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SHORT COMMUNICATION

A note on the robust control of Markov jump linear uncertain systems

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SUMMARY

This note addresses a robust control problem of continuous-time jump linear Markovian systems subject to norm-bounded parametric uncertainties. The problem is expressed in terms of a H_∞ control problem as in the purely deterministic case. The present formulation is simpler and it contains previous results in the literature as particular cases. Robust state feedback controllers are parameterized by means of a set of linear matrix inequalities. The result is illustrated by solving some examples numerically. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: Markov jump linear systems; parametric uncertainties; H_∞-control; linear matrix inequalities method; robust control

1. INTRODUCTION

Markov jump linear systems (MJLS) have been receiving much attention since the important papers such as Wonham [1] and Ji and Chizeck [2], among others. This class of processes has the interesting ability of modelling random parameter changes or failures in system with linear description, and the analysis can drawn from the large body of the deterministic linear theory. The study of stability, control and filtering presents many developments but they still lack behind the deterministic counterpart, mainly because of many inherent questions—MJLS are not mere extensions of deterministic linear systems.

The robustness analysis of MJLS dealing with parameter uncertainties of the Markov state, e.g. see Reference [3] or [4], has no counterpart in the deterministic analysis. However, the problem of parametric uncertainties in the system matrices is akin to the deterministic problems, and it appears previously in References [5–7]. In Reference [5] the authors took the effort to develop conditions in LMI form for the robustness problem of state feedback control, employing an auxiliary linear quadratic control problem defined for the nominal system. In

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Reference [6] the robustness of time-varying discrete-time MJLS is studied, and in Reference [7] an association is made between a H_∞ problem and the parameter uncertainty problem for a fault tolerant control problem.

The H_∞ problem seems to be the natural setting for the norm-bounded matrix parametric uncertainty problem of deterministic systems and likewise, of MJLS. The idea here is to parametrize all possible stabilizing solutions for the robust control problem employing the association with the H_∞ problem in a form that is simpler than any of the previous attempts, and that parallels the linear deterministic case. The resulting stability test and the synthesis of robust feedback control are given in LMI form.

The solution provided here generalizes the parametric uncertainty problem in Reference [7] to deal with system (5), not only with uncertainties in the matrix set A but also in the set B. A direct comparison of the approach here with that in Reference [5] is presented, and we show that the result here parametrizes all possible stabilizing solutions (in the stochastic sense) of the robust control problem, whereas the solution in Reference [5] refers to one specific feedback parametrization expressed as a particular case of the proposed solution.

A simple example is solved to expose the technique.

2. PARAMETRIC UNCERTAINTIES AND THE H$_\infty$-NORM

Let us consider the dynamic system with parametric uncertainties written in the form

$$\dot{\mathbf{x}}_t = (\mathbf{A}(\theta_t) + \mathbf{E}(\theta_t)\mathbf{\Delta}(\theta_t)\mathbf{C}(\theta_t))\mathbf{x}_t \tag{1}$$

where $\mathbf{x}_t \in R^n$ is the system continuous state and θ_t is the jumping state, defined by a continuous-time Markov chain in a probability space $(\Omega, \mathscr{F}, \mathscr{P})$, having the transition matrix $\mathbf{\Lambda} = \{\lambda_{ij}\}$, and taking values in the set $\{1, 2, \ldots, N\}$. The initial state $\mathbf{x}_0$ is known and θ_0 is a random variable with known probability distribution. The sets $\mathbf{A} = (\mathbf{A}_1, \ldots, \mathbf{A}_N)$, $\mathbf{E} = (\mathbf{E}_1, \ldots, \mathbf{E}_N)$, $\mathbf{\Delta} = (\mathbf{\Delta}_1, \ldots, \mathbf{\Delta}_N)$ and $\mathbf{C} = (\mathbf{C}_1, \ldots, \mathbf{C}_N)$ are of compatible dimensions and whenever $\theta_t = i$, for some $i = 1, \ldots, N$, the corresponding matrices $\mathbf{A}_i, \mathbf{E}_i, \mathbf{\Delta}_i$ and $\mathbf{C}_i$ describe the evolution of $t \to \mathbf{x}_t$, according with (1).

