Analytical modeling for the threshold service differentiation mechanism in asynchronous optical buffers

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With the emergence of different kinds of applications, service differentiation has become an important issue to be considered in current and future networks. We propose an exact analytical model for evaluating the performance of feed-forward delay-line buffers when the threshold mechanism is used for service differentiation in asynchronous optical packet-switched networks. The analytical model was derived assuming Poisson arrivals and any packet length distribution. Parameters such as buffer size, load, and others do not affect the accuracy of the model. In addition, it can use an arbitrary number of service-differentiated classes and traffic partitioning within them. To check the exactness of the model, we compared the buffer modeling with simulation results when exponential and uniform distributions are considered for the packet length. The analysis presented here shows that by using the threshold mechanism, it is possible to effectively differentiate the per class packet blocking probability. © 2007 Optical Society of America

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1. Introduction

The Internet has operated on the best-effort concept, which means that traffic is randomly discarded whenever necessary, without any attempt to do any kind of intelligent filtering as, for example, the use of fields in the packet header to make decisions whether to delay, discard, or forward the packet. This implies that all applications are treated in the same way and delivered only if the network has enough resources to support them. Today, however, as a consequence of the emergence of a variety of applications with different quality requirements, new concepts able to provide service differentiation are becoming a very important issue on the Internet.

Integrated service (IntServ) and differentiated service (DiffServ) are two proposals for providing quality of service (QoS) in IP networks. IntServ is a way to offer a deterministic, per flow QoS. However, such an approach is not scalable, as it introduces significant network overhead due to the necessity of reserving, for each flow, transmission capacity in every router along the path [1,2]. Conversely, DiffServ does not rely on reserving bandwidth for each flow, but establishes per hop behavior (PHB) of relatively differentiated QoS levels using the differentiated code point (DHCP) bits within the IP header. Therefore, different treatment is given for IP packets depending on the class they belong to. Although an absolute QoS cannot be guaranteed, such an approach has been shown to be a very plausible and suitable way to support different traffic requirements in large networks [1,2]. For this reason, DiffServ is seen as a viable mechanism for future optical packet-switched networks.

Filtering procedures for service differentiation usually utilize queues coming into the electrical switches, which are serviced depending on each priority class. However, electrical switches are not to be used in all-optical networks, and buffering in optics cannot be implemented in the same way as with electronic memories. Until now, the most convenient optical functionality that resembles queueing (buffering) results from the use of delay-line arrays that provide, for each packet, a delay from a discrete, normally small, set of delays. Such optical delay-line banks are called optical delay line buffers or, for short, optical buffers [3–6].

Optical buffer architectures are classified as feed-forward, feedback, and hybrid [4]. In feed-forward architectures, the decision of transmitting or blocking a packet is made at once, since packets will have just one opportunity of being either directly transmitted to the output, transmitted to one of the delay lines, or blocked if there is not an appropriate delay. In feedback architectures, also known as recirculating buffers, the delay lines connect an output port of a switching element to an input port. Therefore, packets are directly transmitted to the output or sent to a delay line to have another opportunity to be routed to the appropriate output; the number of recirculations is limited by loss and accumulation of optical noise. Hybrid architectures would be the combination of the previous two. Since almost all the loss that a signal experiences in a switching node is related to passing through the switch, the feedforward architecture has the advantage of attenuating all signals almost equally [7]. In this paper we will focus on the feed-forward architecture under the FIFO discipline, so that the decision of transmitting or blocking a packet is made at once, and the order of arrival of incoming packets is preserved at the output, as will be explained later.

In general, packet-based optical networks have been studied in two categories: synchronous networks with fixed-size packets and asynchronous with fixed- or variablesize packets [7,8]. Although synchronous networks may lead to higher performance, it is more difficult to implement with extremely high data rates because it implies that packets coming from different links have to be aligned to the node time reference [5,8], and this is not a simple task in the optical domain [7]. In addition, it is more natural and convenient for variable-size packets to match varying IP packet sizes [9,10]. For these reasons, it is worth considering asynchronous operation with variable-sized packets.

In the literature, some works have focused on the problem of building optical packet switches able to manage QoS. Both the time and wavelength domains have been explored to provide different levels of QoS [10–13], whereas both domains may still be combined. In this paper we will focus on the time domain problem, for which new methods must be applied, as delay-line buffers operate completely differently from traditional electronic buffers and usually do not allow a new coming packet to overcome other packets already queued, a function used in conventional queuing operation for service differentiation [11]. In this case, mechanisms based on a priori access control of packets to the optical buffer are necessary. Here we assume a threshold-based technique, which was proposed for providing service differentiation in feed-forward delay-line buffers. Such a technique restricts the buffer capacity differently for each class of service. References [10,11] have shown, by means of simulations, the effectiveness of the threshold-based technique in providing packet discrimination in delay-line buffers. In addition, the small amount of additional processing makes the threshold mechanism a very attractive option for service differentiation in optical packetswitched networks.

The main proposal of this paper is to develop an exact analytical modeling for the delay-line buffer when the threshold mechanism is used for service differentiation in asynchronous optical networks. In the past years, some important studies have been concerned with modeling delay-line buffers in asynchronous optical packet-switched networks [14–18]. In [16,17], exact analytical models for the feed-forward delay-line buffer under the FIFO discipline have been proposed. The authors in [18] extended the analytical model proposed in [16] and provided an accurate analytical model for shared delay-line buffers. In this paper, we extend the model proposed in [16] for considering the threshold mechanism in delay-line buffers.

The paper is organized as follows. Section 2 describes the basic considerations used in our analysis. In Section 3 we describe the threshold mechanism. Section 4 derives a finite-state Markov model for evaluating the per DiffServ class packet blocking probability and average delay, whereas Section 5 discusses some numerical results and the accuracy of the proposed analytical modeling. Finally, Section 6 concludes the paper.

