thermal smearing does not contribute significantly to experimental energy resolutions.

C. Electron radiative recombination with free holes

This process is just the reverse of the excitation process. The electron and free hole recombine and emit a photon. The coefficient w_x, in units of photons per area per second, has some factors which are similar to the rate of absorption w given above. They differ in detail because of several factors. The photons' wave vector $\mathbf{q}$ can be in any direction, and one must average in three dimensions over directions and polarizations. The emission rate of photons depends upon the occupation numbers of electrons $[n_c(k) = 1/(\exp\{\beta[\varepsilon_c(k) - \mu_c]\} + 1)]$ and holes $[n_{vi}(k)]$ in the two bands. The index i denotes light- or heavy-hole band. It does not depend upon the number of photons in the state $\mathbf{q}$, since this blackbody background radiation is assumed to be negligible. The optical transition is vertical in k space, since the photon wave vector is negligible. We use ω for the frequency of the laser photon making the original interband transition, and ω' for the frequency of the emitted photon:

$$\begin{aligned} w_x = \frac{2e^2}{m_e^2} \int \frac{d^3q}{(2\pi)^3\omega'} (\hat{\xi}\cdot\mathbf{p}_{cv})^2 & \\ \times \int d^2k\, n_c(k) \sum_i n_{vi}(k) & \\ \times \delta\left[\varepsilon_g - \hbar\omega' + \frac{\hbar^2k^2}{2\mu_i}\right] & \ . \end{aligned} \tag{4}$$

The densities of electrons and free holes are defined as

$$n_e = 2\int \frac{d^2k}{(2\pi)^2} n_c(k) \ , \tag{5}$$

$$n_h = 2\int \frac{d^2k}{(2\pi)^2} \sum_{i=1}^{2} n_{vi}(k) \ . \tag{6}$$

This expression will be evaluated in two limits. The first is for a low density of electrons and holes. Then one can use Maxwell-Boltzmann statistics for their densities. In that case,

$$n_c(k) = n_e \left[\frac{\pi\hbar^2}{m_c k_B T}\right] e^{-\beta\varepsilon_c(k)} \ , \tag{7}$$

$$w_x = \frac{n_e n_h}{n_0 \tau_x} \ , \tag{8}$$

$$1/\tau_x \approx \tfrac{4}{3}\alpha_f \omega' \frac{p_{cv}^2}{m_e^2 c^2} G(\hbar\omega' - \varepsilon_g) \ , \tag{9}$$

$$1/n_0 = \frac{\pi\hbar^2 b}{m_c k_B T} \ , \tag{10}$$

$$b = m_c \sum_i \frac{m_{vi}}{m_{vi} + m_c} \Big/ \left[\sum_j m_{vj}\right] \ . \tag{11}$$

In doing these integrals, we assume the temperature is small, so that all of the particles are at small wave vectors k. All matrix elements are evaluated at $k = 0$. We average over the final directions and polarizations of the emitted photons.

The recombination of electrons and holes is proportional to their joint density, or $w_x \simeq n_e n_h$. This is true when the densities are dilute, and Maxwell-Boltzmann statistics is suitable. In GaAs the constant

$n_0 \approx 1.0\times 10^{10}(T/\mathrm{K})$ cm^{-2}. Maxwell-Boltzmann statistics are suitable whenever $n_{e,h} < n_0(T)$.

However, for high densities of particles, Fermi-Dirac statistics are required. The emission rate is no longer proportional to the product of $n_e n_h$. For Fermi-Dirac statistics,we have

$$2\int \frac{d^2k}{(2\pi)^2} n_c(k) \sum_i n_{vi}(k) = n_m \ . \tag{12}$$

The integral equals just the density n_m of whichever band has the smallest Fermi wave vector ($m=c,v$). If there are equal number of electrons and holes, then $m=v$; since the holes have a higher degeneracy, their Fermi function cuts off the wave-vector integrals before the Fermi function of the electrons. For the sake of discussion, we shall assume $n_m = n_h$. Thus we find, after averaging over photon directions and polarizations,

$$w_x = n_h/\tau_x \ . \tag{13}$$

Again we should include the factor $G(\hbar\omega' - \varepsilon_g)$ for the exciton enhancement at the band edge.

