Analytical solution for the dynamic behavior of erbium-doped fiber amplifiers with constant population inversion along the fiber

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We present an analytical solution for the coupled rate and propagation equations for a dynamic two-level homogeneously broadened system interacting with radiation and with constant population inversion along the longitudinal axis of the fiber, z. We derive an analytical solution for the z dependence of these equations, which greatly simplifies the numerical solution for the output powers' time dependence. Amplified spontaneous emission and background loss influences are considered in the model, in contrast to the previous analytical solution presented by Y. Sun *et al.* The solution is derived, and the importance of each term for the dynamic modeling of typical erbium-doped fiber amplifiers is analyzed. © 2004 Optical Society of America

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1. INTRODUCTION

The rate and propagation equations for a homogeneously broadened two-level system interacting with radiation have been extensively used to model erbium-doped fiber amplifier (EDFA) behavior—see the famous study by Giles and Desurvire,[1] for instance. Saleh *et al.*[2] and Sun *et al.*[3,4] have shown that the z dependence of these equations—z defined as the longitudinal fiber coordinate—has an analytical solution in both the static and the dynamic cases if the terms containing the contributions of the amplified spontaneous emission (ASE) and the background loss of the fiber are neglected in the equations. Moreover, with these approximations the gain of an amplifier was shown to depend only on the average inversion level of the EDFA (the fraction of erbium ions in the excited state), not on the detailed excited-ion distribution along the fiber, a fact found almost at the same time as Sun *et al.*[4] by Bononi and Rusch.[5] The time-dependent gain is described in these papers by a single ordinary differential equation for the average inversion level, which in steady state becomes a transcendental equation.

The fundamental limitation of these analytical solutions is that they do not predict the ASE and the background loss influences on EDFA behavior. ASE self-saturation, for instance, is not predicted. In the static case, one can include these effects by employing the average inversion model,[6,7] which is a powerful tool in the physical understanding, simulation, and design of EDFAs. This model estimates the ASE output spectral power by assuming that the population inversion is constant along the fiber. In other words, it is assumed that in any situation the spectral ASE output power is equal to the spectral ASE output power that the amplifier would have with the same average inversion level but with constant inversion along the fiber, because in this last case the propagation equation has an analytical solution and it is not necessary to perform numerical integration along z to calculate the output powers. The validity of this approach has been studied by Desurvire[7]: Although the easily experimentally observed difference between the forward and the backward ASE spectra is not predicted, the approach correctly predicts qualitatively the amplifiers' self-saturation by ASE.

In this paper we extend the average inversion model (AIM) to the dynamic case and apply it to the study of EDFA dynamic behavior. We show that the time-dependent gain, including the ASE and the background loss contributions, has an analytical solution in z if the excited-state population of the erbium ions is constant along the fiber. We show that, in this case, the dynamic gain behavior can be described by a single ordinary differential equation for the average inversion level. We divide the derivation and discussion of the model into two parts. First, in Section 2 we study how to include ASE effects in EDFA dynamic behavior by using the AIM but still considering the background loss of the doped fiber negligible, and then in Section 3 we extend the model to the inclusion of the background loss. At last, in Section 4 we draw some conclusions and make some final comments.

2. MODEL INCLUDING AMPLIFIED SPONTANEOUS EMISSION INFLUENCE IN DYNAMIC ERBIUM-DOPED FIBER AMPLIFIERS

In this section we first present the full system of coupled rate and propagation equations that will be used through this paper. Then we study the extension of the AIM to the dynamic case when ASE is considered but not the background losses of the fiber. Many conclusions that are not altered by the inclusion of the fiber background loss are presented and discussed. Some characteristics that appear when these losses are considered are left to Section 3.

A. Derivation

Following the notation used in Ref. 4, we find that the rate and propagation equations for a two-level homogeneously broadened system interacting with radiation are

$$\frac{\partial N_2(z, t)}{\partial t} = -\frac{N_2(z, t)}{\tau_0} - \frac{1}{\rho S}\sum_{n=1}^{M} [(\gamma_n + \alpha_n) \times N_2(z, t) - \alpha_n]P_n(z, t), \quad (1)$$

$$\frac{\partial P_n(z, t)}{\partial z} = u_n\{[(\gamma_n + \alpha_n)N_2(z, t) - \alpha_n - \alpha_{\text{loss}}] \times P_n(z, t) + 2\gamma_n\Delta\nu N_2(z, t)\}, \quad (2)$$

where $N_2 + N_1 = 1$ is the normalized population of the upper and lower levels; $P_n(z, t)$ is the optical power at location z and time t of the nth beam with wavelength centered at λ_n, expressed in number of photons per unit time; τ_0 is the spontaneous lifetime of the upper level; ρ is the number density of the active erbium ions; S is the doped region area; and γ_n and α_n are the gain and absorption constants (or Giles coefficients[1]). Beams traveling in the forward direction are indicated by the unit vector $u_n = +1$, and beams traveling in the backward direction are indicated by $u_n = -1$. The parameters α_{loss} and $\Delta\nu$ are not defined in Ref. 4 because neither the ASE nor the background loss is considered in that paper. α_{loss} is the attenuation coefficient (units of inverse meters), given by the background loss of the fiberglass host, and $\Delta\nu$ is the frequency interval between two successive wavelengths considered in the model. Typically, to obtain accurate results for a single-channel C-band EDFA, for instance, it is necessary to consider $\sim$100 wavelengths ($M = 200$, one nth channel for each direction for each wavelength) between 1500 and 1600 nm, $\Delta\nu \sim 133$ GHz in the 1.55-μm region. For L-band EDFAs, or thus operating in dense-wavelength-division-multiplexing conditions, consideration of more than 500 wavelengths in the model may be necessary to obtain similar accuracy.