2. Basic Considerations

In this section, we briefly describe the general working mode of feed-forward delayline buffers and the basic considerations used in this paper. In this study, we focus on the time domain and therefore only one wavelength plane is considered in our analysis.

The output fiber itself and a set of B delay lines will form the buffer, as shown in Fig. 1. D_i , $i=1,2,\ldots,B$, denotes the length of the *i*th delay line in units of time. Upon arrival of a new packet, a switching apparatus will switch it to one of the delay lines or to the output fiber itself, which may be considered a delay line with zero delay $(D_0=0)$. Therefore, B+1 is the number of the possible packet access points to the buffer, as shown in the time axis presented in the bottom part of Fig. 1. The duration of an incoming packet must be known at switching time, so that any succeeding packet, even when it arrives while other packets are still being transferred to their delay lines, may be immediately switched to a contention-avoiding delay line or blocked. The choice of the delay line to be accessed should be such that at the output of the chosen delay line, the packet accesses the output fiber without contention. We will assume a FIFO discipline: incoming packets are always placed in the buffer after the last one in output time, with the shortest possible delay. Therefore, a new arriving packet will be switched to the delay line with the minimum delay that exceeds the time (γ) needed for the previously accepted packet to fully leave the buffer. For example, as shown in Fig. 1, let τ be the packet length (in units of time) of the last accepted packet and t^* be equal to the time between the new arriving and the last accepted packets. At the new packet arrival instant, $\gamma = D_i + \tau - t^*$ and the packet will be switched to delay line j if $D_{j-1} < \gamma \leq D_j$. In practical terms, γ represents the time that the last bit of the last inserted packet will leave the buffer (i.e., the tail of the packet in the buffer). Without service differentiation, blocking occurs if and while the leftmost (last) traveling packet overlaps the leftmost buffer access point ($\gamma > D_B$), which is referred as "buffer fullness." When one considers service differentiation, some limitation on the buffer access will be applied, as will be discussed in Section 3.

For the analysis presented here, we assume that packets arrive to the buffer according to a Poisson process with total arrival rate λ . In addition, we assume that the traffic is composed by DS independent classes, each with an arrival rate of $\alpha_{ds}\lambda$, where α_{ds} ($ds \in [1, \ldots, DS]$) represents the fraction of each DiffServ class. Finally, the packet length distribution in this paper may be generic and will be represented by $p_{\tau}(\tau)$. The applicability of the buffer analytical modeling to generic packet length distributions is of great interest as the buffer performance is extremely dependent on them [15,16]. Regarding the packet arrivals, modeling the traffic is not straightforward as the packet buffering leads to considerable traffic shaping. In this analysis, we have assumed Poissonian arrivals in order to evaluate the effectiveness of the threshold mechanism in delay-line buffers.



Fig. 1. Delay-line buffer under the FIFO discipline.

3. Threshold Mechanism

The threshold mechanism [10,11] defines a delay-line threshold for each DiffServ class $ds = 1, 2, \ldots, DS$, which will be represented by $Th^{\langle ds \rangle}$. On the arrival of a packet, if its class delay-line threshold is smaller than the current buffer access point (the delay line to which the packet will be inserted by the FIFO discipline described above), the packet will be blocked; otherwise it will be accepted. In other words, packets belonging to a DiffServ class ds may be switched only to delay lines $0, 1, \ldots, Th^{\langle ds \rangle}$. This generates a different viewing of the buffer capacity for each DiffServ class, which will provide the intended service differentiation. Notice that, in terms of complexity, the only additional task is the comparison between the packet class threshold and the current buffer access point.

4. Analytical Modeling

In this section we present a Markov analytical model to evaluate the packet blocking probability and average delay of asynchronous delay-line buffers with the threshold mechanism capabilities for service differentiation. We extend the model proposed in [16], which is an exact finite-state queueing model for delay-line buffers under class-less traffic. The model proposed here for the threshold mechanism is also exact and may be used for an arbitrary number of DiffServ classes, any packet length distribution, input load, and delay-line spacings.

From the discussion presented before, notice that the threshold mechanism generates DS+1 distinct regions (indexed here by $0,1,\ldots,DS$) of access points that are restricted to some DiffServ classes and are divided by the chosen class thresholds. In our analysis, without loss of generality, we assume that $Th^{\langle 1 \rangle} > Th^{\langle 2 \rangle} > \cdots > Th^{\langle DS \rangle}$, i.e., the first class has access to more delay lines than the second class, and so on. The region indexes are defined so that delay lines within region $0 < k \leq DS$ will be restricted for packets from DiffServ classes $ds=1,2,\ldots,k$. For example, delay lines within region 1 will accept packets only from DiffServ class 1, those within region 2 will accept packets from DiffServ classes 1 and 2, etc. Figure 2 illustrates such regions when one considers DS=3 DiffServ classes, with $Th^{\langle 1 \rangle}=B=7$, $Th^{\langle 2 \rangle}=5$, and $Th^{\langle 3 \rangle}=2$. The values $0,1,\ldots,B$ in the figure represent the indexes of the buffer delay lines, while $D^{\langle k \rangle}$, $k=1,2,\ldots,DS$ is the limit delay for each region k (which corresponds to the limit delay for DiffServ class ds=k). We assume $D^{\langle 0 \rangle}=\infty$ and $D^{\langle DS+1 \rangle}=-\infty$.

From the definitions above and the explanations presented in Section 2, notice that packets belonging to a DiffServ class ds will be blocked while $\gamma > D^{\langle ds \rangle}$, or, in terms of region, while γ is inside (belongs to) region k < ds. We assume that packets from DiffServ classes that can be accepted while γ belongs to region k are said to be priority to such a region. Otherwise, they are said to be nonpriority. In this paper, we will use the term *belongs to* and the symbol \in interchangeably.