D. Electron radiative recombination with bound holes

At low and moderate intensities of the laser excitation, a moderate density of electrons and holes is produced. An important recombination process is the emission of photons due to electron recombination with hole at acceptors. The rate w_B is proportional to the density of acceptors N_A(number/cm^3). One also has to average in three dimensions over the directions and polarizations of the photons. This brings us to the expression

$$w_B = \frac{8}{3\hbar} N_A \alpha_f \frac{p_{cv}^2}{m_e^2 c^2} J_B \ , \tag{14}$$

$$J_B = 2N(\lambda, z_i)^2 \int \frac{d^2k}{(2\pi)^2} n_c(k) J(z_i,\lambda,k)^2 \times [\varepsilon_g - E_B(z_i) + \varepsilon_c(k)] \ , \tag{15}$$

$$J(z_i,\lambda,k) = \int d^3r\, e^{i\mathbf{k}\cdot\mathbf{p}} \cos^2\left[\frac{\pi z}{L}\right] e^{-|\mathbf{r}-\hat{\mathbf{z}}_i|/\lambda} . \tag{16}$$

The matrix element $J(z_i,\lambda,k)$, normalization $N(\lambda,z_i)$, and binding energy $E_B(z_i)$ of the acceptor variational wave function are given in Refs. 1 and 16. The quantity J_B has the dimensions of length times energy. The variational parameter λ for the acceptor state depends upon both L and z_i.

This expression has some interesting features that are an important part of our results. At small values of the electron density, the integral can be evaluated as

$$J_B \approx n_e \hbar\omega' N(\lambda,z_i)^2 J(z_i,\lambda,0)^2 \ . \tag{17}$$

All of the electrons are near the point $k=0$, so the matrix element is evaluated at this point. Here one finds that $w_B \approx n_e N_A / s_{eA}$, where

$$1/s_{eA} = \tfrac{8}{3}\alpha_f \omega' \frac{p_{cv}^2}{m_e^2 c^2} \langle (NJ)^2 \rangle \ , \tag{18}$$

$$\langle (NJ)^2 \rangle = \int dz_i p(z_i) N^2(\lambda,z_i) J^2(z_i,\lambda,0) \ . \tag{19}$$

Usually we take the probability distribution of impurities $p(z_i)$ to be a constant $p=1/L$ in the region $(-L/2, L/2)$. These results are appropriate for a small concentration of electrons, which happens when the quasi-Fermi-level is small.

Usually at low excitation intensities the interband absorption is balanced by the recombination at acceptors. Then this equation is combined with (1) to give an expression for the density of electrons

$$n_e \approx s_{eA} w_I I / N_A \ . \tag{20}$$

The electron density is proportional to the light intensity. This result is obtained by assuming that either the radiative recombination with free holes or else the nonradiative Auger recombination is less likely than the recombination at acceptors. That is the case at small excitation intensity.

The integral in (15) has the property of giving a finite value in the limit that $n_e \to \infty$. Call this value J_{B0}. The integral over wave vectors converges when $n_c(k)=1$ for all values of k. For higher intensities of the laser excitation, the recombination of electrons at acceptors saturates, and can no longer control the rate of recombination. In this case, other mechanisms take over, such as electron recombination with free holes, or Auger recombination.

E. Auger recombination

Auger recombination of electrons and holes is a nonradiative process which occurs because of Coulomb interactions between the particles. There are six possible processes. The electron can recombine with either (i) a free hole, or (ii) a bound hole, while exciting to high kinetic energy either (a) another electron, (b) a free hole, or (c) a bound hole. Our calculations show that this process may be important for high laser intensities. However, for intensities less than $I=1$ W/cm^2, Auger decay appears to be unimportant. Here we shall sketch the derivation of just one of the six processes: the recombination of electrons with bound holes, while exciting another electron to high kinetic energy.