We first present our result in the case $\alpha_{\text{loss}} = 0$. In this case, Eq. (1) can be written as

$$\frac{\partial N_2(z, t)}{\partial t} = -\frac{N_2(z, t)}{\tau_0} - \frac{1}{\rho S}\sum_{n=1}^{M} \left\{u_n\left[\frac{\partial P_n(z, t)}{\partial z}\right] - 2\gamma_n\Delta\nu N_2(z, t)\right\}. \quad (3)$$

If N_2 is constant along z, Eqs. (2) and (3) have analytical solutions. Integrating from 0 to the doped fiber length L, we find the solution for Eq. (2) is

$$P_n^{\text{out}}(t) = P_n^{\text{in}}(t)G_n(t) + 2n_n^{\text{sp}}[G_n(t) - 1]\Delta\nu, \quad (4)$$

with

$$G_n(t) = \exp\{[(\gamma_n + \alpha_n)N_2(t) - \alpha_n]L\}, \quad (5)$$

$$n_n^{\text{sp}} = \frac{N_2(t)\gamma_n}{(\gamma_n + \alpha_n)N_2(t) - \alpha_n}, \quad (6)$$

and the solution for Eq. (3) is

$$\frac{\mathrm{d}N_2(t)}{\mathrm{d}t} = -\frac{N_2(t)}{\tau_0} - \frac{1}{\rho SL}\sum_{n=1}^{M} [P_n^{\text{out}}(t) - P_n^{\text{in}}(t) - 2\gamma_n\Delta\nu N_2(t)L] \quad (7)$$

or, with Eq. (4),

$$\frac{\mathrm{d}N_2(t)}{\mathrm{d}t} = -\frac{N_2(t)}{\tau_0} - \frac{1}{\rho SL}\sum_{n=1}^{M} \{P_n^{\text{in}}(t)[G_n(t) - 1] + 2n_n^{\text{sp}}[G_n(t) - 1]\Delta\nu - 2\gamma_n\Delta\nu N_2(t)L\}. \quad (8)$$

B. Discussion

1. *Physical Interpretation*

Equation (4) is a well-known result[1,6]: The z dependence of the propagation equations including the ASE contribution has an analytical solution if the population inversion of erbium ions is constant along the fiber. Although this solution is exact only if $N_2(z)$ is constant, Ref. 6 shows that this modeling accurately predicts the EDFA gain spectrum for a broad range of input pump and signal powers.

Equation (8) is presented here for the first time, to our knowledge. There are four terms in the right-hand side of this equation.

i. The spontaneous decay term.

ii. The term that involves the input signal powers. This term was first presented by Sun *et al.*[3] In cases in which the ASE can be neglected, the rate equation analytical solution is reduced to these first two terms in Eq. (8): the spontaneous decay term and the Y. Sun term.

iii. The term containing the spontaneous emission factor, n^{sp}. The inclusion of this term in the rate equation as an approximate solution for the ASE influence in EDFAs, although not rigorously derived, to the best of our knowledge, in the scientific literature, has been used by the two commercially available programs that, as far as we know, simulate EDFA dynamic behavior: software by Virtual Photonics, in which this term was first presented, by Optiwave Corporation, VPItransmissionMaker, and OptiSystem. The user's manuals of these software programs[8,9] do not derive this term from the full rate and propagation equations, including the ASE term. The VPItransmissionMaker user's manual[8] cites "private communication with A. Bononi" as the only reference for the inclusion of this term. The OptiSystem user's manual[9] derives this term as an exact solution for the rate equation when the last term in Eq. (3) is neglected. Here we rigorously derive this term as one of the terms of the rate equation's analytical solution when $N_2(z)$ is constant. From now on we call this term the Bononi term because Alberto Bononi seems to have been the first person to propose it. The experimental validity for EDFA dynamic modeling of the simulation results, including in the solution for the rate equation just these three terms (spontaneous decay, Y. Sun, and Bononi terms), without the inclusion of the last term, has been studied by Dimopoulos in his Ph.D. thesis.[10] The conclusions we are going to draw in this paper agree well with his observations that, for most situations of practical importance and, particularly, for EDFAs operating in typical multichannel conditions, the model that considers just these three terms predicts EDFA dynamic behavior with good accuracy (~from 0.05- to 0.5-dB difference when compared with experimental data).