For the analytical modeling, we will focus on the status of the system when a new packet successfully enters the buffer. Notice that the arrival rate of packets that can be accepted by the buffer depends on the region that γ belongs to, which represents the priority packets' arrival rate to each region. For the analytical modeling, it is necessary to quantify the position that γ will be on the arrival of a packet, and also the



Fig. 2. States and regions for the threshold mechanism modeling with DS=3 DiffServ classes. B=7; $Th^{(1)}=B=7$, $Th^{(2)}=5$ and $Th^{(3)}=2$.

changes of the priority packets' arrival rate each time that γ crosses a region limit. Therefore, we define the following set of states for representing the packet insertion in a delay line.

• $i=0,1,\ldots,B-1$, where state *i* is reached by the buffer when a packet is switched to delay line *i* and its tail (which is situated at $D_i + \tau$) does not exceed the region limit. This happens if $\tau \leq D^{\langle r(i) \rangle} - D_i$, where $D^{\langle r(i) \rangle}$ is the delay boundary of region r(i) and r(i) denotes the region to which state *i* belongs. We say that state *i* belongs to region r(i) = k if $D^{\langle k+1 \rangle} \leq D_i < D^{\langle k \rangle}$. For example, in Fig. 2, r(0)=r(1)=3, $D^{\langle 3 \rangle}=D_2$; r(2)=r(3)=r(4)=2, $D^{\langle 2 \rangle}=D_5$; and r(5)=r(6)=1, $D^{\langle 1 \rangle}=D_7$.

• $f_i^{\langle k \rangle}$, $k=1,2,\ldots,DS$; $i=0,1,\ldots,if(k \neq 1,Th^{\langle k \rangle}-1,B)$, where if(x,a,b) returns a if the condition x is true and b otherwise. State $f_i^{\langle k \rangle}$ is reached by the buffer when a packet is switched to delay line i and its tail reaches region k-1. This happens when the inserted packet length $D^{\langle k \rangle} - D_i < \tau \leq D^{\langle k-1 \rangle} - D_i$.

Consequently, depending on the value of γ immediately after the packet insertion on delay line i ($\gamma=D_i+\tau$), the buffer may be led to r(i)+1 distinct conditions: state i or $f_i^{(k)}$, $k=r(i), r(i)-1, \ldots, 1$. A sequence of events and its corresponding state transitions are shown in Fig. 3.

From our state definition, notice that if the packet size is upper bounded by some maximum size $\tau_{\max} < D^{\langle k \rangle} - D_i$, then states $f_i^{\langle k \rangle}, f_i^{\langle k-1 \rangle}, \ldots, f_i^{\langle 1 \rangle}$ will never be reached, i.e., they do not exist. Likewise, if the packet size is lower bounded by some minimum size $\tau_{\min} \geq D^{\langle r(i) \rangle} - D_i$, then state *i* will never be reached, except only if equality is achieved with positive probability. Therefore, on the maximum, only a total of $B + 1 + \Sigma_{ds=1}^{DS} Th_{ds}$ states are necessary to completely model the delay-line buffer under the threshold mechanism. In the equations described hereon, when a state does not exist, we simply set its probability equal to zero.

To conduct the analysis, we further define the following parameters.

• $\lambda^{\langle k \rangle}$, k = 1, 2, ..., DS, represents the arrival rate of packets that may be accepted while γ belongs to region k (i.e., the arrival rate of priority packets to region k). Therefore

$$\lambda^{\langle k \rangle} = \sum_{ds=1}^{k} \alpha_{ds} \lambda.$$
 (1)

• $p_{T^{*(k)}}(t)$ denotes the interarrival time distribution of priority packets to region k, given by



Fig. 3. Sequence of events and its corresponding state transitions.

$$p_{T^{*\langle k \rangle}}(t) = \lambda^{\langle k \rangle} e^{-\lambda^{\langle k \rangle} t} u(t), \qquad (2)$$

where u() is the unit step function.

• τ_i denotes the random variable for the packet length when the state of the buffer is *i*, i.e., when the packet is switched to delay line *i* and $\tau \leq D^{\langle r(i) \rangle} - D_i$.

$$p_{\tau_{i}}(\tau) = \frac{p_{\tau}(\tau)}{\int_{0}^{D^{(r(i))} - D_{i}} p_{\tau}(\xi) \mathrm{d}\xi} [u(\tau) - u(\tau - (D^{\langle r(i) \rangle} - D_{i}))].$$
(3)

• $\tau_{f_i}^{\langle k \rangle}$ denotes the random variable for the packet length when the state of the buffer is $f_i^{\langle k \rangle}$, i.e., when the packet is switched to delay line i and $D^{\langle k \rangle} - D_i < \tau \leq D^{\langle k-1 \rangle} - D_i$.

$$p_{\tau_{l_{i}}^{(k)}}(\tau) = \frac{p_{\tau}(D^{\langle k \rangle} - D_{i} + \tau)}{\int_{D^{\langle k \rangle} - D_{i}}^{D^{\langle k-1 \rangle} - D_{i}} p_{\tau}(\xi) \mathrm{d}\xi} [u(\tau) - u(\tau - (D^{\langle k-1 \rangle} - D^{\langle k \rangle}))].$$
(4)

• h_i denotes the probability that no priority packet arrives when the buffer state is i and while γ belongs to region r(i) [$\gamma \in r(i)$]. For Poisson processes with rate λ , the probability that no arrivals occur in a time interval ξ is given by $e^{-\lambda\xi}$ [19]. Therefore, since the time that $\gamma \in r(i)$ is given by $\tau_i + (D_i - D^{\langle r(i)+1 \rangle})$:

$$h_{i} = \left(\int_{0}^{D^{\langle r(i) \rangle} - D_{i}} e^{-\lambda^{\langle r(i) \rangle} \xi} p_{\tau_{i}}(\xi) \mathrm{d} \xi \right) e^{-\lambda^{\langle r(i) \rangle} (D_{i} - D^{\langle r(i) + 1 \rangle})}.$$
(5)