The kinetic-energy states are quantized in the z direction. For a quantum well with walls of infinite height, the possible energies and eigenfunctions are

$$E_0 = \frac{\hbar^2}{2m_c}\left[\frac{\pi}{L}\right]^2 , \tag{21}$$

$$\cos\left[\frac{\pi z}{L}(2n+1)\right] , \quad E_n = E_0(2n+1)^2 \ , \tag{22}$$

$$\sin\left[\frac{2\pi n z}{L}\right] , \quad E_n = E_0(2n)^2 \ . \tag{23}$$

Here the symbol m_c denotes the effective mass of the band. All of these states vanish at $z=\pm L/2$. Usually the energy E_0 is large enough so that all particles are in the band with the lowest subband energy: $E=E_0$ with

$\cos(\pi z/L)$. However, in the Auger process, one electron is excited to states of high kinetic energy. This high energy permits the electron to be in many possible subbands with different values of n. Here we shall calculate the transition to the final state in the symmetric subband. The calculation to the antisymmetric subband is similar.

The main part of the calculation is the evaluation of the Coulomb matrix element:

$$V=\frac{e^2}{\epsilon_0}\int \frac{d^3r_1 d^3r_2}{|\mathbf{r}_1-\mathbf{r}_2|}\psi^\dagger_{\mathbf{k}_2+\mathbf{q}}(\mathbf{r}_2)\psi_{\mathbf{k}_2}(\mathbf{r}_2)\psi^\dagger_A(\mathbf{r}_1)\psi_{\mathbf{k}_1}(\mathbf{r}_1) , \tag{24}$$

$$\psi_A(\mathbf{r})=N(\lambda,z_i)\cos\left[\frac{\pi z}{L}\right]e^{-|\mathbf{r}-\mathbf{z}_i|/\lambda} , \tag{25}$$

$$\psi_{\mathbf{k}}(\mathbf{r})=\left[\frac{2}{LA}\right]^{1/2}\cos\left[\frac{\pi z}{L}(2n+1)\right]e^{i\mathbf{k}\cdot\rho} . \tag{26}$$

In the wave functions with two-dimensional wave vector $\mathbf{k}$, the state with $n=1$ is used except for $\psi_{\mathbf{k}_2+\mathbf{q}}$. First, evaluate the integral over d^3r_2, which gives

$$V=N(\lambda,z_i)\left[\frac{2}{LA}\right]^{3/2}\frac{e^2}{2\epsilon_0}\int d^3r_1\cos^2\left[\frac{\pi z_1}{L}\right]e^{-|\mathbf{r}_1-\mathbf{z}_i|/\lambda}e^{i(\mathbf{k}_1-\mathbf{q})\cdot\rho_1}\left[\cos(\theta_n z_1)\frac{4\pi}{q^2+\theta_n^2}+\cos(\theta_{n+1}z_1)\frac{4\pi}{q^2+\theta_{n+1}^2}\right] , \tag{27}$$

$$\theta_n=\frac{2\pi n}{L} . \tag{28}$$

The remaining integral is similar to $J(z_i,\lambda,k)$ in (15); the difference is the additional factor of $\cos(\theta_n z_1)$. This factor actually reduces the integral by a significant amount for large values of n. This integral we call J_n. The matrix element for the Auger process is

$$V=\frac{2\pi e^2}{\epsilon_0}N(\lambda,z_i)\left[\frac{2}{LA}\right]^{3/2}\left[\frac{J_n}{q^2+\theta_n^2}+\frac{J_{n+1}}{q^2+\theta_{n+1}^2}\right] , \tag{29}$$

$$J_n(z_i,\lambda,\mathbf{k}_1-\mathbf{q})=\int d^3r\, e^{i(\mathbf{k}_1-\mathbf{q})\cdot\rho}\cos^2\left[\frac{\pi z}{L}\right]\cos(\theta_n z)e^{-|\mathbf{r}-\mathbf{z}_i/\lambda} . \tag{30}$$