We assume that the sets $\mathbf{A}, \mathbf{E}$ and $\mathbf{C}$ are precisely known; however, the only information available on $\mathbf{\Delta}$ is that

$$||\mathbf{\Delta}_i||^2 = \lambda_{\max}(\mathbf{\Delta}_i\mathbf{\Delta}_i') \leqslant \gamma_i^{-2}, \quad i = 1, \ldots, N \tag{2}$$

where $\lambda_{\max}(\cdot)$ denotes the largest eigenvalue of $(\cdot)$. Since one can always re-scale the sets $\mathbf{E}$ and $\mathbf{\Delta}$, we assume without loss that $\gamma_i = \gamma > 0,\ i = 1, \ldots, N$. We wish to guarantee stochastic stability of (1) in an appropriate sense.

Definition 1 (Asymptotic mean square stability).

The system (1) is asymptotically mean square stable if for any $\mathbf{x}_0 \in R^n$ and any distribution of θ_0, one has that

$$\lim_{t\to\infty} E|\mathbf{x}_t|^2 = 0 \tag{3}$$

Here $E(\cdot)$ denotes the expectation with respect to the process $\mathbf{x}$. It has been shown for MJLS with finite Markov state that the concept in Definition 1 is equivalent to others second moment stability such as *stochastic stability* and *exponential mean square stability*, see Reference [8].

Let us recall the definition of the H_∞-norm for jump linear systems. This concept was presented in Reference [9] related to state feedback control; for simplicity we specialize it to open-loop systems in the form

$$\mathscr{S}:\begin{cases}\dot{\mathbf{x}}_t = \mathbf{A}(\theta_t)\mathbf{x}_t + \mathbf{E}(\theta_t)\mathbf{w}_t\\ \mathbf{z}_t = \mathbf{C}(\theta_t)\mathbf{x}_t\end{cases}$$

Definition 2 (H_∞-norm).

Consider $\mathscr{S}$, a stochastically stable system. The H_∞-norm $\|\mathscr{S}\|_\infty$ of $\mathscr{S}$ is the smallest γ such that

$$\|\mathbf{z}\|_2 < \gamma\|\mathbf{w}\|_2$$

for all $\mathbf{w}\in\mathbf{L}^2[0,+\infty)$ with $\|\mathbf{w}\|_2\neq 0$ and $\mathbf{x}_0 = 0$.

Regarding systems with uncertainties, the *Small Gain Theorem* gives sufficient conditions for the stability of the deterministic counterpart of system (1), cf. Reference [10]. The following lemma is a result in the same vein, providing stochastic stability conditions for (1), and a connection with the H_∞-norm of $\mathscr{S}$. For the proof see References [9,11].

Lemma 1.

Suppose that the coupled matrix inequalities

$$\mathbf{A}_i'\mathbf{P}_i + \mathbf{P}_i\mathbf{A}_i + \gamma^{-2}\mathbf{P}_i\mathbf{E}_i\mathbf{E}_i'\mathbf{P}_i + \mathbf{C}_i'\mathbf{C}_i + \sum_{j=1}^{N}\lambda_{ij}\mathbf{P}_j < 0 \tag{4}$$

have a positive definite solution $\mathbf{P}_i$ for each $i = 1,\ldots,N$. Then (1) with (2) and $\mathscr{S}$ are stochastically stable systems. In addition, $\|\mathscr{S}\|_\infty < \gamma$.

A connection between stochastic stability under parametric uncertainties and the H_∞-norm of jump systems can thus be made, by writing system (1) in the form

$$\tilde{\mathscr{S}}:\begin{cases}\dot{\mathbf{x}}_t = \mathbf{A}(\theta_t)\mathbf{x}_t + \mathbf{E}(\theta_t)\mathbf{w}_t\\ \mathbf{z}_t = \mathbf{C}(\theta_t)\mathbf{x}_t\\ \mathbf{w}_t = \mathbf{\Delta}(\theta_t)\mathbf{z}_t\end{cases}$$

Indeed, if (4) is satisfied, it is clear from the above that $\|\tilde{\mathscr{S}}\|_\infty < \gamma$.

3. STATE FEEDBACK DESIGN

Consider now the controlled system

$$\begin{cases} \dot{\mathbf{x}}_t = \mathbf{A}(\theta_t)\mathbf{x}_t + \mathbf{E}(\theta_t)\mathbf{w}_t + \mathbf{B}(\theta_t)\mathbf{u}_t \\ \mathbf{z}_t = \mathbf{C}(\theta_t)\mathbf{x}_t + \mathbf{D}(\theta_t)\mathbf{u}_t \end{cases} \tag{5}$$

with $\mathbf{B} = (\mathbf{B}_1, \ldots, \mathbf{B}_N)$ and $\mathbf{D} = (\mathbf{D}_1, \ldots, \mathbf{D}_N)$ of compatible dimensions.

