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Superluminal tunneling through two successive barriers(*)

V. S. Olkhovsky[1](**), E. Recami[2,3,4](***) and G. Salesi[3,2](***)

[1] *Institute for Nuclear Research - Kiev-03028, Ukraine, and Research Center "Vidhuk", Kiev, Ukraine*

[2] *Facoltà di Ingegneria, Università Statale di Bergamo - 24044 Dalmine (BG), Italy*

[3] *INFN-Sezione di Milano - Milano, Italy*

[4] *CCS, State University at Campinas - Campinas, S.P., Brazil*

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Abstract. – We study the phenomenon of one-dimensional non-resonant tunneling through two successive (opaque) potential barriers, separated by an intermediate free region $\mathcal{R}$, by analyzing the relevant solutions to the Schroedinger equation. We find that the total traversal time does *not* depend not only on the barrier widths (the so-called "Hartman effect"), but also on the $\mathcal{R}$ width: so that the effective velocity in the region $\mathcal{R}$, between the two barriers, can be regarded as practically infinite. This agrees with the results known from the corresponding waveguide experiments, which simulated the tunneling experiment herein considered due to the known formal identity between the Schroedinger and the Helmholtz equation.

Introduction. – It is known within quantum mechanics, with regard to the tunneling processes, that the tunneling time —either evaluated as a simple "phase time" [1] or calculated through the analysis of the wavepacket behaviour [2]— does not depend on the barrier width in the case of opaque barriers. Such a phenomenon, sometimes called "Hartman effect" [3], implies Superluminal and arbitrarily large (group) velocities v inside long enough barriers [2]. Experiments that may verify this prediction by, say, electrons are difficult. Luckily enough, however, the Schroedinger equation in the presence of a potential barrier is mathematically identical [4] to the Helmholtz equation for an electromagnetic wave propagating, *e.g.*, down a metallic waveguide along the x-axis: and a barrier height V bigger than the electron energy E corresponds (for a given wave frequency) to a waveguide transverse size smaller than a cut-off value. A segment of *undersized* guide does therefore behave as a barrier for the wave (photonic barrier): So that the wave assumes therein —like an electron inside a quantum barrier— an imaginary momentum or wave-number and gets exponentially damped along

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(**) E-mail: olkhovsk@kinr.kiev.ua
(***) E-mail: recami@mi.infn.it
(***) E-mail: salesi@ct.infn.it

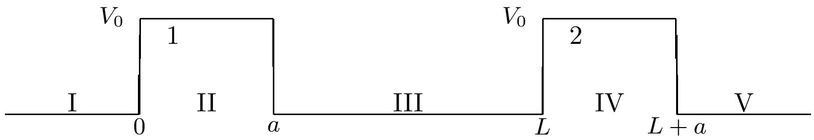

Fig. 1

Fig. 1 – The non-resonant tunneling process, through *two* successive (opaque) potential barriers, considered in this paper. We show that, far from resonances, the (total) phase time for tunneling through the two barriers does depend neither on the barrier widths *nor on the distance between the barriers.*

x, as a consequence. In other words, it becomes an *evanescent* wave (going back to normal propagation, even if with reduced amplitude, when the narrowing ends and the guide returns to its initial transverse size). Thus, a tunneling experiment can be simulated by having recourse to evanescent waves (for which the concept of group velocity can be properly extended [5]).

And the fact that evanescent waves travel with Superluminal speeds has been actually *verified* in a series of famous experiments. Namely, various experiments —performed since 1992 onwards by R. Chiao's and A. Steinberg's group at Berkeley [6], by G. Nimtz at Cologne [7], by A. Ranfagni and colleagues at Florence [7], and by others at Vienna, Orsay, Rennes [7]— verified that "tunneling photons" travel with Superluminal group velocities; in other words, they confirmed, directly or indirectly, the occurrence of the Hartman effect.