The rate of Auger transitions ($\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$) is given by the golden rule

$$w_A=\frac{2^5\pi}{\hbar}\frac{N_A}{L^2}N^2(\lambda,z_i)\int\frac{d^2k_1d^2k_2d^2q}{(2\pi)^6}n_c(k_1)n_c(k_2)\times\left[\frac{2\pi e^2}{\epsilon_0}\right]^2\sum_n\left[\frac{J_n}{q^2+\theta_n^2}+\frac{J_{n+1}}{q^2+\theta_{n+1}^2}\right]^2\delta(\varepsilon_g-E_B+\varepsilon_{k_1}+\varepsilon_{k_2}-\varepsilon_{k_2+q}) ,$$

$$\varepsilon_{k_1}=\frac{\hbar^2}{2m_c}\left[k_1^2+\left[\frac{\pi}{L}\right]^2\right] , \tag{31}$$

$$\varepsilon_{k_2+q}=\frac{\hbar^2}{2m_c}(\mathbf{k}_2+\mathbf{q})^2+E_n \tag{32}$$

$$\approx\frac{\hbar^2}{2m_c}(q^2+\theta_n^2) . \tag{33}$$

There should also be a factor $1-n_c$ to ensure that the final state $\mathbf{k}_2+\mathbf{q}$ is unoccupied. However, this state is always empty, since its kinetic energy is approximately the energy gap ε_g.

The numerical value of this expression is estimated by making some drastic assumptions. The wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ are assumed to be quite small compared with $\mathbf{q}$. The integrals over d^2k_1 and d^2k_2 just give the density of particles. When n is large, the difference between n and $n+1$ can be ignored. These approximations bring us to the expression

$$w_A=\frac{n_e^2N_A}{s_{ee}} , \tag{34}$$

$$\frac{1}{s_{ee}}=\frac{16\pi^2e^4\hbar}{\epsilon_0^2m_cL^2\varepsilon_g^2}N^2(\lambda,z_i)\sum_{n=0}^{\sqrt{\varepsilon_g/4E_0}}J_n^2 . \tag{35}$$

This expression would have to be evaluated numerically for an accurate answer. The important factor is J_n, which we estimate to be

$$J_n \approx \frac{8\pi\lambda^3}{[1+\lambda^2(q^2+\theta_n^2)]^2} \tag{36}$$

$$\approx \frac{8\pi}{\lambda}\left[\frac{\hbar^2}{2m_c\varepsilon_g}\right]^2 , \tag{37}$$

$$\frac{1}{s_{ee}} = \frac{1024\pi^3 e^4 \langle N^2\rangle}{\hbar\epsilon_0^2\varepsilon_g L\lambda^2}\left[\frac{\hbar^2}{2m_c\varepsilon_g}\right]^{9/2} . \tag{38}$$

This estimate causes the quantity J_n to be independent of n. Then the summation is given by J_n^2 time the number of terms. The final result for the Auger rate s_{ee}in units of s/cm^5 is given above. We assume that a similar rate of Auger decay applies to the process where a free hole is excited to high kinetic energy. The total rate of Auger decay, for exciting either a free electron or a free hole, is

$$w_A = \frac{n_e N_A}{s_{ee}}(n_e+n_h) . \tag{39}$$

Another process is the excitation of a bound hole to high kinetic energy. The evaluation of this term requires a knowledge of the correlation between positions of the acceptors. Two bound holes are involved: one is recombined with an electron, and the other is excited. They initially must start on different acceptor sites. So this term can only be evaluated with a knowledge of these correlations. Since the acceptors are dilute, and far apart on the average, we assume the process involving two bound holes can be neglected.

One way to understand this expression is to compare it with the radiative recombination to bound holes. The ratio of these two processes, from (18), (38), and (39) is

$$\frac{w_A}{w_B} = \alpha_f \frac{384\pi^3(m_e c^2)^2\hbar\lambda}{\varepsilon_g p_{cv}^2\omega' L\epsilon_0^2}\left[\frac{\lambda^3}{J}\right]^2 \times\left[\frac{\hbar^2}{2m_c\varepsilon_g\lambda^2}\right]^{9/2}(n_e+n_h) . \tag{40}$$

Using numbers for GaAs, with $\varepsilon_G=1.5$ eV, $E_D=0.01$ eV, $E_A=0.03$ eV, $n_e\hbar^2\pi/m_c\approx 0.03$ eV, and $J\approx 4\pi\lambda^3$, the ratio w_A/w_B is about $O(1)$. Thus the two decay rates are comparable when the quasi-Fermi-level is 20–30 meV. The Auger decay is negligible for quasi-Fermi-levels less than this.