Our goal is to determine the family of robust linear state feedback control laws $\mathbf{u} = \mathbf{K}_\theta \mathbf{x}$, in such way that the MJLS in (5) remains stochastically stable for each $\mathbf{w} = \mathbf{\Delta}_\theta \mathbf{z}$, whenever $\|\mathbf{\Delta}_i\| < \gamma^{-1}$, $i = 1, \ldots, N$. For this purpose, let us introduce the following set of LMI's:

$$\begin{bmatrix} \mathbf{A}_i\mathbf{Y}_i + \mathbf{Y}_i\mathbf{A}_i' + \mathbf{B}_i\mathbf{F}_i + \mathbf{F}_i'\mathbf{B}_i' + \gamma^{-2}\mathbf{E}_i\mathbf{E}_i' + \lambda_{ii}\mathbf{Y}_i & \mathbf{R}_i(\mathbf{Y}) & \mathbf{Y}_i\mathbf{C}_i' + \mathbf{F}_i'\mathbf{D}_i' \\ \mathbf{R}_i(\mathbf{Y})' & \mathbf{S}_i(\mathbf{Y}) & 0 \\ \mathbf{C}_i\mathbf{Y}_i + \mathbf{D}_i\mathbf{F}_i & 0 & -\mathbf{I} \end{bmatrix} < 0 \tag{6}$$

$\mathbf{Y}_i > 0$, for each $i = 1, \ldots, N$, with

$$\mathbf{R}_i(\mathbf{Y}) := [\sqrt{\lambda_{i1}}\mathbf{Y}_i, \ldots, \sqrt{\lambda_{i(i-1)}}\mathbf{Y}_i, \sqrt{\lambda_{i(i+1)}}\mathbf{Y}_i, \ldots, \sqrt{\lambda_{iN}}\mathbf{Y}_i]$$

$$\mathbf{S}_i(\mathbf{Y}) := -\mathrm{diag}(\mathbf{Y}_1, \ldots, \mathbf{Y}_{i-1}, \mathbf{Y}_{i+1}, \ldots, \mathbf{Y}_N)$$

where diag$(\cdot)$ denotes a block diagonal matrix with the blocks on the main diagonal given by $(\cdot)$. The next theorem states the main result of the paper.

Theorem 1.

Each linear state feedback control assuring robust stochastic stability for the closed-loop system in (5) is of form $\mathbf{K}_i = \mathbf{F}_i\mathbf{Y}_i^{-1}$, where the pairs of matrices $(\mathbf{F}_i, \mathbf{Y}_i)$, $i = 1, \ldots, N$ are feasible solutions of the LMI's in (6).

Proof. It follows from Lemma 1 that each control that robustly stabilizes (5) satisfies the constraints

$$\tilde{\mathbf{A}}_i'\mathbf{P}_i + \mathbf{P}_i\tilde{\mathbf{A}}_i + \gamma^{-2}\mathbf{P}_i\mathbf{E}_i\mathbf{E}_i'\mathbf{P}_i + \tilde{\mathbf{C}}_i'\tilde{\mathbf{C}}_i + \sum_{j=1}^{N} \lambda_{ij}\mathbf{P}_j < 0 \tag{7}$$

where

$$\tilde{\mathbf{A}}_i := \mathbf{A}_i + \mathbf{B}_i\mathbf{K}_i, \quad \tilde{\mathbf{C}}_i := \mathbf{C}_i + \mathbf{D}_i\mathbf{K}_i$$

and $\mathbf{P}_i > 0$, $i = 1, \ldots, N$. Let us set $\mathbf{Y}_i := \mathbf{P}_i^{-1}$ and $\mathbf{F}_i := \mathbf{K}_i\mathbf{Y}_i$, and multiply both sides of (7) by $\mathbf{Y}_i$. Finally, applying Schur complement to the non-linear terms in the summation, we can express (7) equivalently as (6). □

The problem of state feedback of MJLS with matrix parametric uncertainties was previously addressed in Reference [5], and let us provide some details of that setting for comparison purposes.