Let us emphasize that the most interesting experimental setup, dealing with evanescent waves, seems to be —however— the one comprehending *two* successive *evanescence regions* ("classical barriers"), separated by a segment of normal region. For suitable frequency bands —*i.e.*, far from resonances— it was found that the total crossing time does not depend on the length of the intermediate (normal) region: namely, that the beam speed along it is infinite. The related experimental results [8] have been already confirmed by numerical simulations, based on Maxwell equations only [9]. But they are so amazing that we want to check whether they agree also with what is predicted by quantum mechanics in the analogous case of two successive potential barriers.

In this note we are actually going to show that, for non-resonant tunneling trough two successive, rectangular (opaque) potential barriers (fig. 1), the (total) phase time does depend neither on the barrier widths *nor on the distance between the barriers.* In other words, far from resonances the tunneling phase time, which does depend on the entering energy, can be shown to be *independent* of the distance between the two barriers.

Phase time evaluation. – Let us consider the (quantum-mechanical) stationary solution for the one-dimensional (1D) tunneling of a non-relativistic particle, with mass m and kinetic energy $E = \hbar^2 k^2/2m = mv^2/2$, through two equal rectangular barriers with height V_0 ($V_0 > E$) and width a, the quantity $L - a \geq 0$ being the distance between them. The Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) \;+\; V(x)\,\psi(x) \;=\; E\,\psi(x)\,, \tag{1}$$

where $V(x)$ is zero outside the barriers, while $V(x) = V_0$ inside the potential barriers. In the various regions I ($x \leq 0$), II ($0 \leq x \leq a$), III ($a \leq x \leq L$), IV ($L \leq x \leq L+a$) and V ($x \geq L+a$), the stationary solutions to eq. (1) are the following:

$$
\begin{cases}
\psi_{\rm I} \;=\; {\rm e}^{+ikx} + A_{1\rm R}\,{\rm e}^{-ikx}\,, & (2a)\\
\psi_{\rm II} \;=\; \alpha_1\,{\rm e}^{-\chi x} + \beta_1\,{\rm e}^{+\chi x}\,, & (2b)\\
\psi_{\rm III} \;=\; A_{1\rm T}\left[{\rm e}^{ikx} + A_{2\rm R}\,{\rm e}^{-ikx}\right], & (2c)\\
\psi_{\rm IV} \;=\; A_{1\rm T}\left[\alpha_2\,{\rm e}^{-\chi(x-L)} + \beta_2\,{\rm e}^{+\chi(x-L)}\right], & (2d)\\
\psi_{\rm V} \;=\; A_{1\rm T}\,A_{2\rm T}\,{\rm e}^{ikx}\,, & (2e)
\end{cases}
$$

where $\chi \equiv \sqrt{2m(V_0-E)}/\hbar$, and quantities $A_{1\rm R}$, $A_{2\rm R}$, $A_{1\rm T}$, $A_{2\rm T}$, α_1, α_2, β_1 and β_2 are the reflection amplitudes, the transmission amplitudes, and the coefficients of the "evanescent" (decreasing) and "anti-evanescent" (increasing) waves for barriers 1 and 2, respectively. Such quantities can be easily obtained from the matching (continuity) conditions:

$$
\begin{cases}
\psi_{\rm I}(0) \;=\; \psi_{\rm II}(0)\,, & (3a)\\
\left.\dfrac{\partial\psi_{\rm I}}{\partial x}\right|_{x=0} \;=\; \left.\dfrac{\partial\psi_{\rm II}}{\partial x}\right|_{x=0}, & (3b)
\end{cases}
$$

$$
\begin{cases}
\psi_{\rm II}(a) \;=\; \psi_{\rm III}(a)\,, & (4a)\\
\left.\dfrac{\partial\psi_{\rm II}}{\partial x}\right|_{x=a} \;=\; \left.\dfrac{\partial\psi_{\rm III}}{\partial x}\right|_{x=a}, & (4b)
\end{cases}
$$

$$
\begin{cases}
\psi_{\rm III}(L) \;=\; \psi_{\rm IV}(L)\,, & (5a)\\
\left.\dfrac{\partial\psi_{\rm III}}{\partial x}\right|_{x=L} \;=\; \left.\dfrac{\partial\psi_{\rm IV}}{\partial x}\right|_{x=L}, & (5b)
\end{cases}
$$

$$
\begin{cases}
\psi_{\rm IV}(L+a) \;=\; \psi_{\rm V}(L+a)\,, & (6a)\\
\left.\dfrac{\partial\psi_{\rm IV}}{\partial x}\right|_{x=L+a} \;=\; \left.\dfrac{\partial\psi_{\rm V}}{\partial x}\right|_{x=L+a}. & (6b)
\end{cases}
$$