F. Hole capture by ionized acceptors

The main radiative decay mode is the recombination of electrons with holes bound to acceptors. This process makes the acceptors become ionized. Free holes from the valence band will become captured in acceptor states at these ionized acceptors. The energy lost by the holes, in this process, is transferred either to phonons or to other particles. The latter is another kind of Auger process. The excess energy in the Auger process is dissipated by exciting either a free electron or a free hole. It can be estimated using (35) with $n=0$ and ε_g replaced by E_A. The result is reduced by a factor of 4 if we neglect the J_{n+1} term (with $n=0$) in (28),

$$w_{hA} = \frac{n_h(n_e+n_h)N_A'}{s_{hA}} , \tag{41}$$

$$\frac{1}{s_{hA}} = \frac{4\pi^2 e^4\hbar\langle N^2J^2\rangle}{m_v\epsilon_0^2 E_A^2 L^2} . \tag{42}$$

The symbol N_A' denotes the density of ionized acceptors. This Auger process becomes important as the density n_h of free holes increases. The other process for capturing holes at ionized acceptors has the excess hole energy dissipated through the emission of one or several phonons. We estimate this phonon process to be smaller, although this estimate can vary considerably from solid to solid. The important factor is the value of the phonon density of states at the energy of the acceptor binding energy. The phonon process is important only if this density of states is large.

III. RATE EQUATIONS

The interband absorption creates free electrons and holes. The electrons can emit photons by recombining with holes which are either free or bound to acceptors. Here we write the rate equations for the change in concentration of free electrons assuming that only these two recombination processes are important. We initially neglect the Auger process. We also assume, for this initial discussion, that all of the acceptors are neutral:

$$\frac{dn_e}{dt} = w_I I - \frac{1}{\tau_x}\frac{n_e n_h}{n_e+n_0} - \frac{n_e N_A}{s_{eA}} . \tag{43}$$

The second term on the right is the recombination of free electrons and holes. We have provided a rough interpolation between the Maxwell-Boltzmann result (8) and the Fermi-Dirac result (12) by writing the rate as proportional to $n_e n_h/(n_e+n_0)$. We assume a steady state, so that the time derivative is zero. This conforms to the typical experimental arrangement, where the excitation is by a cw laser. We assume that $n_e=n_h$, and the solution to the above equation for the density of electrons in the conduction band is

$$n_e = \frac{n_0}{2(1+\xi)}\{I/I_0 - \xi + [(\xi+I/I_0)^2 + 4I/I_0]^{1/2}\} , \tag{44}$$

$$\xi = \frac{\tau_x N_A}{s_{eA}} \approx 2N_A\langle(NJ)^2\rangle , \tag{45}$$

$$I_0 = \frac{n_0}{\tau_x w_I} = \frac{\omega^2\omega' k_B T}{3\pi^2 c^2}\left[\frac{m_c}{b\mu}\right] , \tag{46}$$

where b is defined in (11). There are two key parameters in this expression. The factor I_0 has units of intensity. For GaAs at $T=4$ K we find $I_0\approx 20$ W/cm^2 assuming that the effective-mass factor is $b=\frac{1}{2}$. The second important parameter ξ depends upon the density of acceptors. For acceptors in GaAS we estimate that $2\langle(NJ)^2\rangle\approx 100\lambda^3\approx 10^{-16}$ cm^3. Therefore $\xi\approx 0.01$ when the acceptor density has a low value such as $N_A=10^{14}$ cm^{-3}. In this case the solution has three kinds of behavior.