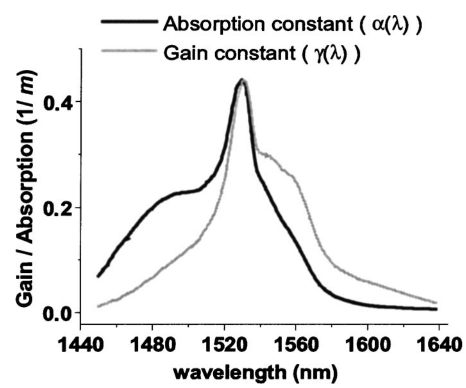
iv. The last term is being presented here for the first time, to our knowledge. The physical interpretation of this term is as follow. Equation (8) represents the conservation of photons: The first term (i) represents the photons that have been emitted per unit time in any direction by spontaneous emission; the Y. Sun term and the Bononi term, (ii) and (iii) above, are the total output photon flux less the total input photon flux, $\Sigma_n^M {P_n}^{\mathrm{out}} - {P_n}^{\mathrm{in}}$ [see Eq. (7)]; and the last term here presented represents the total number of photons per unit time, among those emitted spontaneously, that is emitted in the direction of one of the guided modes, being coupled to the mode. The last term subtracts from the absolute value of the spontaneous decay term (i) those photons that, although emitted spontaneously, couple to the guided modes and are measured as part of the total output power. We call this last term, then, the captured photons term.

2. *Relative Contribution of Each Term*

In this subsection we discuss the relative contribution of each term to the time derivative of the average inversion level. First, we note that an immediate consequence of Eq. (8) is that all the possible values for the derivative of the average inversion level $\mathrm{d}N_2/\mathrm{d}t$ in a given EDFA can be obtained in terms of only two parameters: N_2 and the input power P^{in} of any chosen input channel. All other parameters in this equation are intrinsic parameters of the doped fiber or, in the case of L and α_{loss}, internal parameters of the amplifier. We stress that only one channel (the reference channel) is necessary to simulate any possible values for $\mathrm{d}N_2/\mathrm{d}t$ given by any multichannel possible response to an add or drop operation. The Y. Sun term contribution to $\mathrm{d}N_2/\mathrm{d}t$ in any given situation, which is the only contribution depending on the input powers and not just on N_2, can be simulated by just one channel with an input power such that the total number of absorbed or emitted photons is equal to those in the given situation. All the possible values for $\mathrm{d}N_2/\mathrm{d}t$ in any add or drop operation in a multichannel system can be simulated by a correctly chosen N_2 and P^{in} at the reference wavelength.

Thus, to analyze the contribution of each one of the four terms in Eq. (8) to $\mathrm{d}N_2/\mathrm{d}t$ in a wide range of situations, we performed simulation by using typical amplifier intrinsic parameters (see Fig. 1). We vary N_2 from 0 to 1 (0.1 step) and use three input powers at $\lambda = 1480\,\mathrm{nm}\,({P_{1480}}^{\mathrm{in}})$: 10 μW, 1 mW, and 100 mW. Three graphs for three different EDFAs are shown in Fig. 2. We show the absolute value (positive) of each term contribution to $\mathrm{d}N_2/\mathrm{d}t$ in all the simulated situations. The only difference between the parameters used to simulate each amplifier is the doped fiber length: 10, 20, and 50 m, given a peak gain at 1530 nm of 9.6, 19.2, and 47.9 dB, respectively, for an inversion level $N_2 = 0.75$, which are values for typical *C*-band EDFAs.

Although we show in the graphs in Fig. 2 absolute values, we stress that the spontaneous decay term and the Bononi term contributions to $\mathrm{d}N_2/\mathrm{d}t$ are always negative and the captured photons term contribution is always positive. The Y. Sun term contribution is positive for low N_2 values (which is expected because when N_2 goes to zero the presence of any signal power raises it) and then drops for larger N_2's until it becomes negative, which explains the minimum observed in the Y. Sun term's absolute value curves, corresponding to the point at which this contribution is zero. (The curves connecting the simulated points are just for clarity purposes; the Y. Sun term actually goes to $10^{-\infty}$—zero—near the minimums of the Y. Sun curves shown.) When all contributions are summed, the total value for $\mathrm{d}N_2/\mathrm{d}t$ is positive until a cer-



Other intrinsic parameters

Name	Value
Erbium ions lifetime (τ)	10 (ms)
Doped region radius (S)	1 (μm)
Erbium ions density (ρ)	10^{24} (m^{-3})
Background loss coefficient (α_{loss})	0 (1/m)

Numerical parameters

Start wavelength	1450 (nm)
Stop wavelength	1610 (nm)
Step	1.6 (nm) (which corresponds to Δν~ 200 Ghz in the 1.55 μm region)

Fig. 1. Intrinsic erbium-doped fiber parameters used to simulate the graphs in Fig. 2. Absorption and emission constants and other parameter values are shown.