• $h_{f_i}^{\langle k \rangle}$ is the probability that no priority packet arrives when the buffer state is $f_i^{\langle k \rangle}$ and while γ belongs to region k-1.

$$h_{f_{i}^{(k)}} = \int_{0}^{D^{(k-1)} - D^{(k)}} e^{-\lambda^{(k-1)}\xi} p_{\tau_{f_{i}}^{(k)}}(\xi) \mathrm{d}\xi.$$
(6)

• $g^{\langle k \rangle}$ is the probability that no priority packet arrives during the time that γ crosses all region k. Therefore

$$g^{\langle k \rangle} = e^{-\lambda^{\langle k \rangle} (D^{\langle k \rangle} - D^{\langle k+1 \rangle})}.$$
(7)

4.A. State Transition Probabilities

For the evaluation of the state transition probabilities, consider first that the optical buffer is in state *i*. By the definitions above, this means that $\gamma = D_i + \tau_i$ immediately after the packet insertion at delay line *i*. The probability that the next access point is at $\delta = D_j$, where *j* is any state that can be reached by the buffer while $\gamma \in r(i)$, is given by the probability that a priority packet to region r(i) arrives while $D_{j-1} < \gamma = D_i + \tau_i - t^{*(r(i))} \leq D_j$, where $t^{*(r(i))}$ is the interarrival time of priority packets to region r(i); the distribution of which was defined in Eq. (2). Let $\gamma = D_i + Z_i$. Therefore, $Z_i = \tau_i - t^{*(r(i))}$ represents the analog delay increment up from D_i , with the real increment resulting from an upper quantization of Z_i . Since τ_i and $t^{*(r(i))}$ are independent, the distribution of Z_i while $\gamma \in r(i)$ is given by the convolution of $p_{\tau_i}()$ and $p_{-T^{*(r(i))}}()$, i.e., $p_{Z_i}(z) = p_{\tau_i}() * p_{-T^{*(r(i))}}()$, for $D^{\langle r(i)+1 \rangle} - D_i < z \leq D^{\langle r(i) \rangle} - D_i$.

Now, consider the possibility that the next access point is at $\delta = D_j$, where j is any state that can be reached by the buffer while $\gamma \in r(i) + 1$. First, this means that no priority packet to region r(i) arrived while $\gamma \in r(i)$, which occurs with probability h(i). Due to the memoryless nature of the arrival process, we can assume that the arrival process restarts at $\gamma = D^{\langle r(i)+1 \rangle}$, with the packet arrival rate $\lambda^{\langle r(i)+1 \rangle}$. Therefore, $p_{Z_i}(z) = h_i p_T^{*(r(i)+1)}(-z + D^{\langle r(i)+2 \rangle} - D_i)$, for $D^{\langle r(i)+2 \rangle} - D_i < z \leq D^{\langle r(i)+2 \rangle} - D_i$. Similarly, $p_{Z_i}(z) = h_i g^{\langle r(i)+1 \rangle} p_T^{*(r(i)+2)}(-z + D^{\langle r(i)+2 \rangle} - D_i)$, for $D^{\langle r(i)+3 \rangle} - D_i < z \leq D^{\langle r(i)+2 \rangle} - D_i$, and so on. Equation (8) summarizes $p_{Z_i}(z)$.

$$p_{z_{i}}(z)$$

$$= \begin{cases} 0 \qquad z > D^{\langle r(i) \rangle} - D_{i} \\ p_{\tau_{i}}(\) * p_{-T^{*(r(i))}}(\) \qquad D^{\langle r(i)+1 \rangle} - D_{i} < z \le D^{\langle r(i) \rangle} - D_{i} \\ h_{i}p_{T^{*(r(i)+1)}}(-z + D^{\langle r(i)+1 \rangle} - D_{i}) \qquad D^{\langle r(i)+2 \rangle} - D_{i} < z \le D^{\langle r(i)+1 \rangle} - D_{i} \\ m^{-1} \\ h_{i}\prod_{k=1}^{m-1} g^{\langle r(i)+k \rangle} \qquad D^{\langle r(i)+m+1 \rangle} - D_{i} < z \le D^{\langle r(i)+m \rangle} - D_{i}; \ 1 < m \le DS - r(i) \\ p_{T^{*(r(i)+m)}}(-z + D^{\langle r(i)+m \rangle} - D_{i}) \end{cases}$$

$$\tag{8}$$

 $n(\alpha)$

Consequently, the probabilities that the buffer transits from state *i* either to states *j* or $f_i^{\langle k \rangle}$ are, respectively,

$$P_{i,j} = \left(\int_{D_{j-1}-D_i}^{D_j-D_i} p_{Z_i}(z) \mathrm{d}z \right) \left(\int_0^{D^{\langle r(j) \rangle} - D_j} p_\tau(\tau) \mathrm{d}\tau \right), \tag{9}$$

$$P_{i,f_{j}^{\langle k \rangle}} = \left(\int_{D_{j-1}-D_{i}}^{D_{j}-D_{i}} p_{Z_{i}}(z) \mathrm{d}z \right) \left(\int_{D^{\langle k-1 \rangle}-D_{j}}^{D^{\langle k-1 \rangle}-D_{j}} p_{\tau}(\tau) \mathrm{d}\tau \right).$$
(10)