(i) $1 > \xi^2 > 4I/I_0$: Here the solution of (44) is given approximately by $n_e = n_0 I/I_0 \xi$. This solution applies at very low light intensities where the densities of electrons and free holes are very small. Most of the holes are bound to acceptors, so electrons emit photons by recombining with bound holes. The density of free electrons is given by equating $w_I I = w_B$.

(ii) $1 > 4I/I_0 > \xi^2$: Here the solution to (44) is given approximately by $n_e = n_0 \sqrt{I/I_0}$. Now the electrons recombine mostly with the free holes according to Maxwell-Boltzmann statistics. The results are obtained by setting $w_I I = w_x$ in (8) with $n_h = n_e$.

(iii) $4I/I_0 > 1$: Here the solution to (44) is given as approximately $n_e = n_0 I/I_0 = \tau_x w_I I$. The electrons recombine with the free holes, but one uses the Fermi-Dirac expression (12) for the transition rate.

Figure 1 shows a graph of this behavior. We have plotted $\ln(n_e/n_0)$ versus $\ln(I/I_0)$ for the cases that $\xi = 0.1, 1.0, 10.0$. The first case corresponds to the present discussion, and is the top curve in the figure. At the lowest values of I/I_0, the curve has a slope of 1. At intermediate values, the curve flattens when $n_e \simeq \sqrt{I}$. At the largest intensity values, the curve is linear again.

The other two cases in the figure have large ξ values, which means that there is a larger concentration of acceptors N_A. These cases lack a region where $n_e \simeq \sqrt{I}$. That is, there is no region where electrons recombine with free holes according to Maxwell-Boltzmann statistics. For $\xi > 1$, electrons recombine with bound holes at low intensity, and with free holes using Fermi-Dirac statistics at higher intensities.

In order to have the quasi-Fermi-level of electrons be 20–30 meV, the density of free electrons in the conduction band has to be $n_e \simeq 10^{12}$ cm^{-2}. At low temperatures, the constant n_0 is about 100 times smaller than this, and $n_e < n_0$ for $I < I_0$. Since experimental intensities are typically 1 W/cm$^2 < I_0$, we conclude that the quasi-Fermi-level of electrons is of the order of 1 meV or less. In that case, the Auger process for electron-hole recombination is negligible.

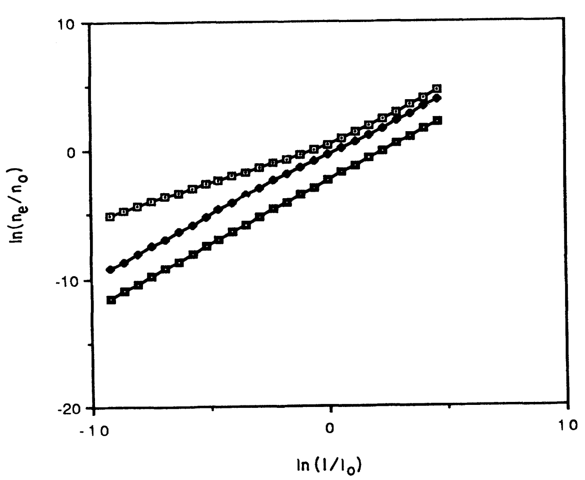

FIG. 1. Log-log plot of the density of conduction-band electrons as a function of the laser power. The density of conduction electrons is normalized to n_0 defined in (10). The laser power is normalized to I_0 defined in (46). The three curves have $\xi = 0.1, 1.0, 10.0$, for the three lines, with the smallest value for the top line. Equation (44) is used to graph these lines.

Next we consider the process whereby the holes bound to acceptors equilibrate with the free holes. We no longer assume that $n_h = n_e$, but try to determine the separate values of these two densities. Let f be the fraction of acceptors which do not have holes in bound states: $n_{hB} = (1-f)LN_A$ and $N_A' = fN_A$. The number of bound holes n_{hB} is determined by the equation for conservation of charge,

$$n_h - n_e + n_{hB} = LN_A \ , \tag{47}$$

$$n_h = n_e + fLN_A \ . \tag{48}$$

For undoped semiconductors, the product $LN_A \simeq 10^8$ cm^{-2} is very small. This is the maximum density of bound holes. The number of unbound holes is larger than this under moderate laser excitation. Since $0 < f < 1$, we have that $n_h \approx n_e$ whenever these densities are much larger than LN_A. For doped semiconductors, then the product LN_A is much larger.