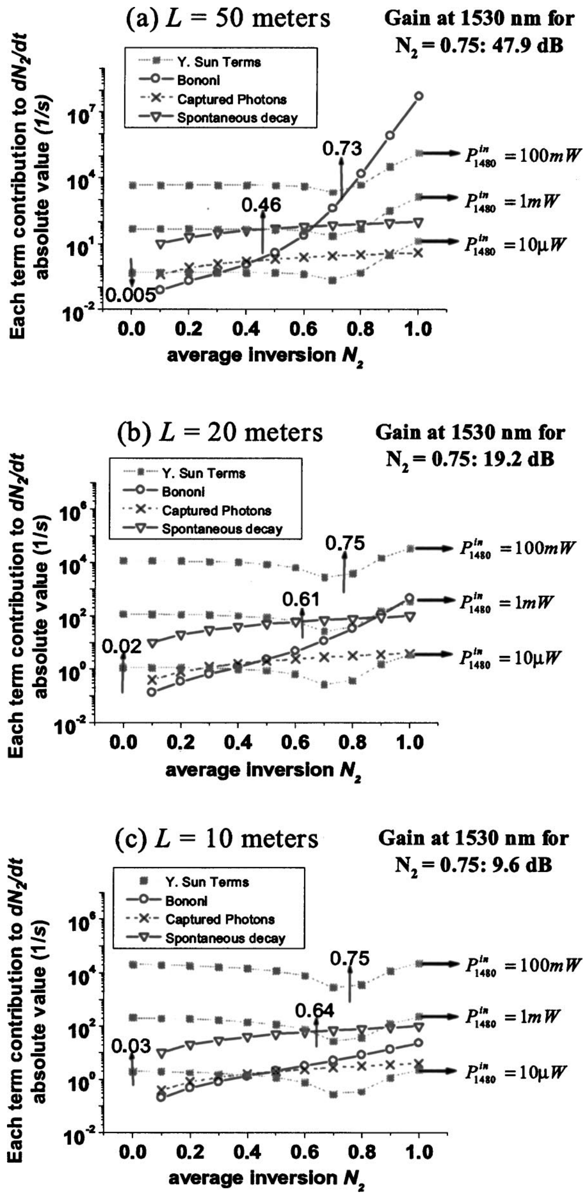


Fig. 2. Each term contribution to dN_2/dt (absolute values) as a function of N_2 and of the input power at $\lambda = 1480\,\mathrm{nm}$, $P_{1480}{}^{\mathrm{in}}$. Observe that just the Y. Sun term contribution depends on the input power. Simulations are obtained with Eq. (8) and intrinsic EDFA parameters shown in Fig. 1. (a), (b), and (c) are for $L = 50$, 20, and 10 m, respectively, which correspond to the amplifier gain at 1530 nm and $N_2 = 0.75$ of 47.9, 19.2, and 9.59 dB, respectively. In each graph we indicate with horizontal arrows in the Y. Sun terms the corresponding $P_{1480}{}^{\mathrm{in}}$ used. The vertical arrows crossing each Y. Sun curve show the value of N_2 at which $dN_2/dt = 0$ for this particular input power at 1480 nm, i.e., the value of N_2 at which the sum of all term contributions is zero, and which corresponds to the N_2 value given by the static solution of the equations when the respective input power at 1480 nm is entering the amplifier.

tain value of N_2 and then becomes negative (unless no channels enter the amplifiers, i.e., $P^{\mathrm{in}} = 0$, in which case dN_2/dt is negative for any N_2 value).

We show, with a vertical arrow in each Y. Sun term curve, the value for N_2 at which $dN_2/dt = 0$ (or, equivalently, at which dN_2/dt becomes negative), for the corresponding input power. Observe that the point at which $dN_2/dt = 0$ corresponds to the value N_2 given by the static solution of the equations when the respective input power at 1480 nm is entering the amplifier. More generally, each graph represents all the possible term contributions to dN_2/dt in an add or drop operation in an amplifier that reaches the final static N_2 value indicated by the vertical arrows and given by the static solution of the input power spectrum at the end of the operation (in the cases represented in Fig. 2, this final input spectrum is simply given by only one input power at 1480 nm). Any value for N_2 below this point will give a dN_2/dt value that corresponds to a drop operation. Any value for N_2 above this point corresponds to an add operation. From Fig. 2(c), for instance, it is possible to say that if all the channels are dropped at the same time in this particular EDFA and just a pump power of 100 mW at 1480 nm survives at the end of the operation, then, no matter which specific drop operation has been performed, no one term's absolute value contribution to dN_2/dt would be larger than $\sim 10^5$ ions/s. Similarly, it is possible to know all the possible dN_2/dt responses to any add or drop operation in any WDM system if one knows the final state of this operation. This can be done by one's performing the same simulations as in Fig. 2 but using for the calculus of the Y. Sun term the input powers given by the spectral input power at the final state of the add–drop operation. In this case, the Y. Sun term contribution to dN_2/dt would not be given by a single input at 1480 nm but by the spectral input powers in the respective final state.