Similarly, when the current buffer state is $f_i^{(k)}$, considering $Z_{f_i^{(k)}}$ the analog delay increment up from $D^{(k)}$:

$$p_{z_{f_{i}^{(k)}}(z)} = \begin{cases} 0 & z > D^{\langle k-1 \rangle} - D^{\langle k \rangle} \\ p_{\tau_{f_{i}^{(k)}}()} * p_{-T^{*\langle k-1 \rangle}()} & 0 < z \le D^{\langle k-1 \rangle} - D^{\langle k \rangle} \\ h_{f_{i}^{(k)}} p_{T^{*\langle k \rangle}(-z)} & D^{\langle k+1 \rangle} - D^{\langle k \rangle} < z \le 0 \\ \\ h_{f_{i}^{(k)}} \prod_{l=0}^{m-2} g^{\langle k+l \rangle} p_{T^{*\langle k+m-1 \rangle}} & D^{\langle k+m \rangle} - D^{\langle k \rangle} < z \le D^{\langle k+m-1 \rangle} - D^{\langle k \rangle}; \ 1 < m \le DS - k + 1. \\ (-z + D^{\langle k+m-1 \rangle} - D^{\langle k \rangle}) \end{cases}$$

$$(11)$$

Consequently, the state transition probabilities from state $f_i^{(k)}$ are given by

$$P_{f_{i}^{(k)},j} = \left(\int_{D_{j-1}-D_{i}}^{D_{j}-D_{i}} p_{Z_{f_{i}^{(k)}}}(z) \mathrm{d}z\right) \left(\int_{0}^{D^{(r(j))}-D_{j}} p_{\tau}(\tau) \mathrm{d}\tau\right),\tag{12}$$

$$P_{f_{i}^{(k)},f_{j}^{(l)}} = \left(\int_{D_{j-1}-D_{i}}^{D_{j}-D_{i}} p_{Z_{f_{i}^{(k)}}}(z) \mathrm{d}z\right) \left(\int_{D^{(l)}-D_{j}}^{D^{(l-1)}-D_{j}} p_{\tau}(\tau) \mathrm{d}\tau\right).$$
(13)

Equations (9), (10), (12), and (13) show that the transition probabilities are completely independent of the past history, therefore enabling the buffer under the threshold mechanism to be modeled as a Markovian, finite-state machine. Thus the steady-state probabilities P_i and $P_{f_i^{(k)}}$ of each state may be calculated exactly by numerically solving the balance equations [20].

4.B. Packet Blocking Probability and Average Delay Evaluation

For the packet blocking probability evaluation, since the model assumes state transitions only when a new packet successfully enters the buffer, we should calculate the average number of arriving and blocked packets before each state transition. Let the buffer be in state *i* or $f_i^{(k)}$. We define

• \overline{T}_i as the average time the buffer stays at state *i* given that a priority packet arrived during the time that γ is at region r(i). This can be obtained by making in Eq. (A2) $\eta = D_i - D^{\langle r(i)+1 \rangle} + \tau_i$, $\lambda_p = \lambda^{\langle r(i) \rangle}$ and integrating on the possible values to τ_i . Therefore

$$\overline{T_{i}} = \frac{1}{\lambda^{\langle r(i) \rangle}} - \int_{0}^{D^{\langle r(i) \rangle} - D_{i}} \frac{[D_{i} - D^{\langle r(i) + 1 \rangle} + \xi] e^{-\lambda^{\langle r(i) \rangle} (D_{i} - D^{\langle r(i) + 1 \rangle} + \xi)}}{1 - e^{-\lambda^{\langle r(i) \rangle} (D_{i} - D^{\langle r(i) + 1 \rangle} + \xi)}} p_{\tau_{i}}(\xi) \mathrm{d}\xi.$$
(14)

• $\overline{T}_{f_i^{(k)}}$ as the average time the buffer stays at state $f_i^{\langle k \rangle}$ given that a priority packet arrived during the time that γ is at region k-1. Make in Eq. (A2) $\eta = \tau_{f_i^{\langle k \rangle}}, \lambda_p = \lambda^{\langle k-1 \rangle}$ and integrate on the possible values to $\tau_{f_i^{\langle k \rangle}}$. This provides

$$\bar{T}_{f_{i}^{(k)}} = \frac{1}{\lambda^{\langle k-1 \rangle}} - \int_{0}^{D^{\langle k-1 \rangle} - D^{\langle k \rangle}} \frac{\xi e^{-\lambda^{\langle k-1 \rangle} \xi}}{1 - e^{-\lambda^{\langle k-1 \rangle} \xi}} p_{\tau_{f_{i}^{(k)}}}(\xi) \mathrm{d}\xi.$$
(15)

• $\overline{T}^{\langle k \rangle}$ is defined as the average time the buffer stays at region k given that a priority packet arrived during the time interval $D^{\langle k \rangle} - D^{\langle k+1 \rangle}$. Make in Eq. (A2) $\eta = D^{\langle k \rangle} - D^{\langle k+1 \rangle}$ and $\lambda_p = \lambda^{\langle k \rangle}$:

$$\bar{T}^{(k)} = \frac{1}{\lambda^{(k)}} - \frac{[D^{(k)} - D^{(k+1)}]e^{-\lambda^{(k)}(D^{(k)} - D^{(k+1)})}}{1 - e^{-\lambda^{(k)}(D^{(k)} - D^{(k+1)})}}.$$
(16)

• $\overline{\tau}_i$ is defined as the average packet length in units of time, when the buffer state is *i*. Therefore

$$\bar{\tau}_{i} = \int_{0}^{D^{(r(i)+1)} - D_{i}} \xi p_{\tau_{i}}(\xi) \mathrm{d}\xi.$$
(17)

• $\overline{\tau}_{f_{i}^{(k)}}$ is defined as the average packet length in units of time, when the buffer state is $f_{i}^{(k)}$:

$$\bar{\tau}_{f_i^{(k)}} = \int_0^{D^{(k-1)} - D^{(k)}} \xi p_{\tau_{f_i^{(k)}}}(\xi) \mathrm{d}\xi.$$
(18)