The starting point in Reference [5] is the solution to the stochastic LQ problem for the nominal system, involving a set of cost matrices $(\mathbf{Q}_i, \mathbf{R}_i)$, $i = 1, \ldots, N$. The following representation for the uncertainties was taken into account therein.

$$\begin{cases} \dot{\mathbf{x}}_t = (\mathbf{A}(\theta_t) + \boldsymbol{\delta}_a(\theta_t))\mathbf{x}_t + (\mathbf{B}(\theta_t) + \boldsymbol{\delta}_b(\theta_t))\mathbf{u}_t \\ \mathbf{u}_t = \mathbf{K}(\theta_t)\mathbf{x}_t \end{cases} \tag{8}$$

where the uncertainties $\boldsymbol{\delta}_a$ and $\boldsymbol{\delta}_b$ are matrices that satisfy $\delta'_{ai}\boldsymbol{\delta}_{ai} \leqslant \mu_{ai}\mathbf{Q}_{0i}$ and $\delta'_{bi}\boldsymbol{\delta}_{bi} \leqslant \mu_{bi}\mathbf{R}_{0i}$, respectively, for some positive scalars μ_{ai}, μ_{bi} and non-negative definite matrices $\mathbf{Q}_{0i}$ and $\mathbf{R}_{0i}$ for all $i = 1, \ldots, N$.

Now, returning back to the robust characterization given here, let us consider the specialized system (5), with

$$\mathbf{C}_i = \begin{bmatrix} \mathbf{Q}_{0i}^{1/2} \\ 0 \end{bmatrix}, \quad \mathbf{D}_i = \begin{bmatrix} 0 \\ \mathbf{R}_{0i}^{1/2} \end{bmatrix}, \quad \mathbf{E}_i = \begin{bmatrix} \mu_{ai}^{1/2}\mathbf{I} & \mu_{bi}^{1/2}\mathbf{I} \end{bmatrix} \tag{9}$$

for all $i = 1, \ldots, N$, and $\gamma = 1$. Moreover, let us assume that (7) has a solution $\mathbf{P}_i > 0$ for all i when the feedback gain is given by $\mathbf{K}_i = -\mathbf{R}_i^{-1}\mathbf{B}_i'\mathbf{P}_i$. In this particular situation the constraints in (7) assume the following form.

$$\begin{aligned} &\mathbf{A}_i'\mathbf{P}_i + \mathbf{P}_i\mathbf{A}_i - 2\mathbf{P}_i\mathbf{B}_i\mathbf{R}_i^{-1}\mathbf{B}_i'\mathbf{P}_i + (\mu_{ai} + \mu_{bi})\mathbf{P}_i^2 + \mathbf{Q}_{i0} \\ &\quad + \mathbf{P}_i\mathbf{B}_i\mathbf{R}_i^{-1}\mathbf{R}_{i0}\mathbf{R}_i^{-1}\mathbf{B}_i'\mathbf{P}_i + \sum_{j=1}^{N} \lambda_{ij}\mathbf{P}_j < 0 \end{aligned} \tag{10}$$

The next lemma states the comparison between the result here and that in Reference [5].

Lemma 2.

The solution for the robust control problem for MJLS in Reference [5] assures stochastic stability for the closed-loop system in (5) with the relevant matrices given by (9), and for the particular feedback gain $\mathbf{K}_i = -\mathbf{R}_i^{-1}\mathbf{B}_i'\mathbf{P}_i$, with $\mathbf{P}_i$, $i = 1, \ldots, N$ that satisfies (10).

Proof. In the Proposition 1 in Reference [5] one of the LMI's constraints is equivalent by a Schur complement, to the inequality $\mathbf{D}_i - \mathbf{F}_i\mathbf{E}_i^{-1}\mathbf{F}_i' > 0$, where

$$\mathbf{D}_i = \begin{pmatrix} -\lambda_{ii}\mathbf{X}_i - \mathbf{A}_i\mathbf{X}_i - \mathbf{X}_i\mathbf{A}_i' + 2\mathbf{B}_i\mathbf{R}_i^{-1}\mathbf{B}_i' - \mu_i\mathbf{I} & \mathbf{B}_i\mathbf{R}_i^{-1}\tilde{\mathbf{R}}_{i0} & \mathbf{X}_i\tilde{\mathbf{Q}}_{i0} \\ \tilde{\mathbf{R}}_{i0}'\mathbf{R}_i^{-1}\mathbf{B}_i' & \mathbf{I}_p & \mathbf{0}_{p\times n} \\ \tilde{\mathbf{Q}}_{i0}'\mathbf{X}_i & \mathbf{0}_{n\times p} & \mathbf{I}_n \end{pmatrix}$$