It is also possible to say from Fig. 2(c) that, as neither the Bononi (ASE) contribution nor the spontaneous decay contribution to dN_2/dt depends on the input powers, these term contributions to dN_2/dt are never larger than $\sim 10^2$ ions/s (in this particular EDFA), no matter which operation is performed or which are the initial and final states of this operation. Another interesting result from Fig. 2 is that, as expected, the absolute value of dN_2/dt in high-gain amplifiers [Fig. 2(a)] is larger in add operations—high N_2 values—than in drop operations—small N_2—as was first observed in Ref. 5. Other expected results that can be observed in Fig. 2 are

i. The captured photons term is in almost all the situations negligible when compared with the other terms. The only one graph in which this term is not at least 2 orders of magnitude smaller than the spontaneous decay term for all N_2 values is for $L = 50\,\mathrm{m}$. In this case, if N_2 is smaller than ~ 0.4 and the Y. Sun term is not considered (for $P^{\mathrm{in}} = 0$, the Y. Sun term is always 0), the captured photons term can have some influence when the erbium ions' lifetime is measured. We return to this particular situation in Subsection 2.B.3. Now we stress that the negligibility of the captured photons term was expected, since, as said in item (iii) in Subsection 2.B.1, it has been shown that the other three terms in Eq. (8) give accurate simulation results for typical EDFAs when compared with experimental data.[10]

ii. For amplifiers with gain <20 dB [Fig. 2(a), the region of Fig. 2(b) where $N_2 < 0.75$, and the region of Fig. 2(c) where $N_2 < 0.5$], the contribution of the Bononi term is negligible in all cases, in that its value is at least 2 orders of magnitude smaller than the contribution of either the spontaneous decay term or the Y. Sun term. This agrees well with the observation in Refs. 3 and 4 that the Y. Sun model [which considers just the spontaneous decay and the Y. Sun term in Eq. (8)] is accurate for amplifiers with gain <20 dB.

iii. In all 100-mW curves, which are a typical total input power (pump and signals) for a *C*-band EDFA, the Y. Sun term contribution is always at least 2 orders of magnitude greater than the spontaneous decay term. This means that the amplifiers' performance depends weakly on the erbium ions' lifetime τ_0. We expected this result because we had done an internal study at our lab, solving numerically the full rate and propagation equations without the approximation $N_2(z) = \text{cte}$, where, for a typical EDFA operating in multi channel conditions, we varied the lifetime of the erbium ions from 9.5 to 10.5 ms; the variation in the gain in relation to the $\tau_0 = 10$-ms case, for a 30-dB peak gain amplifier, was always <0.02 dB. To the best of our knowledge, this is the first time this observation was made.

3. *Influence of the Captured Photons Term in Low-Inversion Cases*

From the fundamental point of view, an interesting result arising from Eq. (8) is the systematic error introduced by the captured photons term when one measures the erbium ions' lifetime τ_0 in an erbium-doped fiber (EDF). The commonly used procedure to determine τ_0 in an EDF is to apply a step-function power excitation to the fiber input port and to fit the exponential ASE power decay at the output port when the step excitation is switched off and $P^{\text{in}} = 0$ (see Ref. 7, Subsection 4.6). An additional care one should take when extracting τ_0 from measured data is to fit just the last tail of the exponential ASE output power decay because the presence of the ASE power itself produces stimulated emission. For ensuring that stimulated emission does not occur, the trick is to start the fitting at the final time points of the measured exponential decay curve, when the ASE powers and the inversion level N_2 are low, because it is commonly understood that the effective measured lifetime, τ_{eff}, goes to τ_0 for low ASE powers and N_2 values. (Observe that the accuracy of the τ_0 measure depends crucially, then, on the photodetector sensitivity, which determines how low the ASE power can be to be detected.) However, it can be easily shown from Eq. (8) that, even when $N_2 \to 0$, $\tau_{\text{eff}} \to \tau_0$ rigorously only for short fiber lengths L. At short fiber lengths, what happens is that the absolute value of the Bononi term equals the value of the captured photons term. Both cancel each other, and, in this case, $\tau_{\text{eff}} \to \tau_0$. [Observe in Fig. 2 that, for low N_2 values, the Bononi term curve approximates the captured photons term curve at shorter fiber lengths; i.e., in Fig. 2(a)—for $L = 50\,\text{m}$—the distance between the curves for both terms is larger than in Fig. 2(c)—for $L = 10\,\text{m}$.] However, at doped fiber lengths commonly used in EDFAs ($L \sim <10\,\text{m}$), it can be shown that, although the Bononi term contribution to τ_{eff} goes to zero when N_2 goes to 0, the captured photons term still has an influence on the measure of the erbium ions' lifetime. This last observation can be easily derived from the fact that the term multiplying N_2 in the right-hand side of Eq. (8) is

$$-\left(\frac{1}{\tau_0} - \frac{1}{\rho S}\sum_{n}^{M} 2\gamma_n \Delta\upsilon\right)$$

and not just $-1/\tau_0$, independently of the L or N_2 values. Then the effective measured lifetime τ_m is

$$\tau_m = \frac{\tau_0}{1 - \dfrac{\tau_0}{\rho S}\displaystyle\sum_{n}^{M} 2\gamma_n \Delta\upsilon}. \tag{9}$$