Equations (3)-(6) are eight equations for our eight unknowns ($A_{1\rm R}$, $A_{2\rm R}$, $A_{1\rm T}$, $A_{2\rm T}$, α_1, α_2, β_1 and β_2). First, let us obtain the four unknowns $A_{2\rm R}$, $A_{2\rm T}$, α_2, β_2 from eqs. (5) and (6) in the case of *opaque* barriers, *i.e.*, when a is large enough (and χ not too small) so that one can assume that $\chi a \to \infty$:

$$
\begin{cases}
\alpha_2 \longrightarrow {\rm e}^{ikL}\,\dfrac{2ik}{ik-\chi}\,, & (7a)\\
\beta_2 \longrightarrow {\rm e}^{ikL-2\chi a}\,\dfrac{-2ik(ik+\chi)}{(ik-\chi)^2}\,, & (7b)\\
A_{2\rm R} \longrightarrow {\rm e}^{2ikL}\,\dfrac{ik+\chi}{ik-\chi}\,, & (7c)\\
A_{2\rm T} \longrightarrow {\rm e}^{-\chi a}{\rm e}^{-ika}\,\dfrac{-4ik\chi}{(ik-\chi)^2}\,. & (7d)
\end{cases}
$$

Then, we may obtain the other four unknowns $A_{1\rm R}$, $A_{1\rm T}$, α_1, β_1 from eqs. (3) and (4). Again in the case of large enough barriers (and $\chi a \to \infty$), one gets:

$$
\begin{cases}
\alpha_1 \longrightarrow \dfrac{2ik}{ik-\chi}\,, & (8a)\\
\beta_1 \longrightarrow {\rm e}^{-2\chi a}(k-i\chi)\dfrac{\sin k(L-a)}{\chi}A\,, & (8b)\\
A_{1\rm R} \longrightarrow \dfrac{ik+\chi}{ik-\chi}\,, & (8c)\\
A_{1\rm T} \longrightarrow {\rm e}^{-\chi a}{\rm e}^{-ikL}A\,, & (8d)
\end{cases}
$$

where

$$A \equiv \frac{2\chi k}{2\chi k \ \cos k(L-a) \ + \ (\chi^2 - k^2) \ \sin k(L-a)} \tag{9}$$

results, incidentally, to be real.

At this point, by applying the well-known definition of phase time (see, for instance, refs. [1–3]), we can derive that the tunneling time

$$\begin{aligned} {\tau^{\rm ph}}_{\rm tun} \ &\equiv \ \hbar \, \frac{\partial \arg \left[A_{1\rm T} A_{2\rm T} {\rm e}^{ik(L+a)} \right]}{\partial E} \ = \ \hbar \, \frac{\partial}{\partial E} \arg \left[\frac{-4ik\chi}{(ik-\chi)^2} \right] = \\ &= \ \hbar \, \frac{\partial}{\partial E} \ \arctan \left[\frac{k^2 - \chi^2}{k \, \chi} \right] = \frac{1}{\hbar\chi} \, \frac{2m}{k} \, , \end{aligned} \tag{10}$$

while depending on the energy of the tunneling particle, *does not depend* on $L + a$ (being it actually independent both of a and of L).