For calculating the packet blocking probability of each DiffServ class ds, we will separate the average number of blocked and arriving packets when the buffer state is *i* or $f_i^{\langle k \rangle}$. The packet blocking probability, therefore, will be obtained by the ratio of the expected number of blocked and arriving packets. The first may be calculated by multiplying the per class packet arrival rate $(\alpha_{ds}\lambda)$ by the time during which the buffer blocks packets of such class. For example, suppose that the current state of the buffer is *i*. If $ds \leq r(i)$, obviously no packet from DiffServ class ds will be blocked while the buffer is in state i. On the other hand, if ds > r(i), the time during which the buffer remains blocking ds packets may be obtained by the probability $(1-h_i)$ that one priority packet arrives while $\gamma \in r(i)$ times the average elapsed time (\overline{T}_i) for the arrival of a priority packet at r(i); plus the probability $(h_i)(1-g^{(r(i)+1)})$ that no priority packet arrives while $\gamma \in r(i)$ and one priority packet to region r(i)+1 arrives while $\gamma \in r(i)+1$ times the total average elapsed time $(\bar{\tau}_i + D_i - D^{\langle r(i)+1 \rangle} + \bar{T}^{\langle r(i)+1 \rangle})$ for the arrival of a priority packet at r(i)+1; and so on, until the probability $(h_i \prod_{k=r(i)+1}^{ds-1} g^{\langle k \rangle})$ that no priority packet arrives before $\gamma \in$ region ds (from that point onwards any arriving packet from DiffServ class ds will be accepted) times the total average elapsed time $(\overline{\tau}_i + D_i)$ $-D^{(r(i)+1)}$). The equation below summarizes the explanation above, where the sums (Σ) and products (Π) will be 0 and 1, respectively, if the inferior limit is higher than the upper limit.

E[number of blocked packets/i] = $\alpha_{ds}\lambda$

$$\times \begin{cases} 0 & ds \leq r(i) \\ (1-h_{i})\overline{T}_{i} + h_{i} \Biggl[\Biggl(\sum_{j=r(i)+1}^{ds-1} [\overline{\tau}_{i} + D_{i} - D^{\langle j \rangle} + \overline{T}^{\langle j \rangle}] (1-g^{\langle j \rangle}) \\ \times \prod_{k=r(i)+1}^{j-1} g^{\langle k \rangle} \Biggr) + [\overline{\tau}_{i} + D_{i} - D^{\langle ds \rangle}] \prod_{k=r(i)+1}^{ds-1} g^{\langle k \rangle} \Biggr] & ds > r(i). \end{cases}$$

$$\tag{19}$$

The expected number of arriving packets for each DiffServ class ds may be calculated by summing the expected number of blocked packets with the number of

accepted packets given that the buffer is in state *i* or $f_i^{\langle k \rangle}$. For example, assume again that the buffer is in state *i*. If $ds \leq r(i)$, the expected number of accepted packets is given by the probability $(1-h_i)$ that a priority packet arrives while $\gamma \in r(i)$ times the probability $(\alpha_{ds}\lambda/\lambda^{\langle r(i) \rangle})$ of such a packet belongs to DiffServ class ds; plus the probability $(h_i)(1-g^{\langle r(i)+1 \rangle})$ that no priority packet arrives while $\gamma \in r(i)$ and one priority packet to region r(i)+1 arrives while $\gamma \in r(i)+1$ times the probability $(\alpha_{ds}\lambda/\lambda^{\langle r(i)+1 \rangle})$ that such a packet belongs to DiffServ class ds; and so on until that $\gamma \in$ region DS. On the other hand, if ds > r(i), the expected number of accepted ds packets may be calculated by quantifying the probability $(h_i \Pi_{l=r(i)+1}^{ds-1}g^{\langle l \rangle})$ that no packet is accepted until that $\gamma \in$ region ds times the probability that a ds packet arrives from that point onwards, which is similar to the previous explanation. Therefore

E[number of accepted packets/i]

$$= \begin{cases} \frac{(1-h_i)\alpha_{ds}\lambda}{\lambda^{\langle r(i)\rangle}} + h_i \sum_{j=r(i)+1}^{DS} (1-g^{\langle j\rangle}) \frac{\alpha_{ds}\lambda}{\lambda^{\langle j\rangle}} \prod_{k=r(i)+1}^{j-1} g^{\langle k\rangle} & ds \leq r(i) \\ \frac{ds-1}{h_i \prod_{l=r(i)+1}} g^{\langle l\rangle} \sum_{j=ds}^{DS} (1-g^{\langle j\rangle}) \frac{\alpha_{ds}\lambda}{\lambda^{\langle j\rangle}} \prod_{k=ds}^{j-1} g^{\langle k\rangle} & ds > r(i) \end{cases},$$

$$(20)$$

When the buffer state is $f_i^{\langle k \rangle},$ a similar procedure can be done, so that

E[number of blocked packets/ $f_i^{(k)}$] = $\alpha_{ds}\lambda$

$$\times \begin{cases} 0 & ds \leq k-1 \\ (1-h_{f_{i}^{(k)}})\bar{T}_{f_{i}^{(k)}} + h_{f_{i}^{(k)}} \left[\left(\sum_{j=k}^{ds-1} \left[\bar{\tau}_{f_{i}^{(k)}} + D^{\langle k \rangle} - D^{\langle j \rangle} + \bar{T}^{\langle j \rangle} \right] (1-g^{\langle j \rangle}) \prod_{l=k}^{j-1} g^{\langle l \rangle} \right) & , \\ + \left[\bar{\tau}_{f_{i}^{(k)}} + D^{\langle k \rangle} - D^{\langle ds \rangle} \right] \prod_{l=k}^{ds-1} g^{\langle l \rangle} \end{bmatrix} & ds > k-1 \end{cases}$$