It can be seen that the difference between the actual and the measured lifetimes is increased if the doped region area S is decreased. This was expected because, from Eq. (8), the captured photons term increases if S decreases while the spontaneous decay term remains unaltered. [The reason we chose a very small doped region radius (1 μm) in our simulations was that we were interested in the study of a situation in which the captured photons term could have some importance.] Using Eq. (9) and the same parameters as for all the preceding presented simulations, we calculated τ_m for three doped region radii: 5, 1, and 0.3 μm. The calculated τ_m in each case were 10.01, 10.41, and 17.76 ms. Interestingly, measuring τ_0 in very small doped region areas in one-dimensional waveguides with large L can induce nonnegligible systematic error and even, for very small doped region areas, a large error. Observe that decreasing the doped region radius, while keeping unaltered the erbium ions' concentration ρ, the gain and absorption constants (γ_n and α_n), and, consequently, the overlap factor between the doped region area and the optical mode,[1] is the same as reducing the optical mode effective area, i.e., concentrating the photons near the longitudinal axis of the fiber. As a typical value for the overlap factor is 0.75, a doped region area radius of, say, 1 μm, implies that 75% of the photons are confined in a region with 1-μm radius. A natural question here is if there exists or can exist some device with such small optical mode effective areas. How small can the diameter for the effective area of an optical mode be? Can it be smaller than the wavelength of the guided light? In principle, there is not any physical limitation for the future construction of waveguides with arbitrary small optical mode areas at any wavelength. The most promising candidates for constructing these devices are small-mode-area photonic crystal fibers, commonly used to obtain large nonlinear coefficients. However, it has been shown in a recent publication[11] that the minimum obtainable optical mode diameter for operation at 1.55 μm in these photonic crystal fibers is 1.07 μm for silica–air fibers. Anyway, a doped region of 1-μm radius, as in the simulations here presented, is a realistic value for the doped region area, and we finish this subsection stressing that in this case the measured lifetime τ_m would be 10.41 ms, for a real value of 10 ms.

3. INCLUSION OF THE BACKGROUND LOSS

A. Derivation

The analytical solution for the system of coupled equations (1) and (2) when α_{loss} is not zero is derived in this subsection. Equation (3) must be substituted by

$$\frac{\partial N_2(z,\,t)}{\partial t} = -\frac{N_2(z,\,t)}{\tau_0} - \frac{1}{\rho S}\sum_{n=1}^{M}\left\{u_n\left[\frac{\partial P_n(z,\,t)}{\partial z}\right] - 2\gamma_n \Delta\nu N_2(z,\,t) + \alpha_{\text{loss}} P_n(z,\,t)\right\}. \quad (10)$$

Equation (4) remains the solution for Eq. (1) with the difference that the gain, instead of Eq. (5), is now given by

$$G_n(t) = \exp\{[(\gamma_n + \alpha_n)N_2(t) - \alpha_n - \alpha_{\text{loss}}]L\}. \quad (11)$$

The n^{sp} factor remains given by Eq. (6), with the substitution of $\alpha_n \rightarrow \alpha_n + \alpha_{\text{loss}}$ in the last term of the denominator. Equation (7) becomes

$$\frac{\mathrm{d}N_2(t)}{\mathrm{d}t} = (\ldots) - \frac{\alpha_{\text{loss}}}{\rho SL}\sum_{n}^{M}\int_0^L P_n(z)\mathrm{d}z, \quad (12)$$

where (...) is the right-hand side of Eq. (8) [with the gain given by Eq. (11)]. By use of Eq. (4) for $P_n(z)$, the integral in Eq. (12) has an analytical solution:

$$\frac{\mathrm{d}N_2(t)}{\mathrm{d}t} = (\ldots) - \frac{\alpha_{\text{loss}}}{\rho SL}\sum_{n}^{M}\left(\frac{P_n{}^{\text{in}}(t)}{\ln[G_n(t)]}[G_n(t) - 1]L + 2n_n{}^{\text{sp}}\Delta\nu L\left\{\frac{G_n(t) - 1}{\ln[G_n(t)]} - 1\right\}\right). \quad (13)$$

B. Discussion

1. *Physical Interpretation*

Two new terms appear in the rate equation analytical solution when the background loss of the doped fiber is considered. The first term represents the number of signal and pump photon loss per unit time because of the attenuation; we call it the signal and pump loss term. The second represents the number of ASE photon loss per unit time because of the same cause; we call it the ASE loss term. The contribution of these terms to $\mathrm{d}N_2/\mathrm{d}t$ is always negative because they subtract from the input power those photons that, although having entered the EDF, have been absorbed without exciting an erbium ion. If these terms were not considered, a wrong number of input power photons that are not measured at the output would be considered photons absorbed by an erbium ion populating N_2. This interpretation is better understood by observing that Eqs. (8) and (13), in the zero-loss and lossy cases, respectively, represent the conservation of photons of the system (see Ref. 4, Subsection III.A).

2. *Relative Contribution of Each Term*

We performed the same set of simulations as in the previous case but just for input powers of 100 mW because it was expected that for such high power the impact of the background loss contribution to $\mathrm{d}N_2/\mathrm{d}t$ would be maximized [as can be seen in Eq. (12)]. We choose as a typical value $\alpha_{\text{loss}} = 0.007\,\text{m}^{-1}$ (or 0.03 dB/m), according to Ref. 1. The amplifiers' gains for 0.75 inversion were very weakly deteriorated by the inclusion of the background loss coefficient and remain 9.6, 19.2, and 47.9 dB for the amplifiers using EDFs of 10, 20, and 50 m, respectively. The simulation results are shown in Fig. 3. We stress again that both terms giving the background loss contribution in Eq. (13) are always negative.