The rate equations for electrons and holes is presented while neglecting the Auger decay of the electrons:

$$\frac{dn_e}{dt} = w_I I - \frac{1}{\tau_x}\frac{n_e n_h}{n_e + n_0} - \frac{n_e N_A (1-f)}{s_{eA}} \ , \tag{49}$$

$$\frac{dn_h}{dt} = w_I I - \frac{1}{\tau_x}\frac{n_e n_h}{n_e + n_0} - \frac{n_h (n_e + n_h) f N_A}{s_{hA}} - \frac{n_h f N_A}{s_p} \ , \tag{50}$$

$$\frac{dn_{Bh}}{dt} = -\frac{n_e N_A (1-f)}{s_{eA}} + \frac{n_h (n_e + n_h) f N_A}{s_{hA}} + \frac{n_h f N_A}{s_p} \ . \tag{51}$$

Equation (49) is similar to (43). They differ in the factor of $1-f$ in the last term, which allows for the fact that some of the acceptors may be ionized. The rate constant s_{hA} is for the Auger capture of free holes at ionized acceptors, while s_p is for the capture by giving the excess energy to phonons. In a steady state, we set all three of these equations to zero. They are not independent, since subtracting the first two equations yields the third. The three unknown variables (n_e, n_h, f) are determined by solving any of the above two equations with the constraint that $n_h = n_e + fLN_A$. In general, these equations are very nonlinear. We solve (51) and find

$$n_e = \frac{n_0}{4fH}\left(1 - f(1 + P + 3fAH) \pm \{[1 - f(1 + P + 3fAH)]^2 - 8AHf^3(P + fAH)\}^{1/2}\right) \ , \tag{52}$$

$$A = \frac{LN_A}{n_0} \ , \tag{53}$$

$$H=\frac{s_{eA}n_0}{s_{hA}}\ ,\qquad (54)$$

$$P=\frac{s_{eA}}{s_p}\ .\qquad (55)$$

For GaAs semiconductors, with quantum wells of width $L \simeq 100-200$ Å, we find that $H \simeq O(10^4)$. This large number dominates the answer. The other constants are generally less than unity. The above equation only has a reasonable solution when $f \simeq 1/H \simeq (10^{-4})$. We conclude that only a small fraction of acceptors are ionized. Any acceptors that are ionized by recombining with conduction electrons are quickly neutralized by the capture of free holes. In that case, the solution we provided in (44) should be accurate.

IV. DISCUSSION

We present a dynamical model for the photoluminescence of quantum wells under continuous laser excitation. We have calculated the values of the quasi-Fermi-levels of electrons and holes due to their steady-state, but nonequilibrium, excitation. Various relaxation processes have been considered to occur after the initial interband absorption which makes electron-hole pairs: electron radiative recombination with free holes, electron radiative recombination with holes bound at neutral acceptors, nonradiative recombination of electrons with holes bound to acceptors, and the trapping of free holes at ionized acceptors.

The results depend upon the density of acceptors. For a large concentration of acceptors, the steady-state concentration of free electrons is proportional to the intensity of the laser excitation. This linearity is valid as for two main processes: the conduction electrons recombine with either free holes or with holes bound to acceptors.

For a very small density of acceptors, on the order $N_A \simeq 10^{14}$ cm^{-3}, the variation of electron density has a more complex dependence upon the intensity of the laser. The dependence is linear at high and low intensities, but there is an intermediate regime where the dependence is proportional to $\sqrt{I}$.

We find that the densities of conduction electrons are small for laser intensities on the order of $I=1$ W/cm^2. The quasi-Fermi-level is on the order of 1 meV. At low temperatures, this low Fermi energy still requires Fermi-Dirac statistics.

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