$$(21)$$

E[number of accepted packets/ $f_i^{\langle k \rangle}$]

$$= \begin{cases} \frac{(1-h_{f_{i}^{(k)}})\alpha_{ds}\lambda}{\lambda^{\langle k-1\rangle}} + h_{f_{i}^{\langle k\rangle}} \sum_{j=k}^{DS} (1-g^{\langle j\rangle}) \frac{\alpha_{ds}\lambda}{\lambda^{\langle j\rangle}} \prod_{l=k}^{j-1} g^{\langle l\rangle} & ds \leq k-1\\ \frac{ds-1}{h_{f_{i}^{\langle k\rangle}}} \prod_{m=k}^{DS} g^{\langle m\rangle} \sum_{j=ds}^{DS} (1-g^{\langle j\rangle}) \frac{\alpha_{ds}\lambda}{\lambda^{\langle j\rangle}} \prod_{l=ds}^{j-1} g^{\langle l\rangle} & ds > k-1 \end{cases}$$

$$(22)$$

Consequently, for each DiffServ class ds, the packet blocking probability will be given by

$$P_{B_{ds}} = \frac{\sum_{s \in S} E[\text{number of blocked packets/s}]P_s}{\sum_{s \in S} (E[\text{number of blocked packets/s}] + E[\text{number of accepted packets/s}])P_s},$$
(23)

where S is the set of all states for the buffer.

The total packet blocking probability, i.e., the blocking probability of any packet sent to the buffer, is given by

$$P_{B_{Th}} = \sum_{ds=1}^{DS} \alpha_{ds} PB_{ds}.$$
 (24)

For the average delay evaluation, notice that (a) we must consider only the packets accepted by the buffer; (b) a packet belonging to a DiffServ class ds may be accepted only while γ belongs to a region k > ds; (c) among the packets accepted in region k > ds, packets from DiffServ class ds will represent the fraction $\alpha_{ds}\lambda/\lambda^{\langle k \rangle}$; and (d) when

there is a transition either to state *i* or $f_i^{(k)}$, the packet will receive a delay D_i . Therefore, the average delay may be calculated by

$$\bar{D}_{ds} = \frac{\sum_{k=ds}^{DS} \frac{\alpha_{ds} \lambda}{\lambda^{\langle k \rangle}} \left[\sum_{i=Th^{\langle k+1 \rangle}+1}^{Th^{\langle k \rangle}} D_i \left(P_i + \sum_{l=1}^{r(i)} P_{f_i^{(l)}} \right) \right]}{\sum_{k=ds}^{DS} \frac{\alpha_{ds} \lambda}{\lambda^{\langle k \rangle}} \left[\sum_{i=Th^{\langle k+1 \rangle}+1}^{Th^{\langle k \rangle}} P_i + \sum_{l=1}^{r(i)} P_{f_i^{(l)}} \right]},$$
(25)

where, for equation simplification, we assume $Th^{(DS+1)}=-1$ and r(B)=1. Finally, the total average delay may be calculated by

$$\bar{D}_{Th} = \sum_{k=1}^{DS} \sum_{i=Th^{(k+1)}+1}^{Th^{(k)}} D_i \left(P_i + \sum_{l=1}^{r(i)} P_{f_i^{(l)}} \right).$$
(26)

5. Numerical Results and Model Validation

Figures 4(a) and 4(b) compare the total and per DiffServ class packet blocking probabilities and average delays when the threshold mechanism is used for service differentiation. PB_{ds} and PB_{Th} denote the individual DiffServ classes and the total packet blocking probabilities, respectively. Similarly, D_{ds} and D_{Th} represent the per DiffServ and total average delays. For comparison purposes, the classless scenario is included from [16] and labeled PB for the packet blocking probability and \overline{D} for the average delay. We assume an optical buffer with B=seven delay lines, uniform packet length distribution between 0 and $2\overline{\tau}$ with average packet length $\overline{\tau}$ =1.0, and total arrival rate λ =0.6, which provides an input load ρ =0.6. Moreover, we made the usual assumption that $D_i=iD$, $i=0,1,\ldots,B$, where D is the delay unit parameter, also called the buffer granularity [14], although the model can handle any combination of delays. Finally, we assumed DS=three DiffServ classes, with the following per DiffServ class traffic partitioning: α_1 =0.2, α_2 =0.3, α_3 =0.5 and delay-line thresholds: Th_1 =7, Th_2 =5, and Th_3 =3.

To validate the accuracy of the proposed analytical modeling, the buffer performance as predicted by the model is compared with simulation results. These were obtained by running an *ad hoc*, event driven simulator. Events are marked by Poisson arrivals of packets to the buffer from the assumed DiffServ classes. On the arrival of a packet, its DiffServ class threshold is compared to the buffer access point to decide, through the threshold mechanism, if the packet should be accepted or blocked. Blocked packets are simply discarded, whereas an accepted packet will be inserted into a delay line and its duration will be used to update γ for deciding the acceptance or blocking of ensuing arriving packets.

With respect to performance, it can be seen that the threshold mechanism is capable of providing an efficient service differentiation in terms of blocking probability, whereas the difference on the average delay is small. For example, in Fig. 4(a), for



Fig. 4. Threshold mechanism performance for uniform packet length distribution and three DiffServ classes. B=7; $\rho=0.6$; $\alpha_1=0.2$, $\alpha_2=0.3$, $\alpha_3=0.5$; $Th_1=7$, $Th_2=5$, $Th_3=3$.

a delay unit D=1 it can be observed that there is almost 1 order of magnitude difference in the packet blocking probability between DiffServ classes 2 and 3 and more than 1 order of magnitude between DiffServ classes 1 and 2. The average delay varies from approximately 1.3 for DiffServ class 3 to 2.0 for DiffServ class 1. Comparing the packet blocking probabilities for the classless (*PB*) and threshold (*PB_{Th}*) scenarios, a slight degradation can be observed in the latter. On the other hand, the per DiffServ class (*D_{ds}*) and total (*D_{Th}*) average delays are lower when compared with the delays in the classless scenario. These are expected due to the buffer capacity segregation produced by the threshold mechanism.