The most notable result is the large contribution of the signal and pump loss term to $\mathrm{d}N_2/\mathrm{d}t$. With other powers at 1480 nm (for instance, 10 μW and 1 mW, as in Fig. 2), this term contribution would be reduced in the same proportion as the Y. Sun term (both depend linearly on P^{in}). It is then easy to have a general idea of what would happen in these cases with the signal and pump loss term by just seeing the evolution of the Y. Sun term curves for different input powers in Fig. 2. It is obvious that in many cases the contribution of the signal and pump loss term cannot be neglected. Moreover, in the cases shown in Fig. 3, for $P^{\text{in}} = 100\,\text{mW}$, this term has a stronger influence than the spontaneous decay, the Bononi, and the captured photons terms in all the situations. In contrast to this unexpected result, some expected results observed in Fig. 3 are as follows:

i. Near the minimum absolute value for the Y. Sun term, the contribution of the background loss term to $\mathrm{d}N_2/\mathrm{d}t$ is the largest among all terms. The strong influence of the background loss of the fiber on the EDFA gain dynamic near the region where the gain is unitary was expected, since it was reported in Ref. 12. [Observe that the minimum of the Y. Sun term absolute value occurs when its value is zero or when $P^{\text{in}} \approx P^{\text{out}}$, and then the gain is almost unitary in this case; it is not unitary because in P^{out} are counted the Bononi term photons, not just $P^{\text{in}}G$.]

ii. The ASE loss term follows the general shape of the Bononi term curve. This was expected, since this term represents the small proportion of ASE photons being lost per unit time, which increases with increasing ASE powers.

iii. The value for N_2 in which $\mathrm{d}N_2/\mathrm{d}t$ is zero (pointed to with a vertical arrow in each graph) is reduced by a small factor in relation to the case in which $\alpha_{\text{loss}} = 0$. In Fig. 3, these values are 0.72, 0.73, and 0.73 for $L = 50$, 20, and 10 m, respectively, whereas in Fig. 2 these values are 0.73, 0.75, and 0.75, respectively.

3. *Background Loss Influence in Typical Erbium-Doped Fiber Amplifiers Operating in Wavelength-Division-Multiplexing Conditions*

To illustrate the large contribution of the background loss terms to $\mathrm{d}N_2/\mathrm{d}t$ and, more generally, to EDFA gain behavior, we simulate a situation of practical interest: a 20-m-fiber EDFA transient response to a drop operation in WDM conditions. Again, we used internal and intrinsic parameters as shown in Fig. 1. We also used a pump power of 100 mW at 1480 nm, a fixed input signal power of 600 μW at 1539.6 nm, and a dropped input signal power of 1.8 mW at 1549.2 nm, this last representing a drop operation of three 600-μW input power channels. We show in Fig. 4 three gain excursion curves when the 1549.2-nm channel is dropped at $t = 0$: the transient response with Eqs. (11) and (13), the same transient response with these equations but when $\alpha_{\text{loss}} = 0$ [Eqs. (4) and (8)], and, third, the response when the Bononi (ASE) contribution is neglected (in this case we set arbitrarily $n_{\text{sp}} = 0$). From Fig. 4 it is clear that neglecting α_{loss} has