for $\mathbf{X}_i = \mathbf{X}_i' > 0$, and $\mu_i = \mu_{ia} + \mu_{ib}$, $\tilde{\mathbf{R}}_{i0}\tilde{\mathbf{R}}_{i0}' = \mathbf{R}_{i0}$, $\tilde{\mathbf{Q}}_{i0}\tilde{\mathbf{Q}}_{i0}' = \mathbf{Q}_{i0}$ and

$$\mathbf{F}_i = \begin{pmatrix} \mathbf{X}_i & \cdots & \mathbf{X}_i \\ \mathbf{0}_{(2n+p)\times n} & \cdots & \mathbf{0}_{(2n+p)\times n} \end{pmatrix}, \quad \mathbf{E}_i = \mathrm{diag}\left\{\frac{1}{\lambda_{ij}}\mathbf{X}_j\right\}_{j=1,\ldots,N,\, j\neq i,\, \lambda_{ij}\neq 0}$$

Applying the Schur complement to the aforementioned constraints, one gets the equivalent form:

$$\begin{aligned} &\mathbf{A}_i\mathbf{X}_i + \mathbf{X}_i\mathbf{A}_i' - 2\mathbf{B}_i\mathbf{R}_i^{-1}\mathbf{B}_i' + \mu_i\mathbf{I} + \mathbf{X}_i\mathbf{Q}_{i0}\mathbf{X}_i \\ &+ \mathbf{B}_i\mathbf{R}_i^{-1}\mathbf{R}_{i0}\mathbf{R}_i^{-1}\mathbf{B}_i' + \sum_{j=1}^{N} \lambda_{ij}\mathbf{X}_i\mathbf{X}_j^{-1}\mathbf{X}_i < 0 \end{aligned} \tag{11}$$

by setting $\mathbf{P}_i = \mathbf{X}_i^{-1}$ one can conclude that (11) is equivalent to (10), completing the proof. □

The comparison is clear from the statements of Lemmas 1 and 2. It is important to stress that the stability conditions in Reference [5] require other constraints besides those in the proof of Lemma 2, and thus being a more conservative result.

4. EXAMPLE

The example is borrowed from Reference [12], having a four-state Markov chain.

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{A}_3 = \begin{bmatrix} -1 & 2 \\ 2 & 5 \end{bmatrix}, \quad \mathbf{A}_4 = \begin{bmatrix} 5 & 2 \\ 2 & -1 \end{bmatrix}$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad \mathbf{B}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{B}_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\mathbf{C}_1 = \mathbf{C}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{C}_3 = \mathbf{C}_4 = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{D}_1 = \mathbf{D}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{D}_3 = \mathbf{D}_4 = \begin{bmatrix} 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$\mathbf{E}_1 = \mathbf{E}_2 = \mathbf{E}_3 = \mathbf{E}_4 = \begin{bmatrix} 2/5 & 0 & 1 \\ 0 & 2/5 & 1 \end{bmatrix}$$

and the transition matrix

$$\mathbf{\Lambda} = \begin{bmatrix} -5 & 2 & 3 & 0 \\ 1 & -4 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The basic idea is to verify if there exists a feedback control $\mathbf{K} = (\mathbf{K}_1, \ldots, \mathbf{K}_N)$ such that $\mathbf{u} = \mathbf{K}_\theta \mathbf{x}$ stabilizes the system (5) for any value of $||\mathbf{\Delta}_i||$ within an specified interval. Here we investigate the largest value for the norm $||\mathbf{\Delta}_i||$ expressed by γ^{-1}, for which the controlled system (5) with the relevant matrices given above is stochastic stabilizable. We implement the set of LMI's in (6) using the software $\mathrm{LMI}_{\mathrm{sol}}$ [13], and set up a problem to minimize the value of γ. The result obtained and the control $\mathbf{K}$ that attains the result are as follows:

$$||\mathbf{\Delta}_i|| \leqslant 0.69662$$

$$\mathbf{K}_1 = [\,-2.6030 \quad -5.0508\,], \quad \mathbf{K}_2 = [\,-3.6233 \quad -1.5994\,]$$

$$\mathbf{K}_3 = [\,-4.2902 \quad -12.578\,], \quad \mathbf{K}_4 = [\,-16.126 \quad -5.1297\,]$$