This result does *not only* confirm the so-called "Hartman effect" [2, 3] for the two opaque barriers —*i.e.*, the independence of the tunneling time from the opaque barrier widths— but it does *also* extend such an effect by implying the total tunneling time to be independent even of L (see fig. 1). This might be regarded as a further evidence of the fact that quantum systems seem to behave as non-local; but is has a more general meaning, being it associated with the properties of any waves (and, in fact, something very similar happens also, *e.g.*, with electromagnetic waves: see below). It is important to stress once more that the previous result holds, however, for non-resonant (nr) tunneling: *i.e.*, for energies far from the resonances that arise in region III due to interference between forward and backward travelling waves (a phenomemon quite analogous to the Fabry-Pérot one in the case of classical waves). Otherwise it is known that the expression for the time delay τ near a resonance is rather larger: for example, for a Gaussian resonance at $E_{\rm r}$ with half-width Γ, it would be $\tau = \hbar\Gamma[(E - E_{\rm r})^2 + \Gamma^2]^{-1} + \tau_{\rm nr}$.

Discussion. – The tunneling-time independence from the width (a) of each one of the two opaque barriers is itself a generalization of the Hartman effect, and might be *a priori* understood —following refs. [4, 6]— on the basis of the reshaping phenomenon which takes place inside a barrier.

With regard to the even more interesting tunneling-time independence from the distance $L - a$ between the two barriers, it may be understood on the basis of the interference *between the waves* coming out of the first barrier (region II) and traveling in region III *and the waves* reflected from the second barrier (region IV) back into the same region III.

Such an interference has been shown [2] to cause an "advancement", *i.e.*, an effective acceleration of the forward-traveling waves, even in region I: Namely, going on to the wavepacket language, we noticed in refs. [2] that the arriving wavepacket does interfere with the reflected waves that start to be generated as soon as the packet forward tail reaches the first barrier edge: In such a way that, already before the barrier, the backward tail of the initial wavepacket decreases —because of destructive interference with those reflected waves— at a larger degree than the forward one. This simulates an increase of the average speed of the entering packet; hence, the effective (average) flight-time of the approaching packet from the source to the barrier does decrease.

So, a reshaping and "advancement" of the same kind (inside the barriers, as well as to the left of the barriers) may qualitatively explain why the tunneling-time is independent of the barrier widths and of the distance between the two barriers. Phenomena of this kind, actually, do not seem to be at variance with Special Relativity, as has already been discussed in a number of papers (cf., *e.g.*, refs. [9] and [5], and refs. therein). It remains impressive, nevertheless, that in region III —where no potential barrier is present, the current is non-zero and the wave function is oscillatory— the effective speed (or group-velocity) is practically *infinite*(1). After some straightforward but rather bulky calculations, one can moreover see that the same effects (*i.e.*, the independence from the barrier widths and from the distances between the barriers) are still valid for any number of barriers, with different widths and different distances between them.

Finally, let us recall that the known similarity between photon and (non-relativistic) particle tunneling [2, 4, 10, 11] implies that our previous results hold also for photon tunneling through successive "barriers": For example, for photons in the presence of two successive band gap filters, like two suitable gratings or two photonic crystals. Experiments should be easily realizable; while indirect experimental evidence seems to come from papers such as [12].

Let us also repeat that the classical, relativistic (stationary) Helmholtz equation for an electromagnetic wavepacket in a waveguide is known to be formally identical to the quantum, non-relativistic (stationary) Schroedinger equation for a potential barrier(2); so that, for instance, the tunneling of a particle under and along a barrier has been simulated [2, 4, 7–11, 13] by the traveling of evanescent waves along an undersized waveguide. Therefore, the results of this paper are to be valid also for electromagnetic wave propagation along waveguides with a succession of undersized segments (the "barriers") and of normal-sized segments. This confirms the results obtained, within the classical realm, directly from Maxwell equations [9, 13], as well as by the known series of "tunneling" experiments performed —till now— with microwaves (see refs. [7] and particularly [8]).

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(1)Loosely speaking, one might say that the considerd two-barriers setup *can* behave as a "space destroyer" with reference to its intermediate region.

(2)These equations are, however, different (due to the different order of the time derivative) in the time-dependent case. Nevertheless, it can be shown that they still have in common classes of analogous solutions, differing only in their spreading properties [2, 11].

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