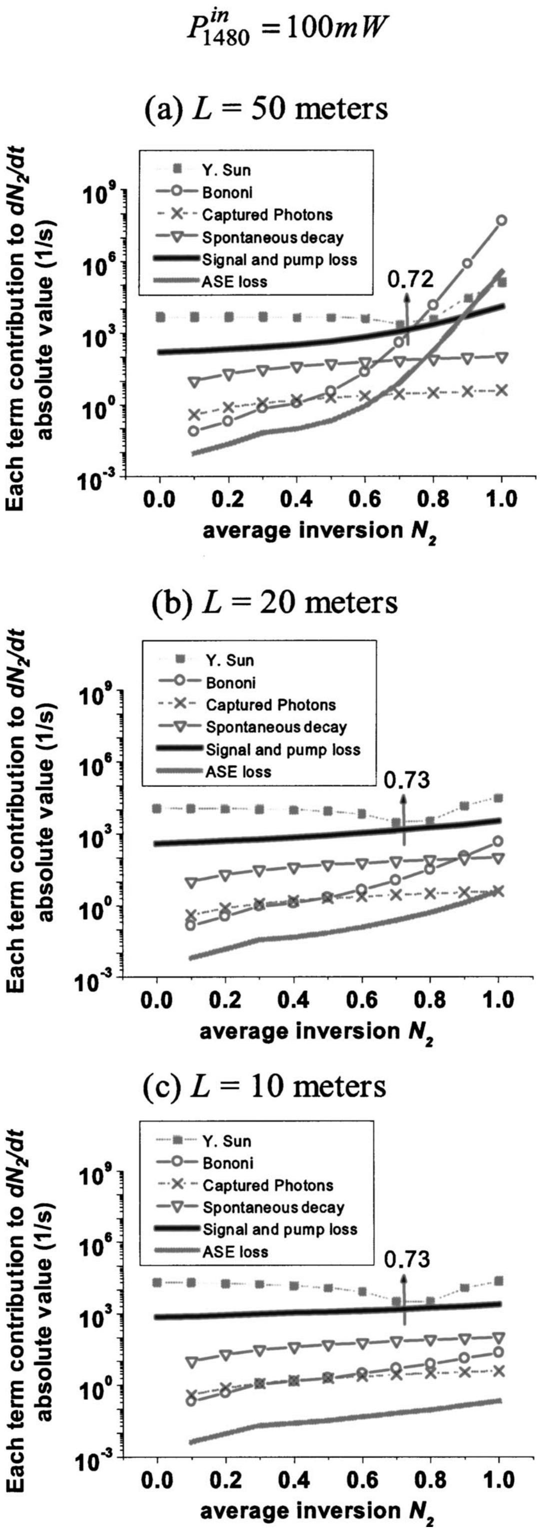


Fig. 3. Each term contribution to $\mathrm{d}N_2/\mathrm{d}t$ absolute value as a function of N_2. Simulations obtained with Eq. (13) and intrinsic EDFA parameters shown in Fig. 1. (a), (b), and (c) are for $L = 50$, 20, and 10 m, respectively, which correspond to the amplifier gain at 1530 nm and $N_2 = 0.75$ of 47.9, 19.2, and 9.59 dB, respectively. As in Fig. 2, the vertical arrows crossing each Y. Sun curve show the value of N_2 at which $\mathrm{d}N_2/\mathrm{d}t = 0$ for this particular input power at 1480 nm, i.e., the value of N_2 at which the sum of all term contributions is zero, and which corresponds to the N_2 value given by the static solution of the equations when the respective input power at 1480 nm is entering the amplifier.

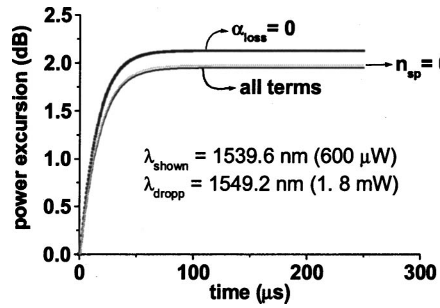


Fig. 4. WDM surviving channel's output power excursion when three of four channels are dropped at $t = 0$ in a typical EDFA employing a 20-m doped fiber. We used for the simulations a pump power of 100 mW at 1480 nm, a constant surviving channel power of 600 μW at 1539.6 nm, and a dropped channel power of 1.8 mW at 1549.2 nm. The gain at 1549.2 nm before the drop operation is ~10 dB in each one of the three curves. The lower curve is the simulation result obtained with Eqs. (11) and (13), and the other two curves are simulation results obtained when the background losses or the ASE contributions are neglected.

a larger impact on the EDFA response than neglecting the ASE influence (the case $n_{\mathrm{sp}} = 0$).

To perform the simulations shown in Fig. 4, we implement, on the basis of the code presented in Ref. 13, a MATLAB Simulink code for the resolution of Eqs. (11) and (13).[14]

4. CONCLUSIONS

We derived an analytical solution for the z dependence of the system of rate and propagation equations describing EDFA dynamic behavior, including the ASE and background loss influences. In the analytical solution for the upper-level population rate equation, we identified a new term, the captured photons term, which modifies the spontaneous decay and needs to be included when the erbium ions' lifetime in realistic EDFs is measured. When the background loss is included, we identify two new terms in the rate equation. One of these terms corresponds to the signal and pump photon loss due to the background loss, and the other corresponds to the ASE photon loss because of the same effect. We also showed that the background loss influence in typical EDFA gain behavior is larger than the ASE influence.

Even though the above equations were derived for two-level homogeneously broadened systems, they can also be applied to EDFAs pumped at 980 nm (a three-level system), as long as $N_3 \ll N_1, N_2$, where N_3 is the upper-level fractional population and $N_1 + N_2 + N_3 = 1$. This condition is guaranteed by the fast relaxation time from level three to level two (~10 μs), when compared with τ (~10 ms). Thus the two-level approximation can be used for 980-nm pumping. In fact, Eqs. (1) and (2) have been extensively used to model either 980- or 1480-nm pumped EDFAs.

Finally, we stress that the fact that the coupled rate and propagation equations, including the ASE and the background loss effects, have an analytical solution in z when the population of excited ions constant along the fiber is not obvious *a priori*, because even in this case the solution could depend, in principle, on the detailed powers distribution along z. Observe in Eqs. (2) and (3) that, even if N_2, in the right-hand side, does not depend on z, dN_2/dt and dP_n/dz depend on z, through the terms being multiplied by $P_n(z, t)$. It is not obvious, for instance, that the gain is the same for pumps copropagating or counterpropagating with the signals, a fact that arises naturally from the average inversion model. The main utility of the AIM, either in the dynamic case (here presented) or in the static case (presented in Ref. 6), is the physical insights that can be obtained from it.

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