Assume now, in a second test that the set of matrices $\mathbf{E}$ and $\mathbf{\Delta}$ are not scaled as assumed in Section 2. In this situation, the maximum value for the norm $||\mathbf{\Delta}_i||$ can be explored independently by replacing γ by γ_i for each i in (6). We implement in this case an optimization problem that minimizes the sum $\sum_{i=1}^{4} \gamma_i$, thus providing the largest admissible uncertainty intervals and the control that attains the result:

$$||\mathbf{\Delta}_1|| \leqslant 0.49202, \;\; ||\mathbf{\Delta}_2|| \leqslant 0.89141, \;\; ||\mathbf{\Delta}_3|| \leqslant 0.96708, \;\; ||\mathbf{\Delta}_4|| \leqslant 0.96693$$

$$\mathbf{k}_1 = [\,-2.6028 \quad -4.5045\,], \quad \mathbf{k}_2 = [\,-5.4017 \quad -3.4360\,]$$

$$\mathbf{k}_3 = [\,-5.1857 \quad -13.8723\,], \quad \mathbf{k}_4 = [\,-25.3628 \quad -8.0049\,]$$

Notice that the two results are compatible; the uncertainty intervals for the Markov states ranging from two to four increase, whereas the interval for state one contracts when compared with the solution for the first test.

5. CONCLUSION

This note generalizes previous results on robust control for MJLS to be found in References [5,7], concerning matrix parametric uncertainties in the following aspects:

(i) The stability conditions presented here are less conservative than that in Reference [5], and lead to an LMI feasibility problem of smaller dimensions.
(ii) A larger class of parametric uncertainties is considered here, since we do not impose any particular structure of matrices or a specific control, as indicated in (9) and (10).
(iii) We deal with uncertainties in the control matrix set **B**, thus extending the results in Reference [7].

An important result is that each feedback control that robustly stabilizes the system (5) corresponds to a feasible solution of (6) and conversely. This equivalence in not present in the previous works.

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REFERENCES

1. Wonham WM. On a matrix Riccati equation of stochastic control. *SIAM Journal on Control* 1968; **6**:681–697.
2. Ji Y, Chizeck HJ. Controllability, stabilizability, and continuous-time Markovian jump linear quadratic control. *IEEE Transactions on Automatic Control* 1990; **35**:777–788.
3. Costa, OLV, do Val JBR, Geromel JC. A convex programming approach to H_2 control of discrete-time Markovian jump linear systems. *International Journal of Control* 1997; **66**:557–579.
4. ElGhaoui L, Rami MA. Robust state-feedback stabilization of jump linear systems via LMIs. *International Journal of Robust Nonlinear Control* 1996; **6**:1015–1022.
5. Bernard F, Dufour F, Bertrand P. On the JLQ problem with uncertainty. *IEEE Transactions on Automatic Control* 1997; **42**:869–872.
6. Morozan T. Parametrized Riccati equations associated to input–output operators for discrete-time systems with state-dependent noise. *Stochastic Analysis and Applications* 1998; **16**:915–931.
7. Shi P, Boukas EK. H_∞-control for Markovian jumping linear systems with parametric uncertainty. *Journal of Optimization Theory and Applications* 1997; **95**:75–99.
8. Feng X, Loparo KA, Ji Y, Chizeck HJ. Stochastic stability properties of jump linear systems. *IEEE Transactions on Automatic Control* 1992; **37**:38–53.
9. de Souza CE, Fragoso MD. H_∞ control for linear systems with Markovian jumping parameters. *Control Theory and Advanced Technology* 1993; **9**:457–465.
10. Colaneri P, Geromel JC, Locatelli A. *Control Theory and Design—An* RH_2/RH_∞ *Viewpoint*. Academic Press: New York, 1997.
11. Hinrichsen D, Pritchard AJ. Stochastic H_∞. *SIAM Journal on Control and Optimization* 1998; **36**:1504–1538.
12. Ji Y, Chizeck HJ. Jump linear quadratic Gaussian control in continuous time. *IEEE Transactions on Automatic Control* 1992; **37**:1884–1892.
13. Oliveira MC, Faria DP, Geromel JC. LMIsol User's Guide, 1998. available at `http://www.dt.fee.unicamp.br/˜mauricio/#soft`.