Throughout this paper, we assumed an average packet length $\bar{\tau}$ =1.0. Therefore, both the average delay and the delay unit are already normalized to the packet duration. Consequently, an average delay equal to one corresponds to about 0.4 μ s when considering typical applications with average packet sizes of 500 bytes and transmission rates of 10 Gbits/s. From Fig. 4(b), it can be observed that in this case the total average delay in the classless and threshold scenarios will be approximately 1.2 and 0.8 μ s for a single unit delay (D=1.0).

Figures 5(a) and 5(b) compare the total and per DiffServ class packet blocking probabilities and average delays for a larger optical buffer (B=31 delay lines), exponential packet length distribution, and offered load $\rho=0.6$. We assume DS=three DiffServ classes, but now $\alpha_1=0.1$, $\alpha_2=0.2$, $\alpha_3=0.7$ and the delay-line thresholds are $Th_1=31$, $Th_2=26$, and $Th_3=21$. Similar to the previous case, the calculations fit the simulations very well, and an effective service differentiation in terms of blocking probability as well as a small difference in the average delay is observed.

Following the methodology proposed in [13], the efficiency of a QoS differentiation mechanism, S, is defined as the relative decrease in throughput when introducing service differentiation. The throughput in the threshold and classless scenarios may be obtained as $1-PB_{Th}$ and 1-PB, respectively. Therefore, $S = (1-PB_{th})/(1-PB)$, which varies with the delay unit (D). In the examples shown in Figs 4(a) and 5(a), we observed S = 0.915 and 0.964, respectively, for the worst case, i.e., the lowest value of S. Such values provide an insight into the efficiency of the threshold service differentiation mechanism, which is in agreement with the service differentiation scheme based on access restriction on the wavelength domain analyzed in [13]. Perhaps the major disadvantage of any kind of approach that exploits the buffer time domain is that the optimal value of the delay unit D is different for each DiffServ class. The delay unit is optimal where the blocking probability is minimal. As can be observed, the optimum delay unit for the highest-priority traffic is larger than the optimum delay unit for the second priority class, and so on. Given that higher-priority traffic in some regions does not compete with lower-priority traffic, high-priority traffic "sees" a lower effective load offered to the buffer. Combined with the fact that the optimal delay unit increases as the offered load to the buffer is reduced [15] offers an explanation for the increase in the optimal delay unit with higher traffic priorities.

Regarding the operational characteristics of the threshold mechanism, it is important to observe that it is very simple to be implemented, although the performances of the classes are correlated and it is possible to attribute just a finite set of thresholds for service differentiation. However, these thresholds can be dynamically tuned to the



Fig. 5. Threshold mechanism performance for exponential packet length distribution and three DiffServ classes. B=31; $\rho=0.6$; $\alpha_1=0.1$, $\alpha_2=0.2$, $\alpha_3=0.7$; $Th_1=31$, $Th_2=26$, $Th_3=21$.

instantaneous traffic profiles to guarantee that traffic variations may be properly treated. The analytical modeling presented here can be used in future planning for the threshold mechanism in delay-line buffers.

6. Conclusions

In this paper we propose a Markov chain analytical model for FIFO delay-line buffers under the threshold mechanism and validate its accuracy through numerical simulations. The use of optical buffer thresholds has been proposed to offer differentiated quality of service in optical packet-switched networks. The model is valid for a variety of input parameters such as packet length distribution and input load (in regard to Poissonian arrivals); number of DiffServ classes, as well as class traffic partitioning as a percentage of total traffic; class thresholds within the buffer and uniformnonuniform delay-line spacings.

The results indicate that the threshold mechanism provides an effective service differentiation in terms of blocking probability while the average packet delay exhibits a small difference among the classes. The analysis also confirms, when comparing the threshold and classless scenarios, that the total packet blocking probability is slightly higher in the threshold scenario, whereas the per class and average delays are lower. The different blocking probabilities for each class of traffic can be optimized by appropriate choice of thresholds and delay unit values. It is clear that proper buffer design is necessary for performance optimization and this can be realized using the proposed analytical model. For low packet blocking probabilities, numerical simulation performance analysis is time consuming and the availability of accurate analytical models is clearly beneficial.

Appendix A: Average Elapsed Time Between Packet Arrivals

Let η be an interval of interest and assume that a priority packet arrived during such an interval. Assume that η is totally contained in a region of the buffer so that the arrival rate of priority packets does not change. The conditional elapsed time distribution for the arrival of a priority packet in η is given by

$$p_{T_p^*/\eta}(t^*) = \frac{p_{T_p^*}(t^*)[u(t^*) - u(t^* - \eta)]}{\int_0^{\eta} p_{T_p^*}(\xi) \mathrm{d}\xi} = \frac{\lambda_p e^{-\lambda_p t^*}[u(t^*) - u(t^* - \eta)]}{1 - e^{-\lambda_p \eta}},$$
(A1)

where $p_{T_p^*}()$ and λ_p represent, respectively, the interarrival time distribution and arrival rate of priority packets.

Therefore, the average elapsed time, \overline{T}_e , for the arrival of a priority packet in η may be written as

$$\bar{T}_{e} = \int_{0}^{\eta} \underline{\xi} p_{T_{p}^{*}/\eta}(\underline{\xi}) d\underline{\xi} = \frac{1}{\lambda_{p}} - \frac{\eta e^{-\lambda_{p}\eta}}{1 - e^{-\lambda_{p}\eta}}.$$
 (A2)

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